Static Analysis
and
Dataflow Analysis

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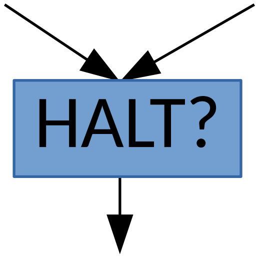
  - Does my program terminate?

Static Analysis

Brief Review of Undecidability



HALT? “Does my program terminate?”

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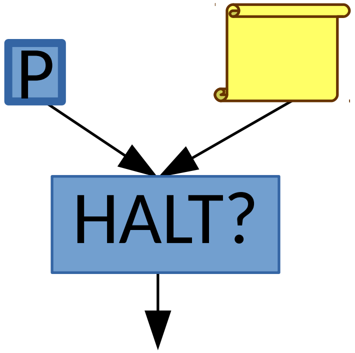

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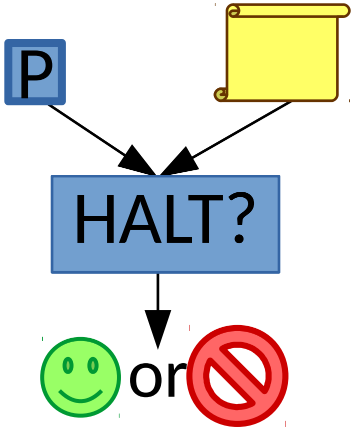

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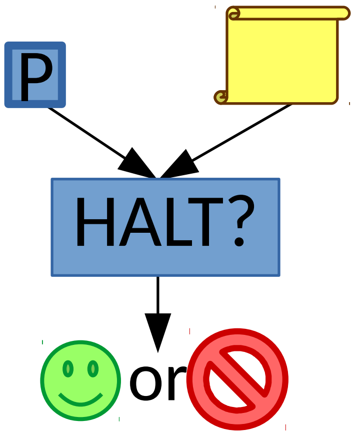

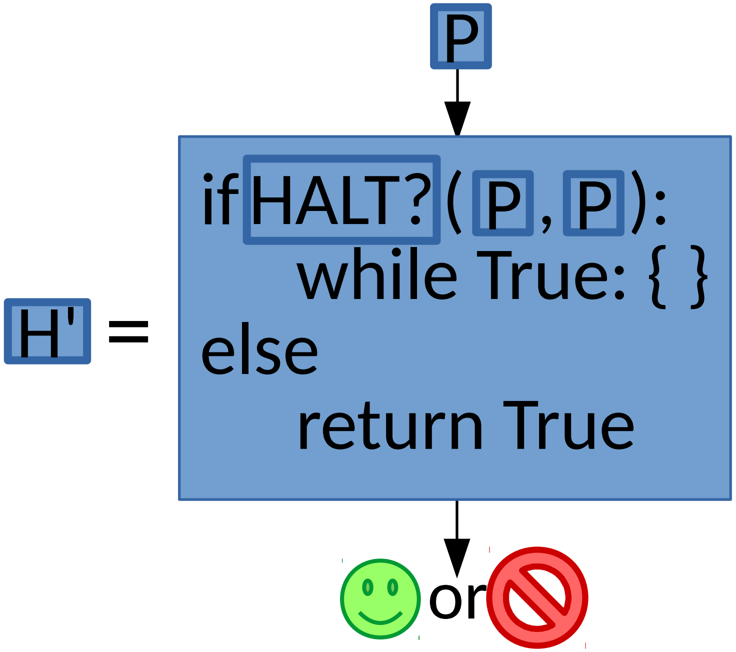

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if HALT?(P, P):
    while True: { }
else
    return True
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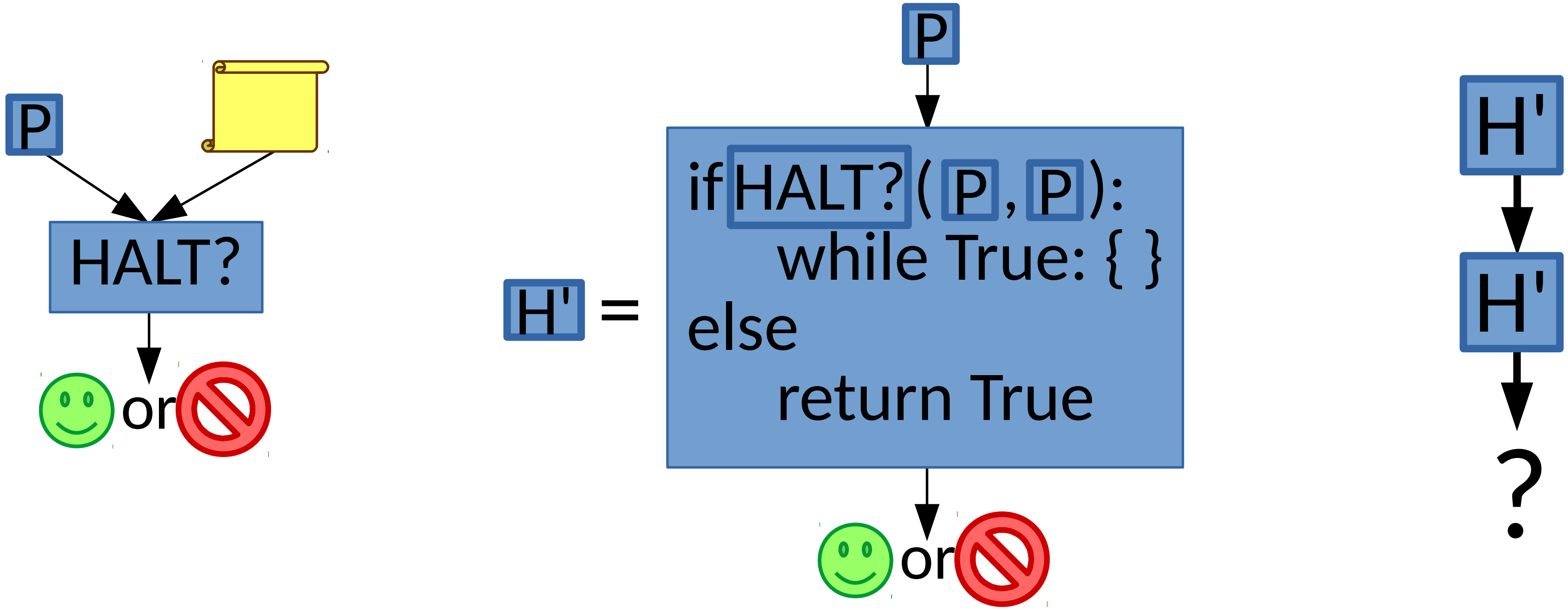

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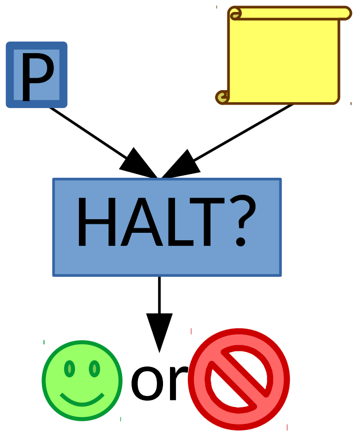

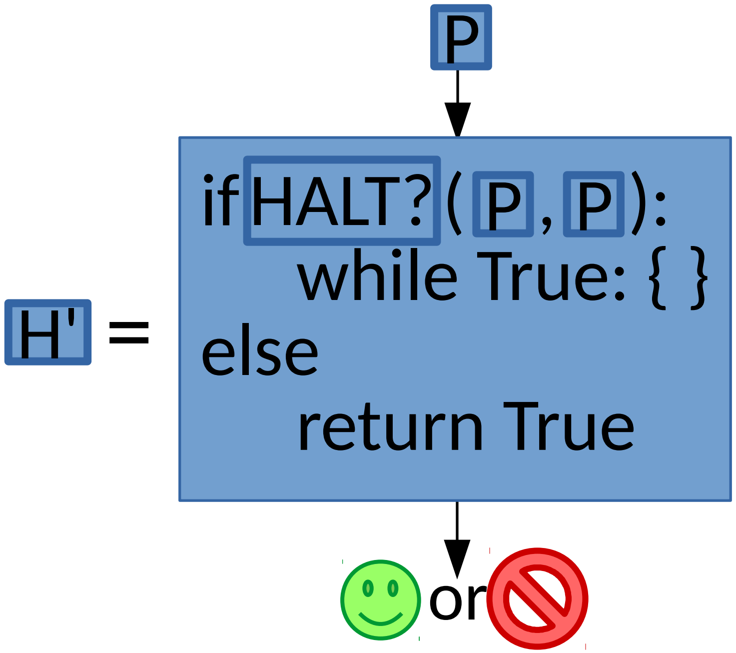

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if HALT?(P, P):
        while True: { }
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```

It's a classic paradox!

Static Analysis

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- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
  - Does my program terminate?

But wait? Isn't that impossible?

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- Only if answers must be perfect.

Static Analysis

Overapproximate or *underapproximate* the problem, and try to solve this simpler version.

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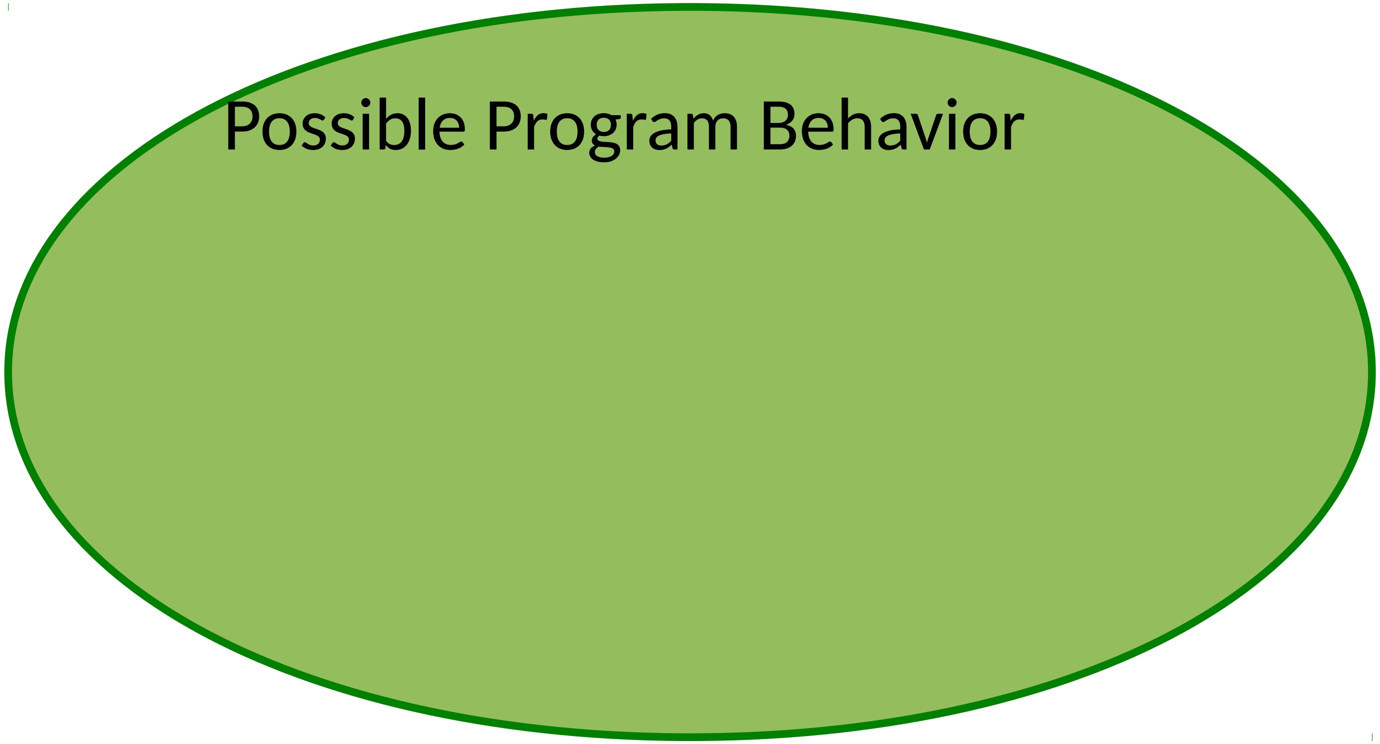
Striking the right balance is key to a useful analysis

Static Analysis

Modeled program behaviors



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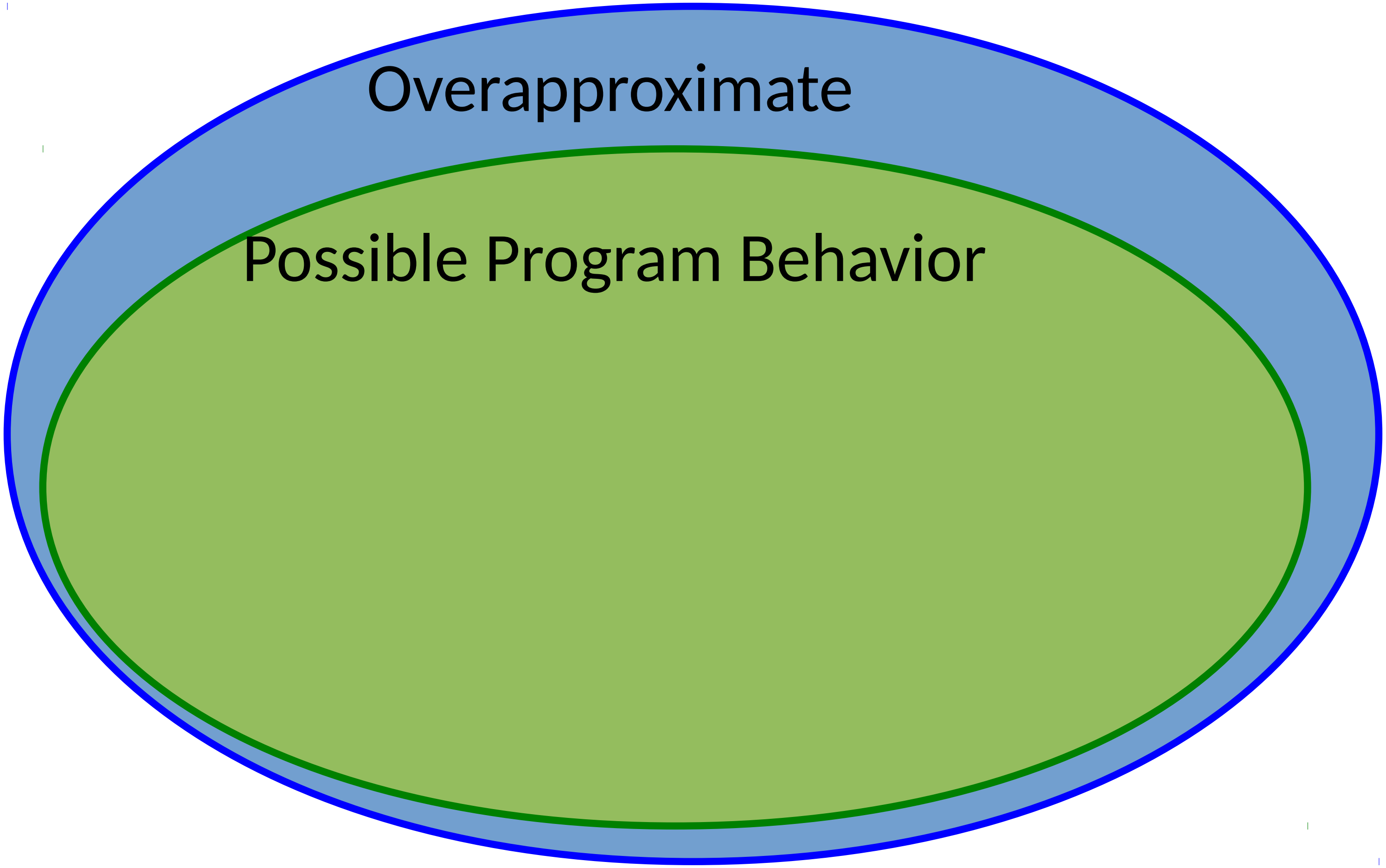
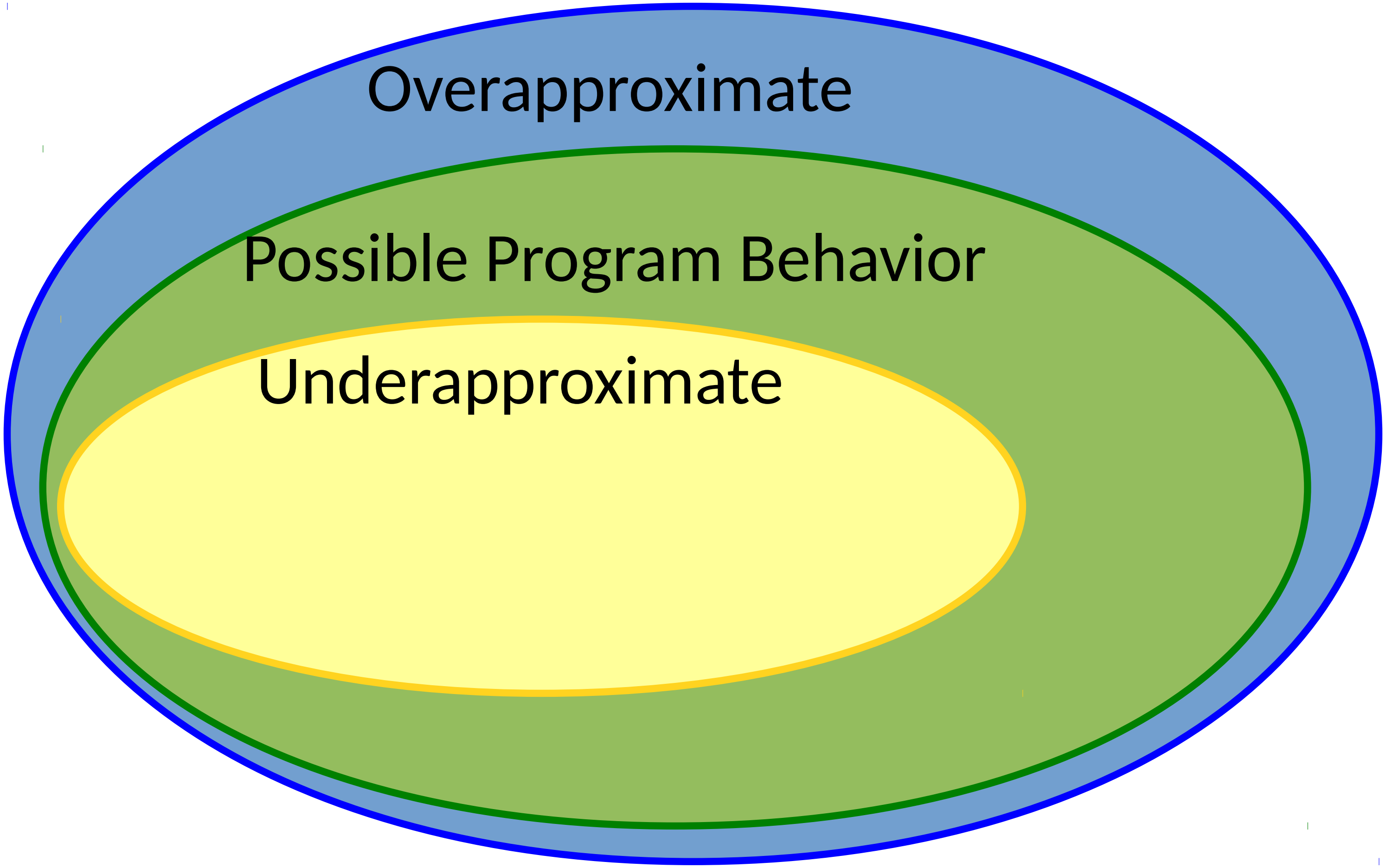


Consider some behaviors possible when they are not.

Static Analysis

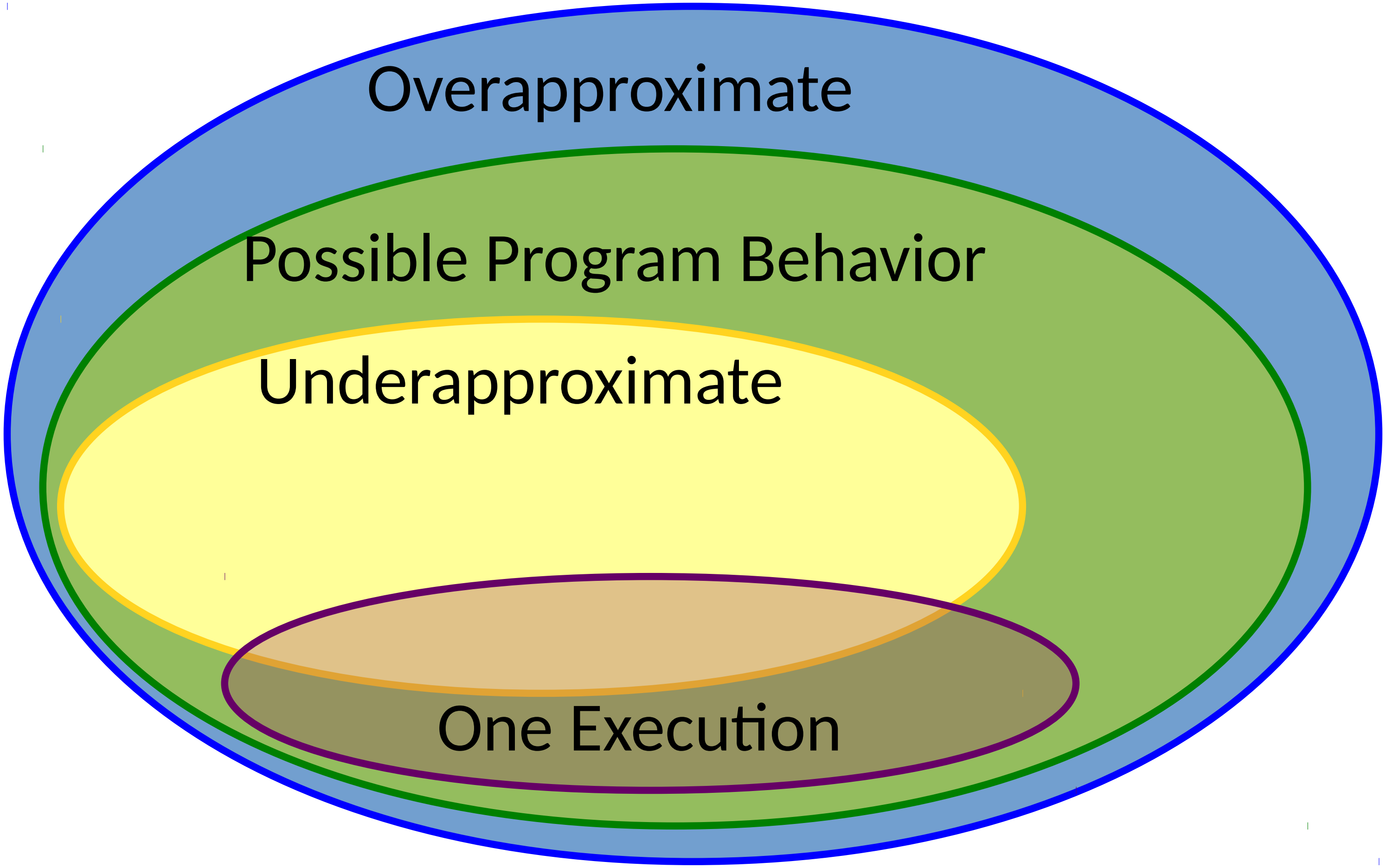
Modeled program behaviors



Ignore some behaviors that *are* possible.

Static Analysis

Modeled program behaviors



A Simple Example – Dataflow Analysis

Q: Is a particular number ever negative?
- Might be an offset into invalid memory!

Approximate the program's behavior

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Approximate the program's behavior

- Concrete domain: integers
- Abstract domain: {-,0,+} $\bigcup$ {$\top$,$\bot$}

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Q: Is a particular number ever negative?

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Approximate the program's behavior

- Concrete domain: integers
- Abstract domain: {-,0,+} ⋃ {⊤,⊥}

concrete(x) = 5 ↦ abstract(x) = +
concrete(y) = -3 ↦ abstract(y) = -
concrete(z) = 0 ↦ abstract(z) = 0

Combines sets of the concrete domain

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This type of approximation is called *abstract interpretation*.

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- Can be subtle.
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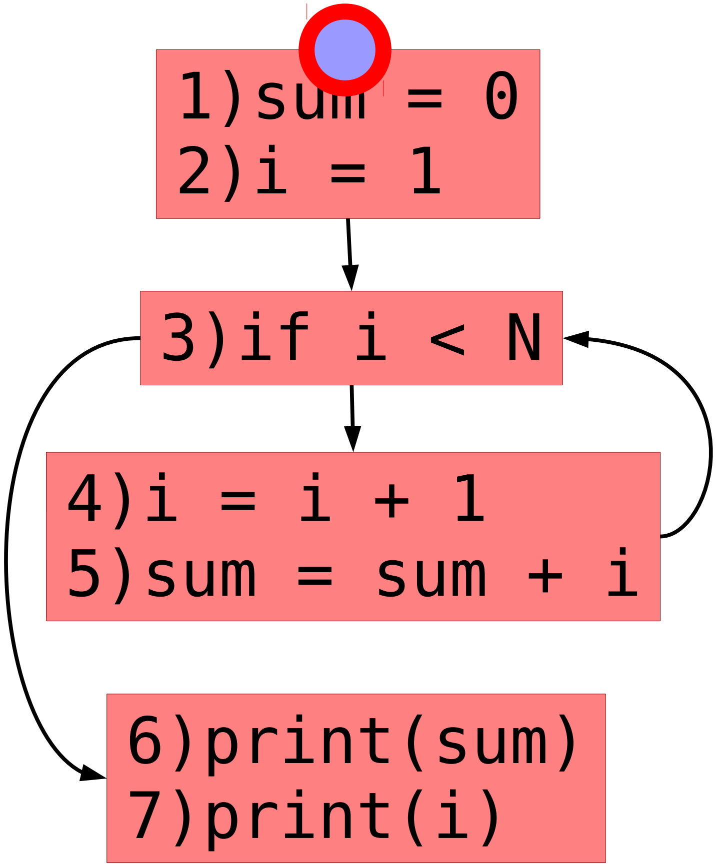
Consider a divide by 0 analysis.
What are:
    True Positives
    False Positives
    True Negatives
    False Negatives

Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.



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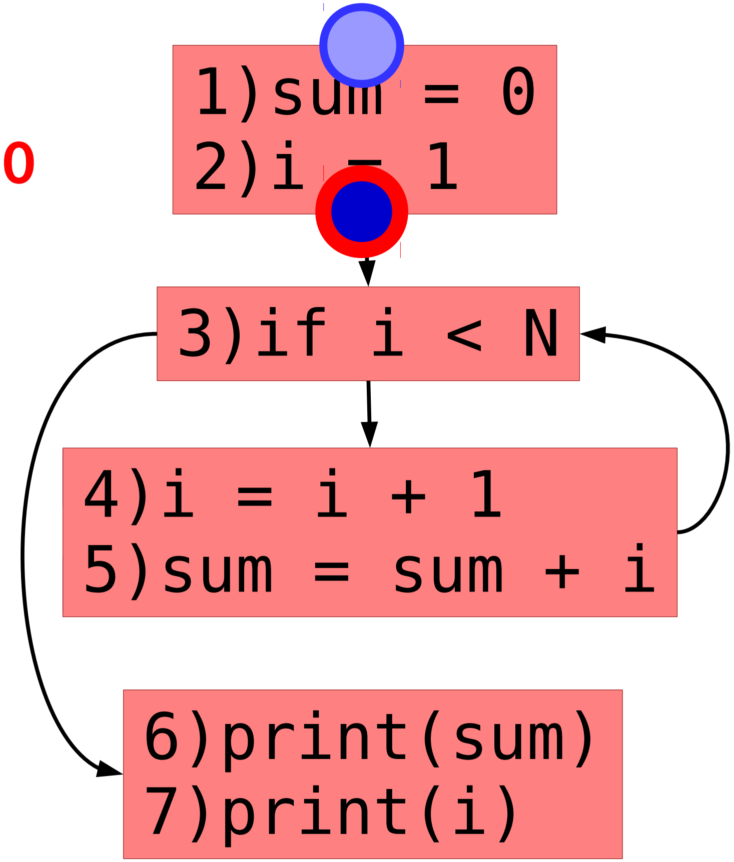

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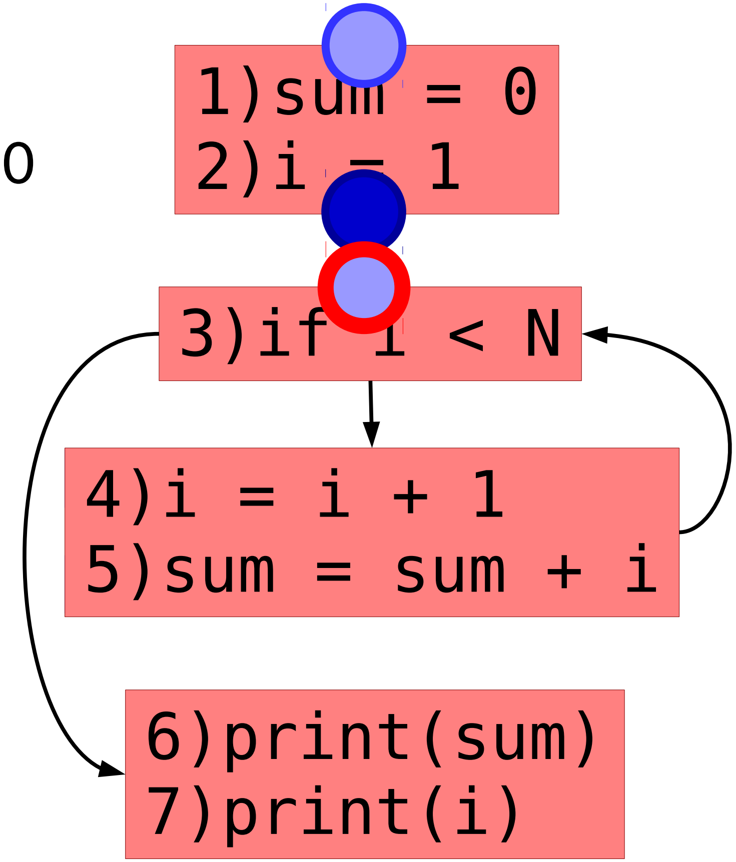

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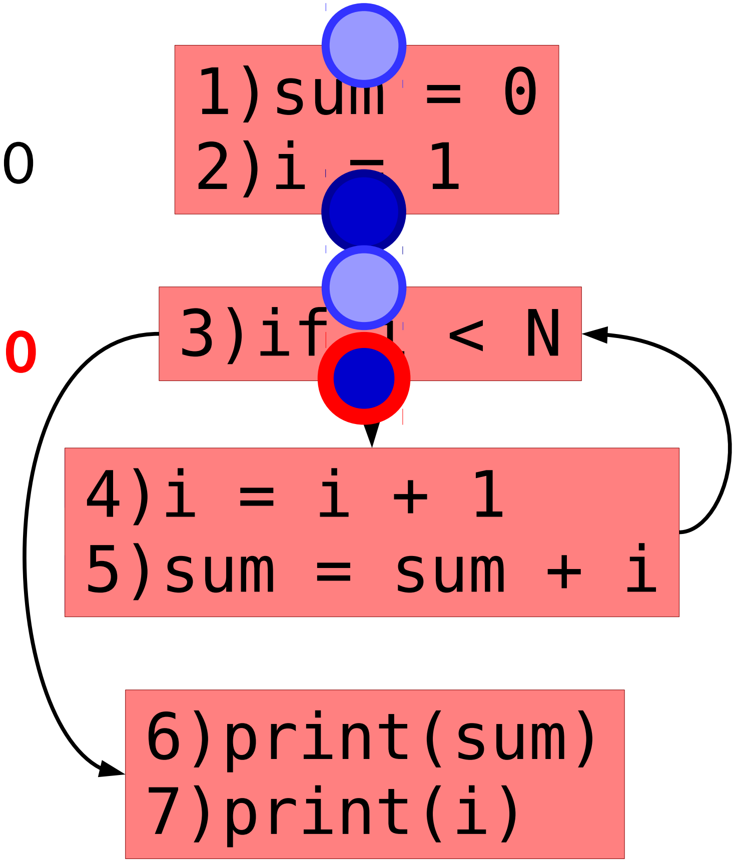

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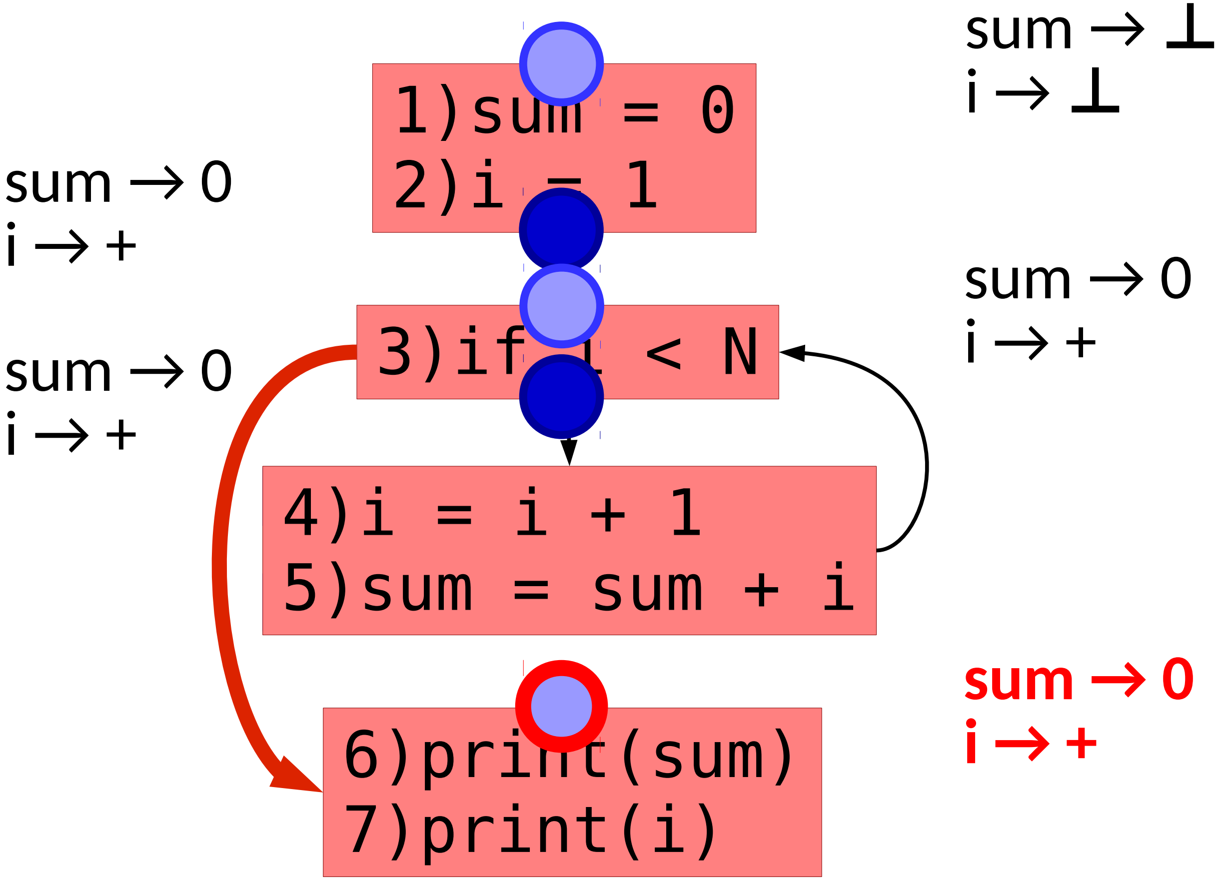

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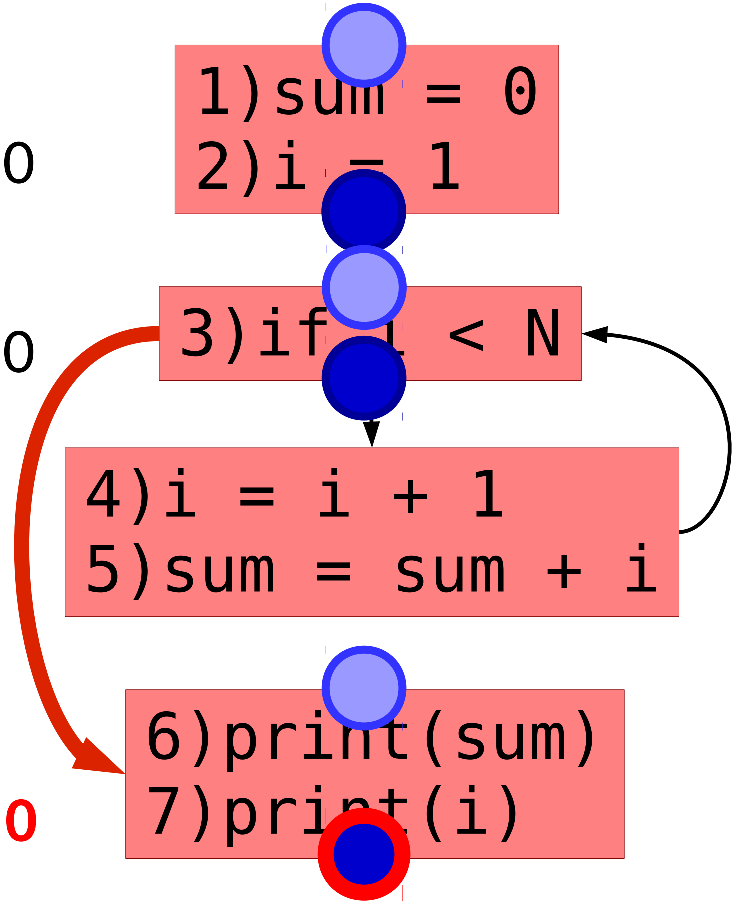

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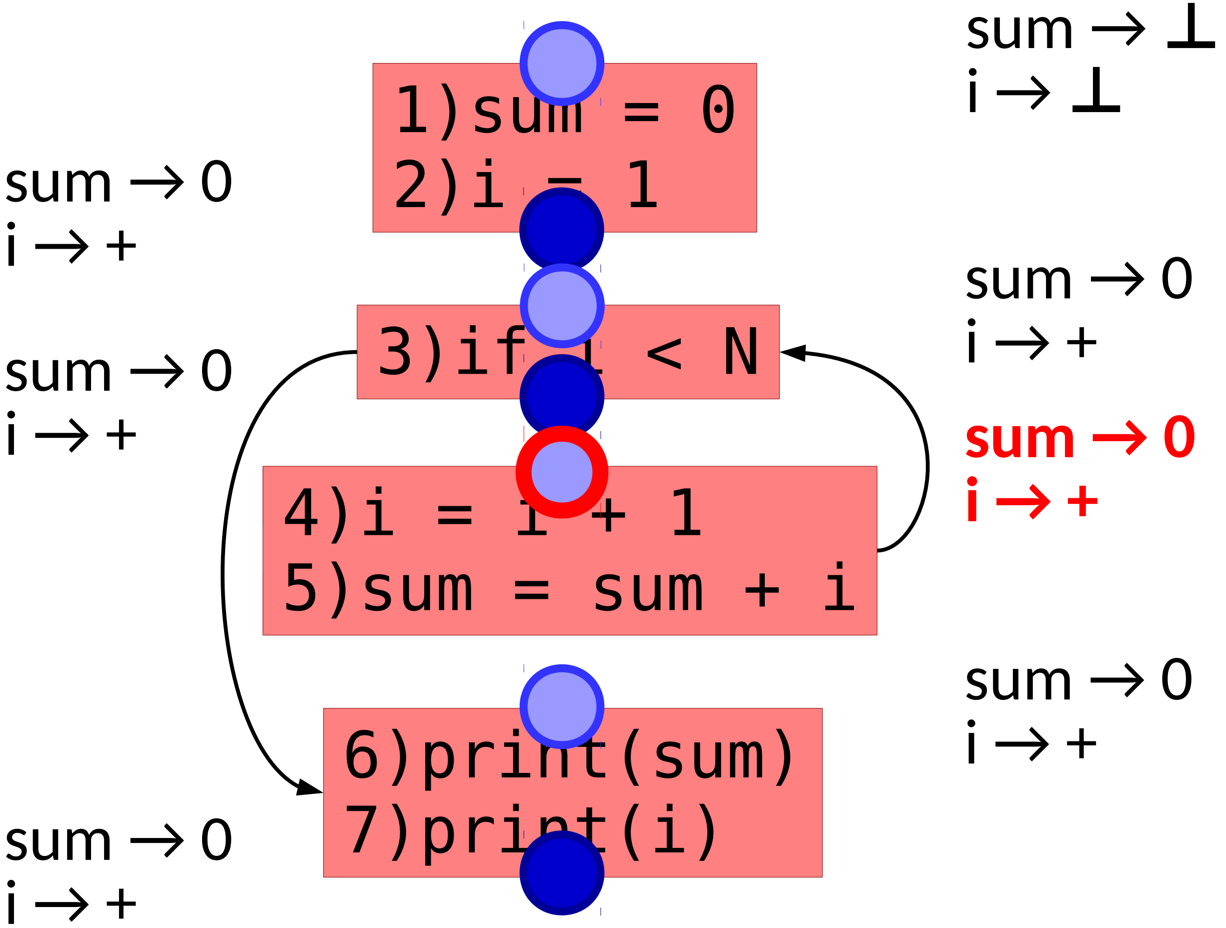

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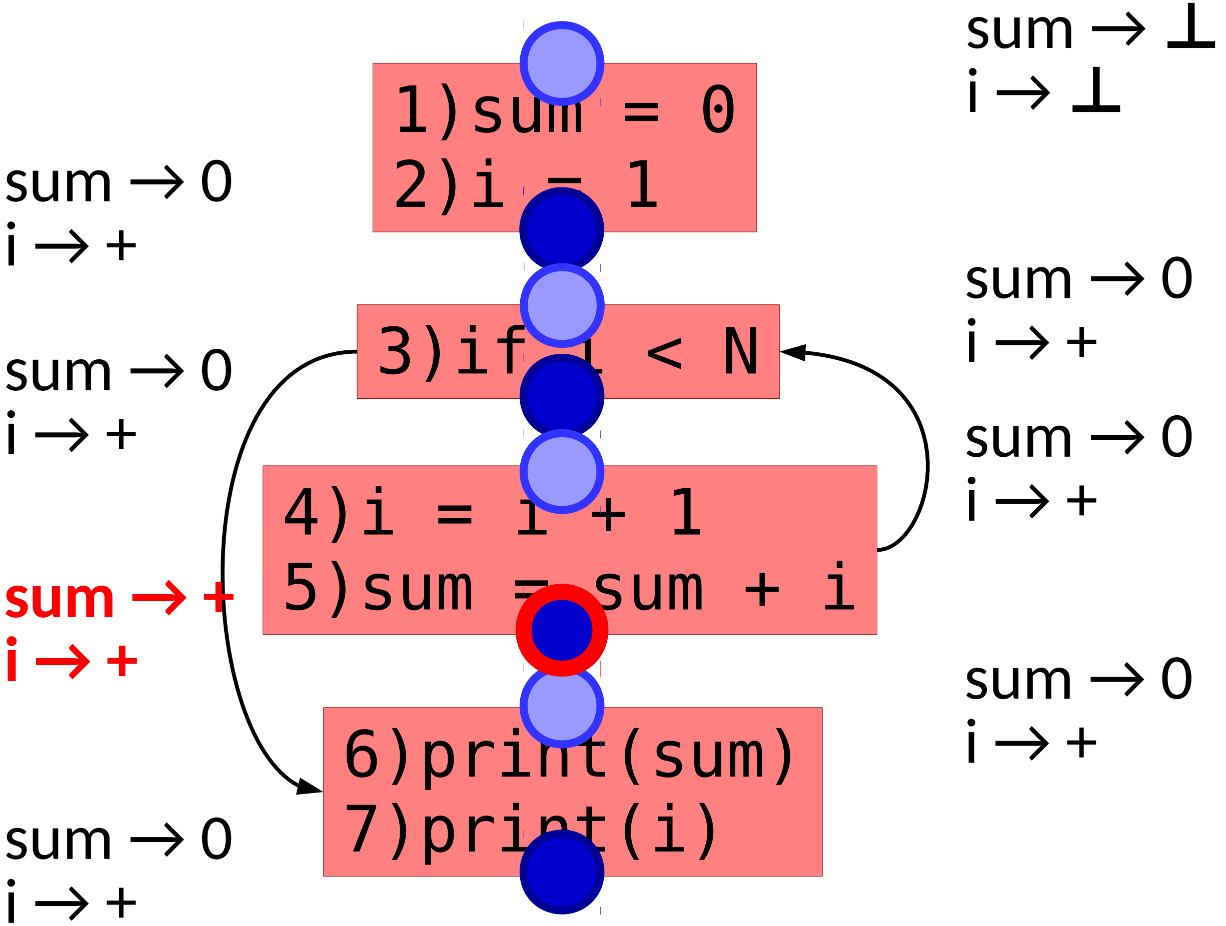

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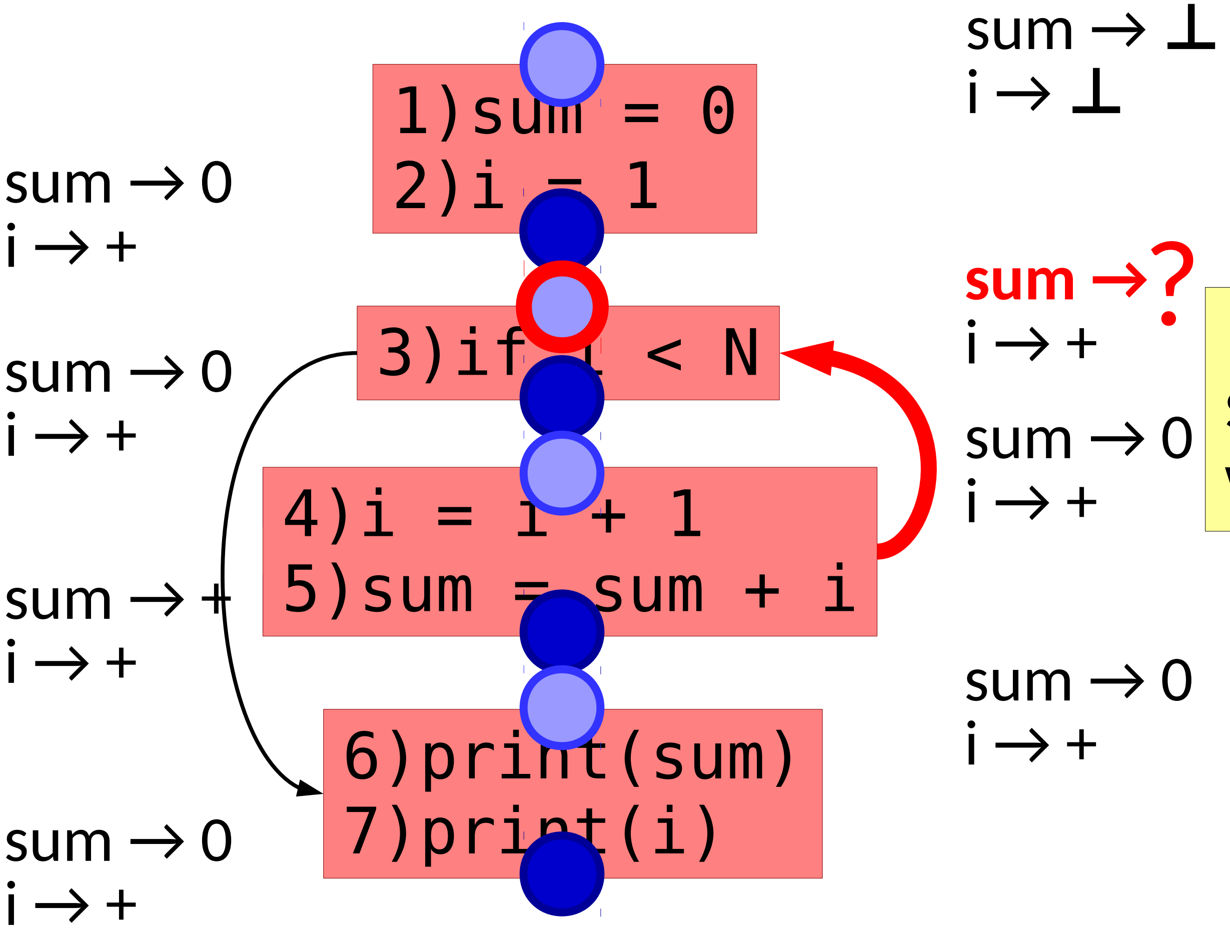

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Meet Operator
sum was 0, but
what should it be now?

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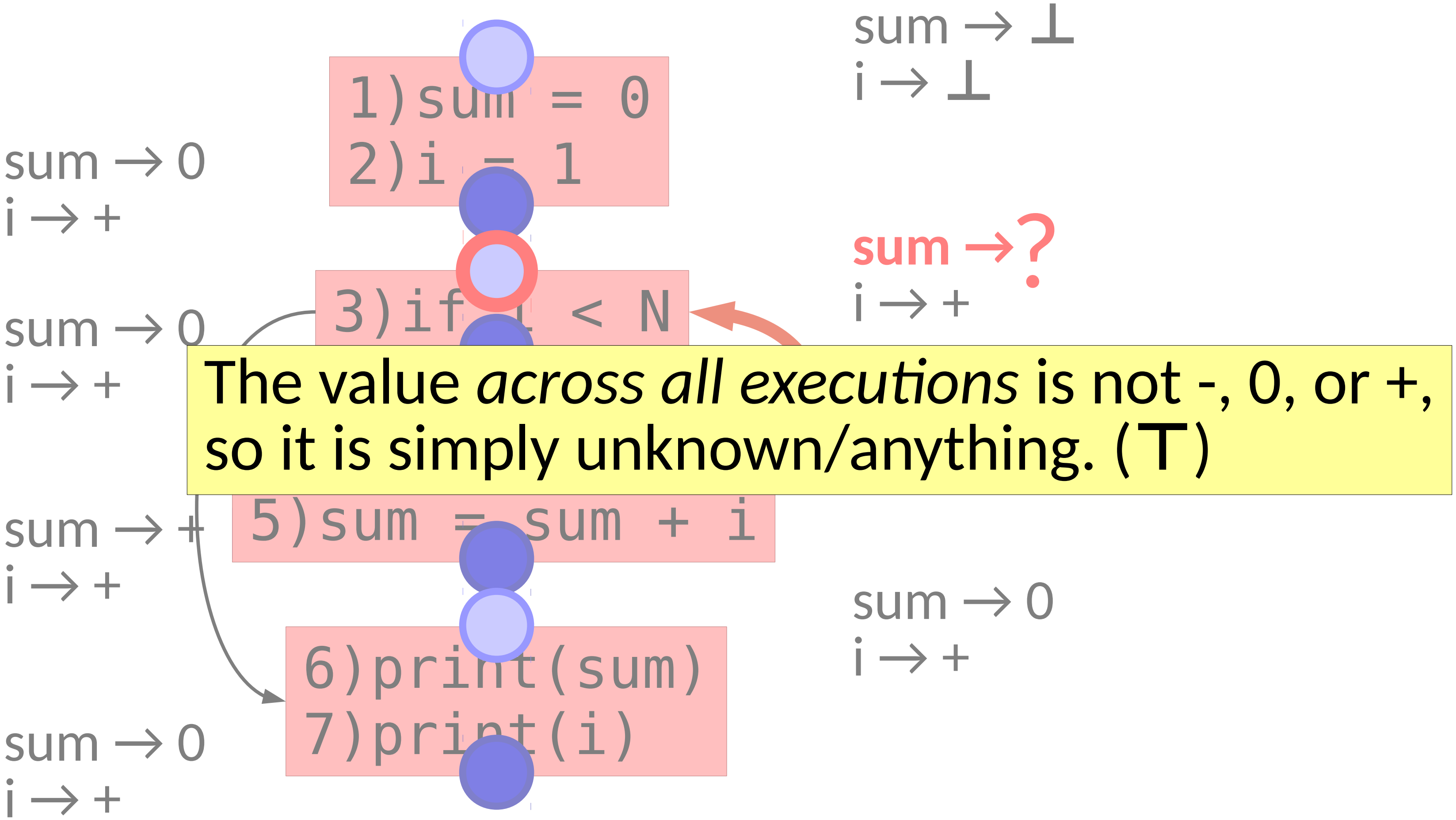

sum → ⊥
i → ⊥

1)sum = 0
2)i = 1

sum → 0
i → +

sum →?
i → +

3)if i < N

sum → 0
i → +

The value *across all executions* is not -, 0, or +, so it is simply unknown/anything. (⊤)

5)sum = sum + i

sum → +
i → +

sum → 0
i → +

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sum → 0
i → +

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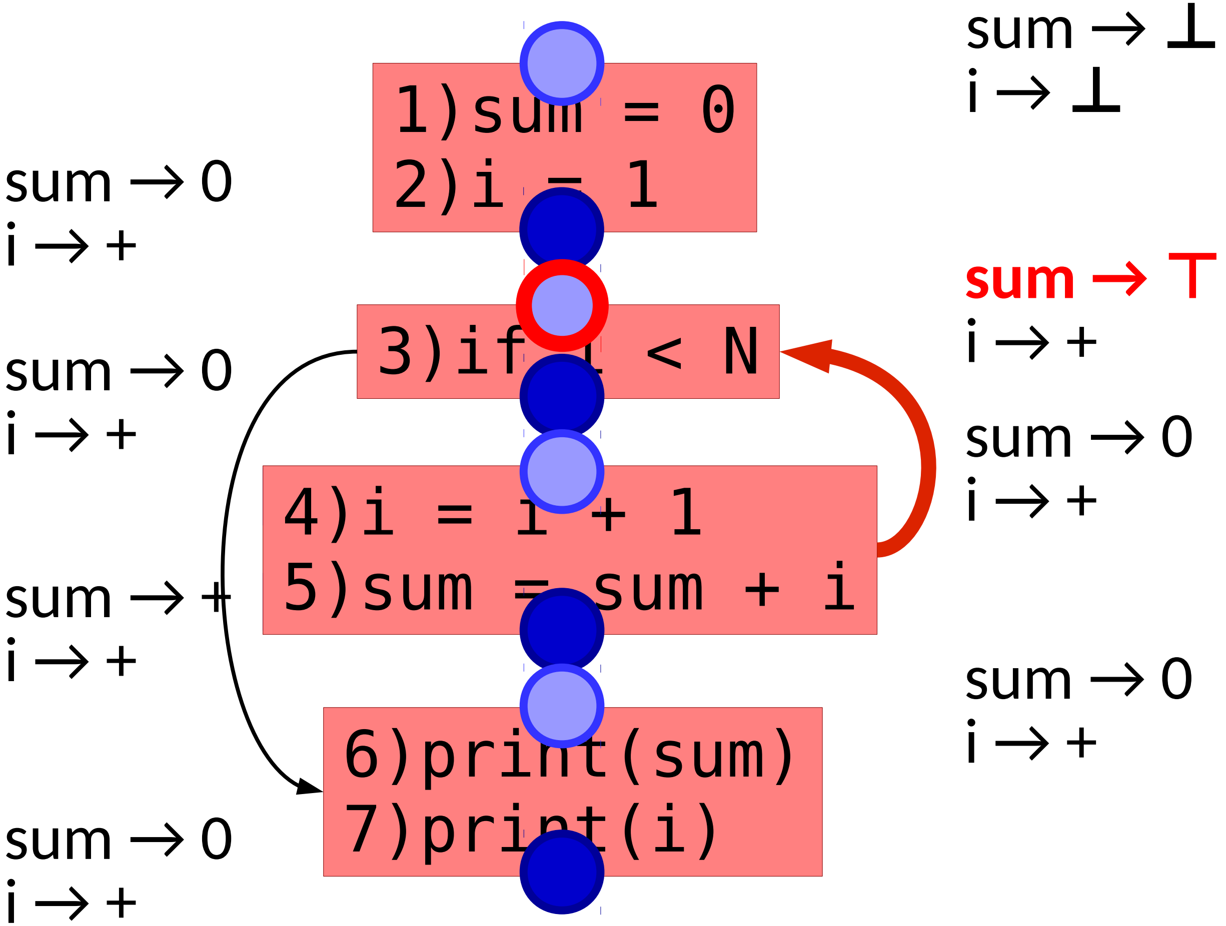

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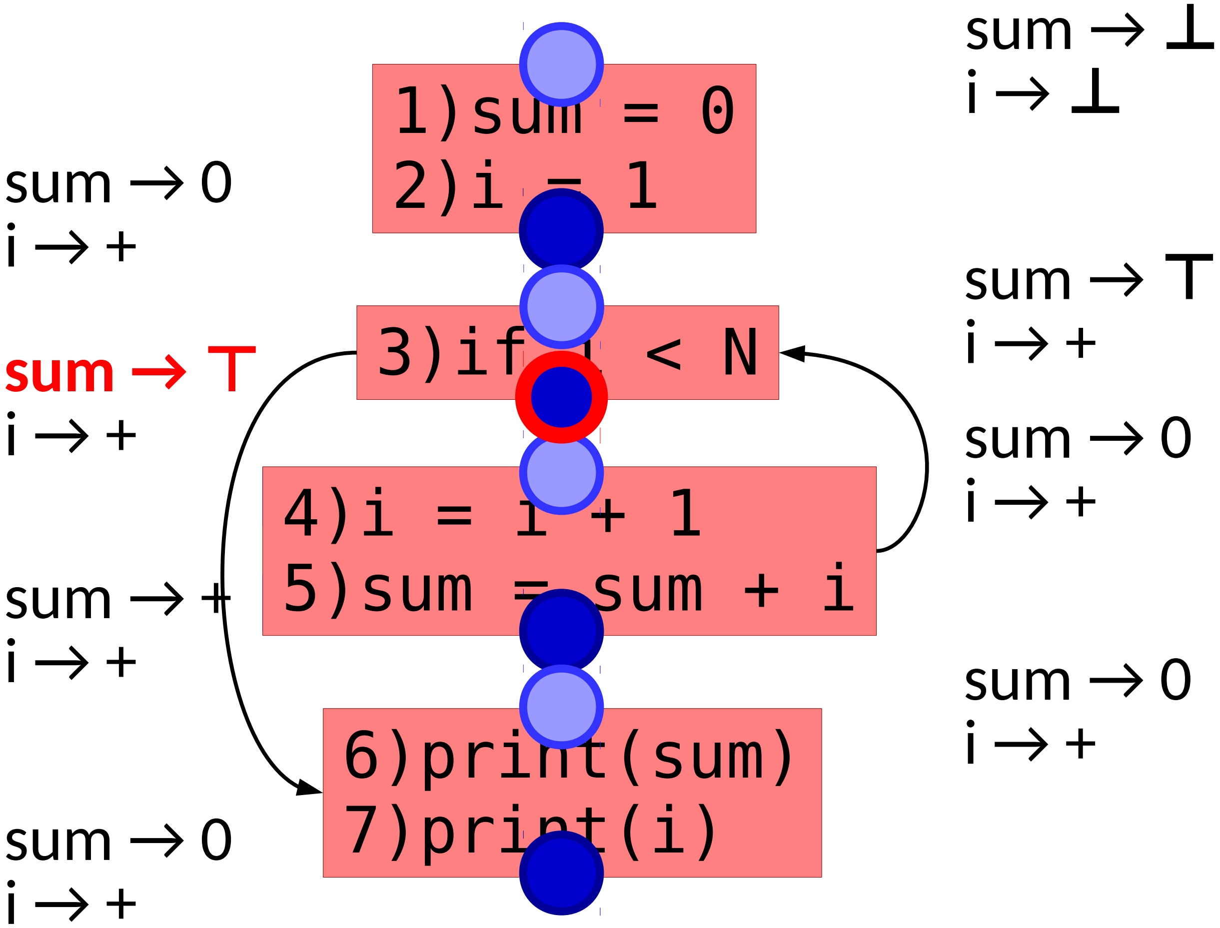

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i → ⊥

sum → 0
i → +

sum → ⊤
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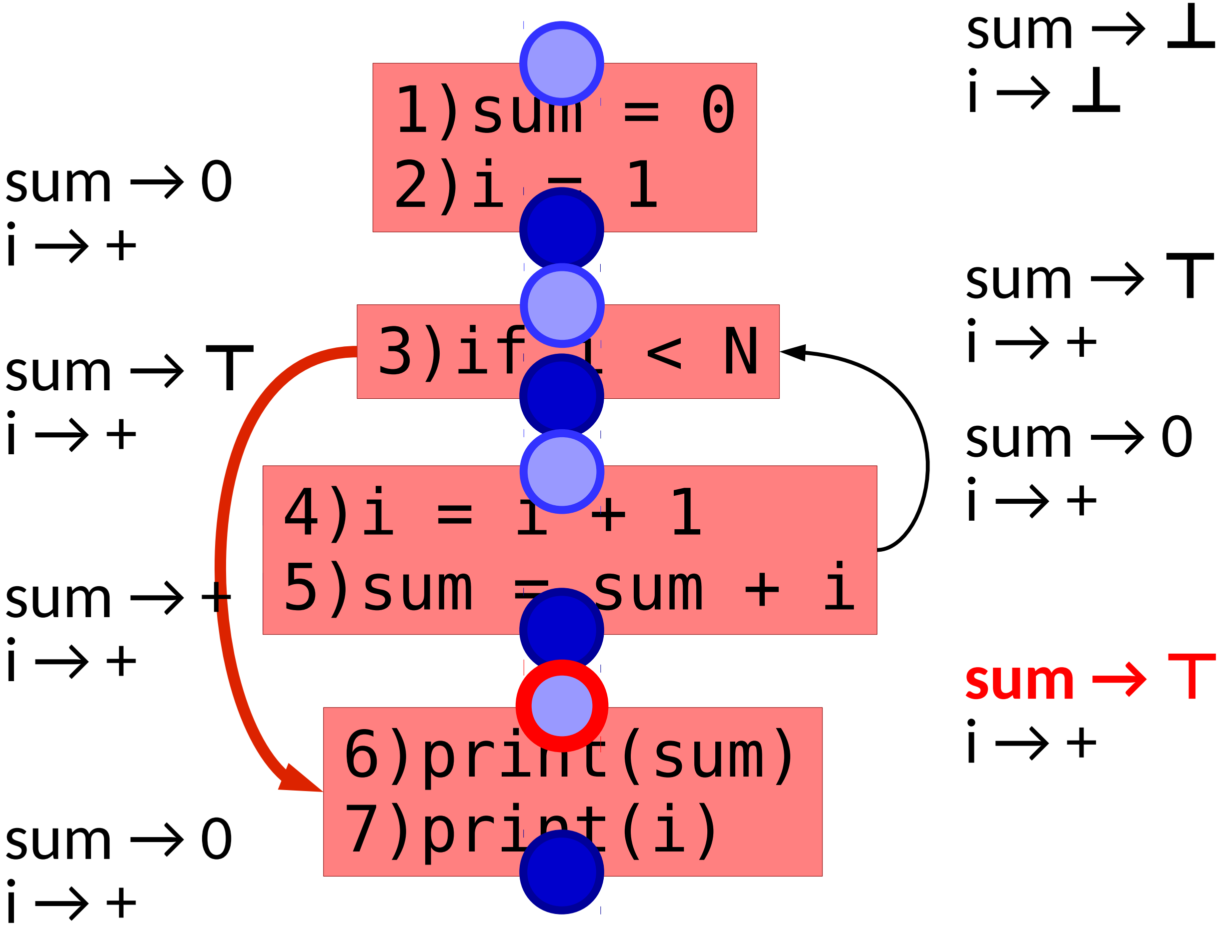

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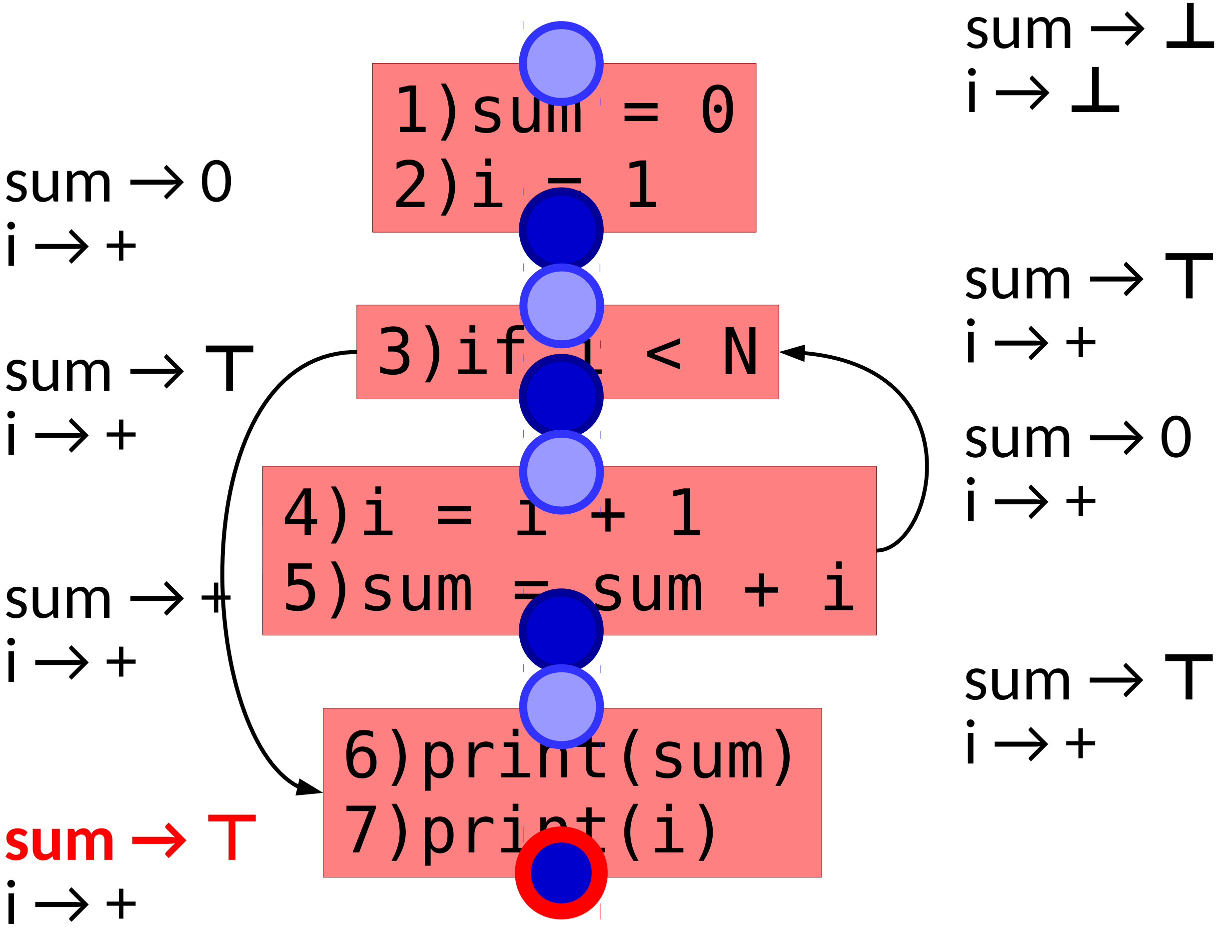

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- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point

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    (Topological Order, Strongly Connected Components)

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Will it always terminate?

Dataflow Analysis

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function

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  Loosely: <CFG, Transfer Function, Lattice Abstraction>

Dataflow Analysis

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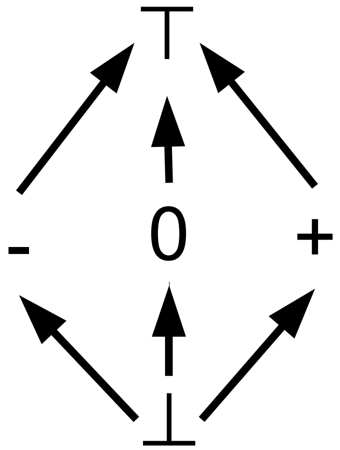
- For basic analyses, use a monotone framework
  - $\{-,0,+\} \cup \{\top,\bot\}$
  - They define a partial order
  - Abstract state can only move ***up*** lattice at a statement



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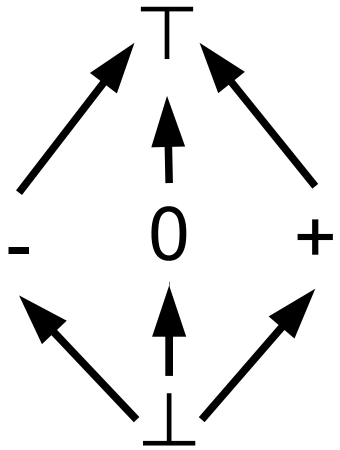

Why does this specific example terminate?

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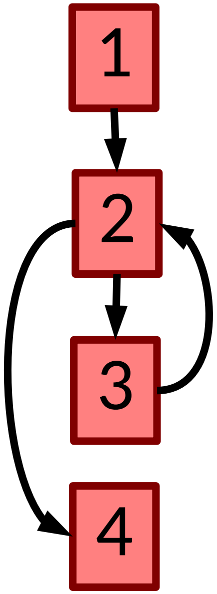
- For basic analyses, use a monotone framework
- But in theory a lattice need not be finite!
  - Widening operators can still make it feasible (e.g., heuristically raise to $\top$)

Dataflow Analysis

- Note: need to model program state before and after each statement
- Proper ordering & a work list algorithm improves the efficiency

Worklist Algorithms

```
work = nodes()
state(n) = $\bot$ $\forall$ n $\in$ nodes()
while work $\neq \emptyset$:
    unit = take(work)
    old = state(unit)
    before = $\prod$state(p)
                 $\forall$ p $\in$ preds(unit)
    new = transfer(before, unit)
    if old $\neq$ after:
        work = work $\cup$ succs(unit)
        state(unit) = new
```



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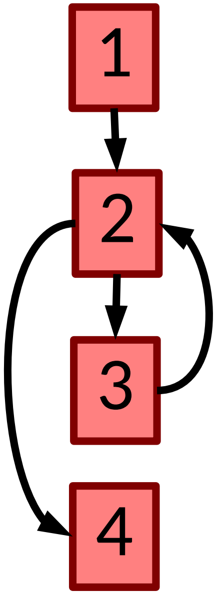
        state(unit) = new
```



work: 1 2 3 4

state: { (1 ↦ ⊥) (3 ↦ ⊥)
(2 ↦ ⊥) (4 ↦ ⊥) }

Worklist Algorithms

```
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
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    before = ∏state(p)
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    new = transfer(before, unit)
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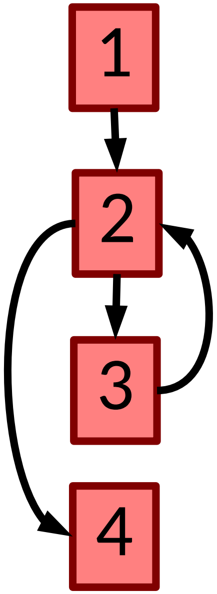
```

unit = 1



work: 2 3 4

state: $\{ (1 \mapsto \bot) \quad (3 \mapsto \bot)$
$(2 \mapsto \bot) \quad (4 \mapsto \bot) \}$

Worklist Algorithms

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```

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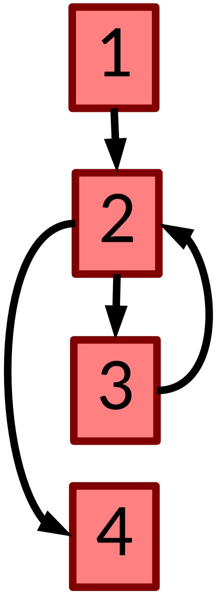
old = ⊥



work: 2 3 4

state: $\left\{ \begin{matrix} (1 \mapsto \bot) & (3 \mapsto \bot) \\ (2 \mapsto \bot) & (4 \mapsto \bot) \end{matrix} \right\}$

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    new = transfer(before, unit)
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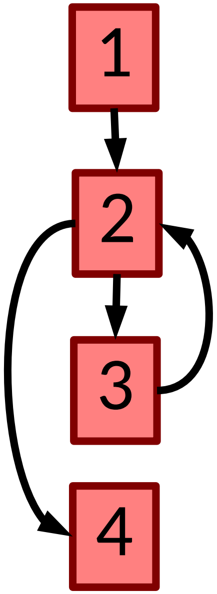
        state(unit) = new
```

unit = 1

old = ⊥

new = sum → 0
i → +



work: 2 3 4

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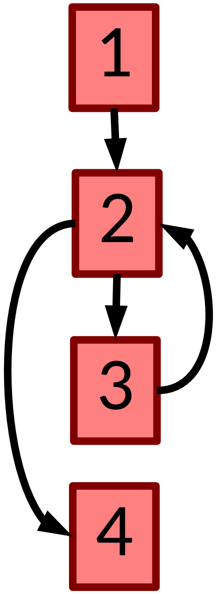

work: 2 3 4

state: {(1 ↦ sum → 0
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(2 ↦ ⊥) (4 ↦ ⊥) }

Worklist Algorithms

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    new = transfer(before, unit)
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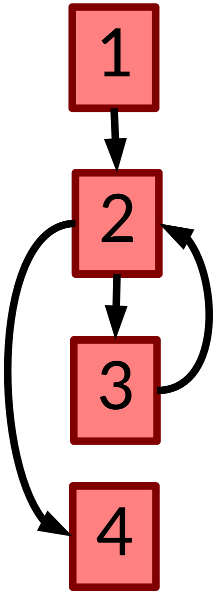
```

unit = 2

old = ⊥

new = sum → 0
i → +



work: 3 4

state: { (1 ↦ sum → 0, i → +) (3 ↦ ⊥)

(2 ↦ sum → 0, i → +) (4 ↦ ⊥) }

Worklist Algorithms

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    new = transfer(before, unit)
    if old ≠ after:
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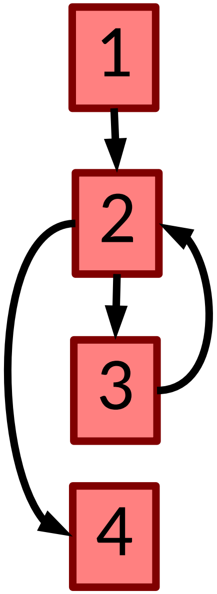
        state(unit) = new
```

unit = 3

old = ⊥

new = sum → +
i → +



work: ⬚ 4 2

2 was added back to the list

state: {(1 ↦ sum → 0, i → +) (2 ↦ sum → 0, i → +) (3 ↦ sum → +, i → +) (4 ↦ ⊥)}

Worklist Algorithms

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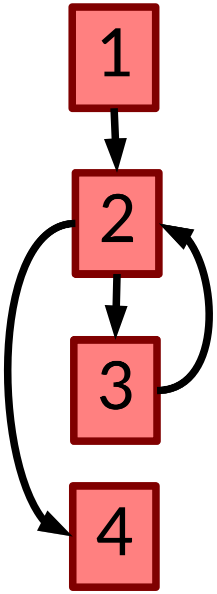
        state(unit) = new
```

unit = 4

old = ⊥

new = sum → 0
i → +



work: 2

state: {(1 ↦ sum → 0, i → +) (3 ↦ sum → +, i → +)
(2 ↦ sum → 0, i → +) (4 ↦ sum → 0, i → +)}

Worklist Algorithms

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work = nodes()
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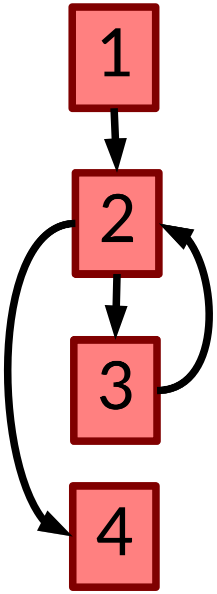
```

unit = 2

old = sum → 0
i → +

new = sum → ⊤
i → +



4,3 were added back to the list

work: 4 3

state: {
(1 ↦ sum → 0, i → +)
(2 ↦ sum → ⊤, i → +)
(3 ↦ sum → +, i → +)
(4 ↦ sum → 0, i → +)
}

Worklist Algorithms

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    if old ≠ after:
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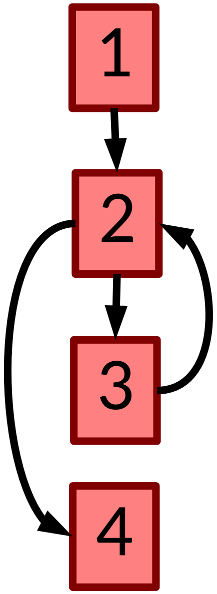
        state(unit) = new
```

unit = 4

old = sum → 0, i → +

new = sum → ⊤, i → +



work: 3

state: { (1 ↦ sum → 0, i → +) (2 ↦ sum → ⊤, i → +) (3 ↦ sum → +, i → +) (4 ↦ sum → ⊤, i → +) }

Worklist Algorithms

```
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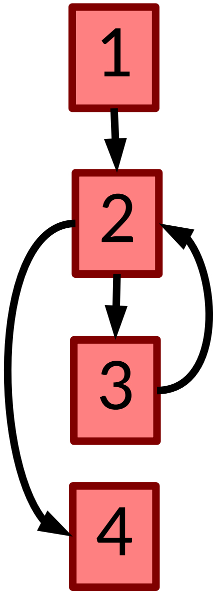
```

unit = 3

old = sum → +, i → +

new = sum → +, i → +

No change



work:

state: {(1 ↦ sum → 0, i → +), (2 ↦ sum → ⊤, i → +), (3 ↦ sum → +, i → +), (4 ↦ sum → ⊤, i → +)}

Worklist Algorithms

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work = nodes()
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    new = transfer(before,
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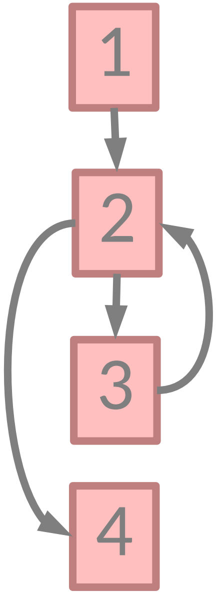
```

unit = 4

old = sum → 0
i → +

new = sum → ⊤

Done!



work:

state: {
(1 ↦ sum → 0, i → +)
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}

Effect of Approximation

- There are several possible sources of imprecision

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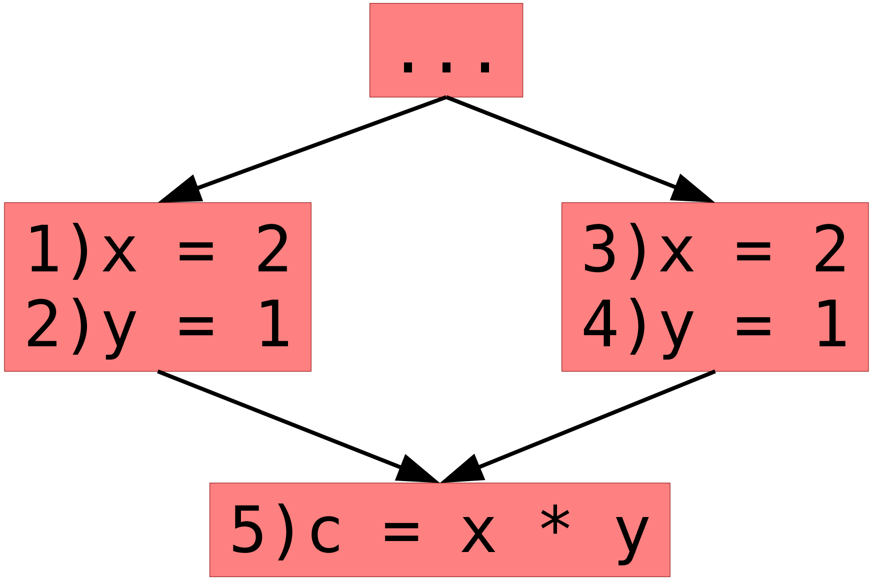

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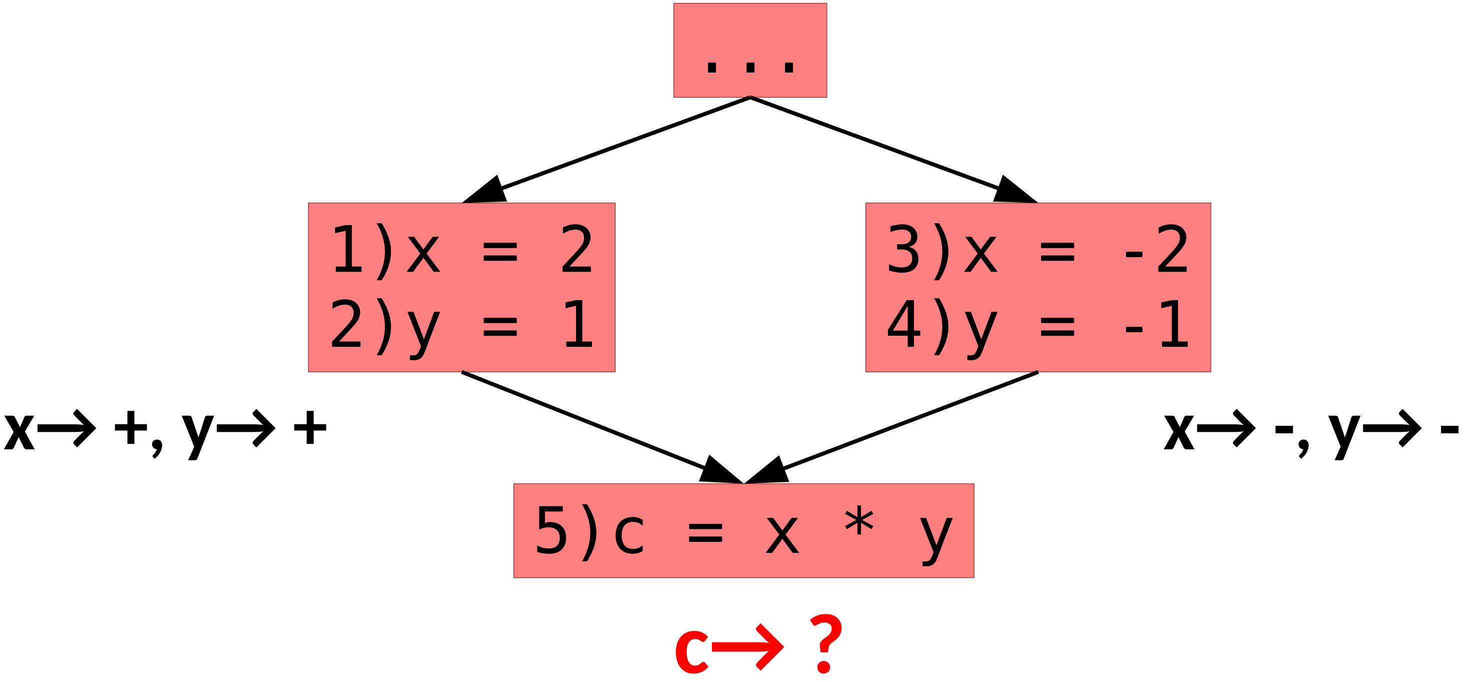

```
...
```

```
1)x = 2
2)y = 1
```

```
3)x = -2
4)y = -1
```

x→ +, y→ +

x→ -, y→ -

```
5)c = x * y
```

c→ ?

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- 2 Key sources are
  - Control flow
    - Many different paths are summarized together

Effect of Approximation

- There are several possible sources of imprecision
- 2 Key sources are
  - Control flow
    - Many different paths are summarized together
  - Abstraction
    - Deliberately throwing away information
    - Granularity of program state affects correlations across variables

Effect of Approximation

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For one path p: $f_p(\perp) = f_n(f_{n-1}(\ldots f_1(f_0(\perp))))$
For all paths p: $\prod_p f_p(\perp)$

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  - When f() is distributive, MFP=MOP

    $f(x \prod y \prod z) = f(x) \prod f(y) \prod f(z)$
  - This applies to an important class of problems called bitvector frameworks.

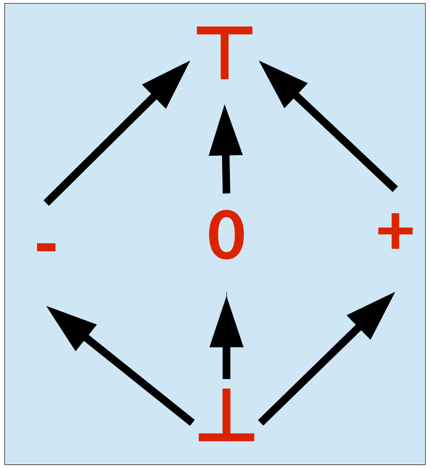
Effect of Approximation

- If approximation yields imprecise results, why do we do it?

Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- Abstract domain: e.g. {**-**,**0**,**+**} ⋃ {⊤,⊥}
- Transfer functions: **-** + **+** → ⊤
- Bounded domain lattice height:
- Concern for false + & -



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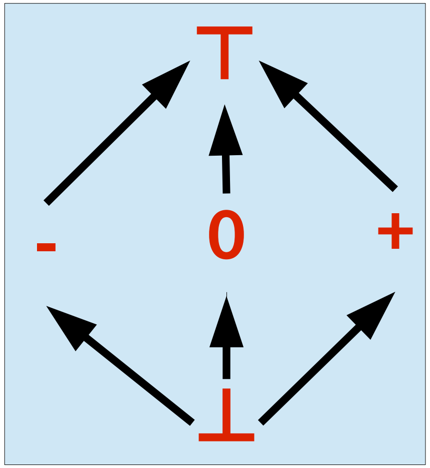

Implementation:

- Computing using work lists
- Speeding up by sorting CFG nodes

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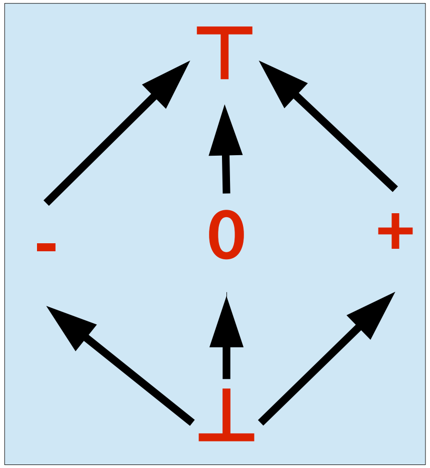

Implementation:

- Computing using work lists
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Let's see an example

File Policy Analysis

Goal: Identify potential misuses of open/closed files

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e.g. read, write, print, flush, close, …

File Policy Analysis

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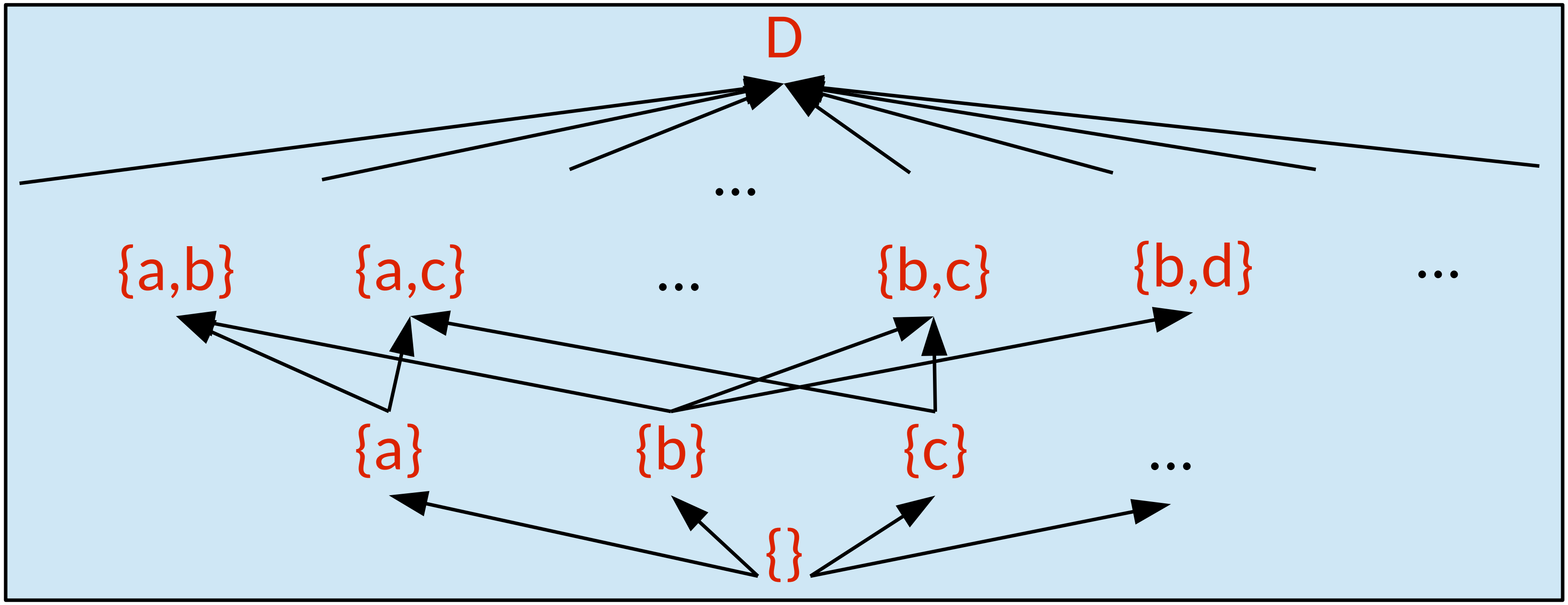
e.g. read, write, print, flush, close, …

What should our design actually be?

- Abstract domain?
- Transfer functions?
- Lattice?

Bitvector Frameworks

- When the property concerns subsets of a finite set, the abstract domain & lattice are easy:
  - Concrete: D = {a, b, c, d, … }
  - Abstract: $\wp$(D) = { {}, {a}, {b}, …, {a, b}, {a, c}, …}
  - Lattice: Defined by subset relation:



Bitvector Frameworks

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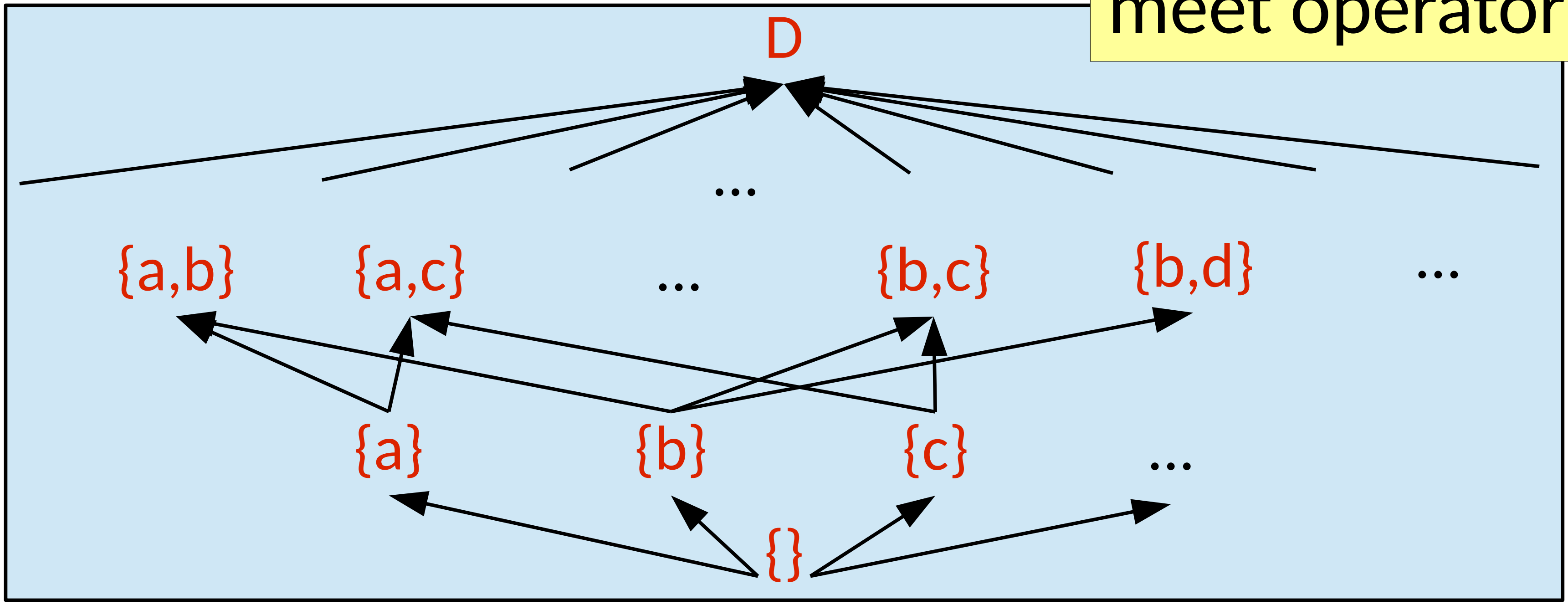
  - Lattice: Defined by subset relation:

What would the meet operator be?



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- Why is this convenient?
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  - X={a,b}, Y={c,d} → $X \sqcup Y$ = {a,b}∪{c,d} = {a,b,c,d}
  - We can implement the abstract state using efficient bitvectors!

Bitvector Frameworks

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  - X={a,b}, Y={c,d} → X⊔Y = {a,b}∪{c,d} = {a,b,c,d}
  - We can implement the abstract state using efficient bitvectors!

Let's see how we might implement the file policy framework in LLVM…

[DEMO]

Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care

Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care
- *Flow insensitive* analyses aggregate into a global state
  - Better scalability
  - Less precision
  - “Does this function modify global variable X?”

Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
  - “If bar() is called by foo(), then it is exception free.”
  - Can enable more precise *interprocedural* analyses

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Can you imagine how to solve this?
What problems might arise?

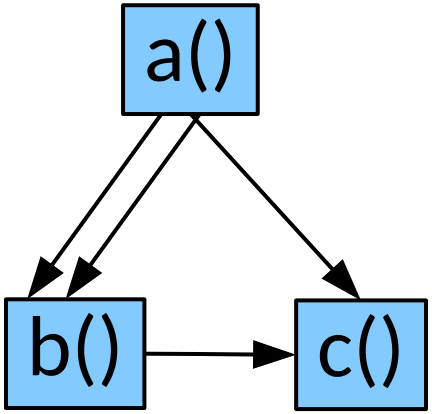
Context Sensitivity

- Recall that we can extract a call graph
  - Just as you are doing in your first project!

```
def a():
   b()
   …
   b()

def b():
   …
   c()

def c():
   …
```



The behavior of c() could be affected by each "…"

Modeling them can make analysis more precise.

Context Sensitivity

- Simplest Approach
  - Add edges between call sites & targets
  - Perform data flow on this larger graph

```
def main():
  x = 7
  r = p(x)
  x = r
  z = p(x+10)
```

```
def p(a):
  if a < 9:
    y = 0
  else:
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```

Example from Stephen Chong

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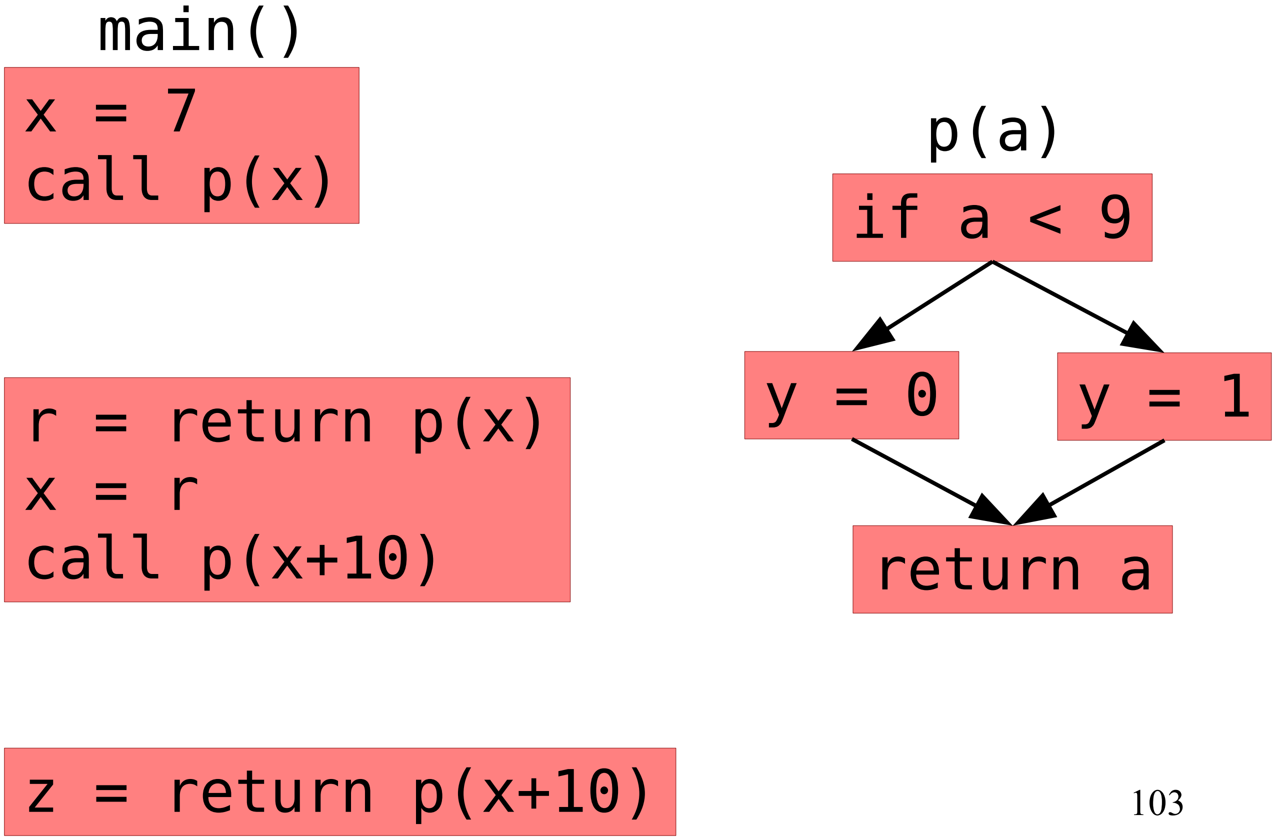

main()

```
x = 7
call p(x)
```

```
r = return p(x)
x = r
call p(x+10)
```

```
z = return p(x+10)
```

p(a)

```
if a < 9
```

```
y = 0
```

```
y = 1
```

```
return a
```

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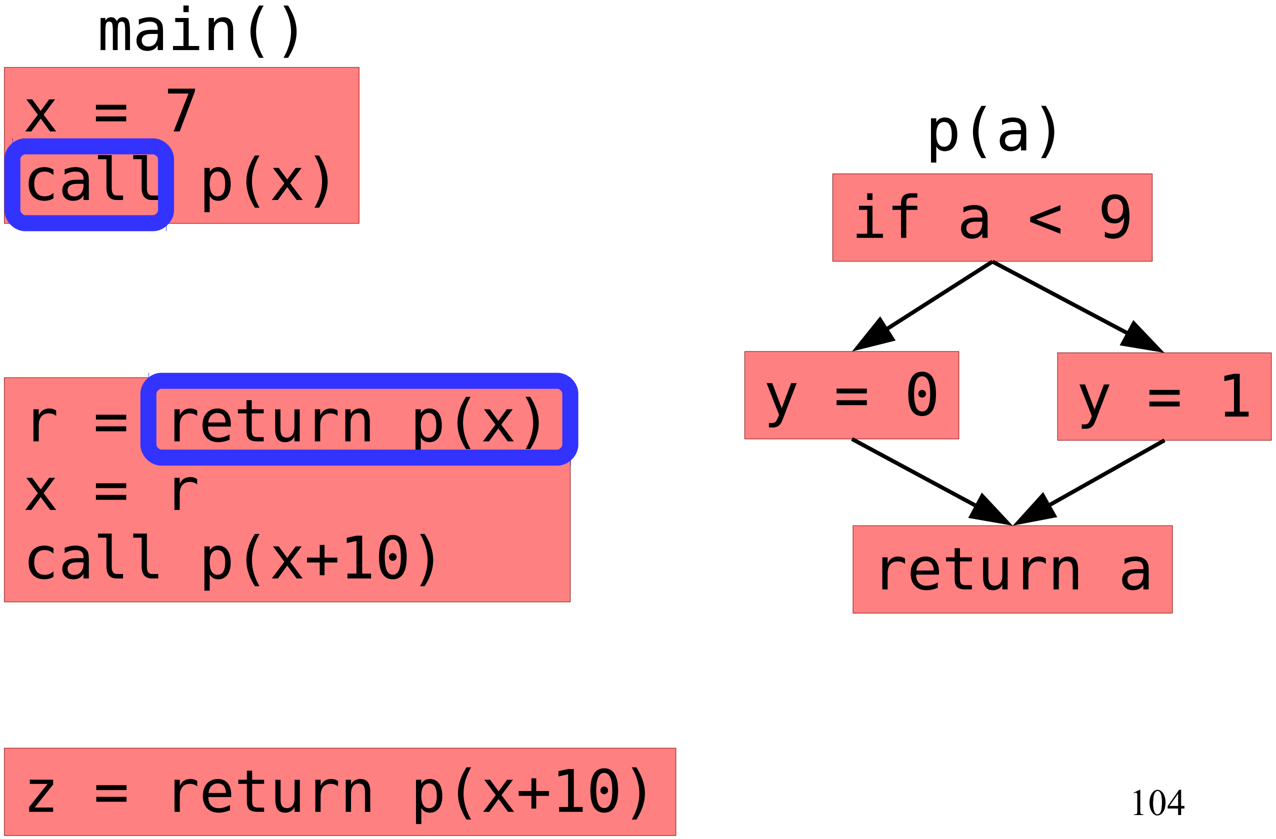

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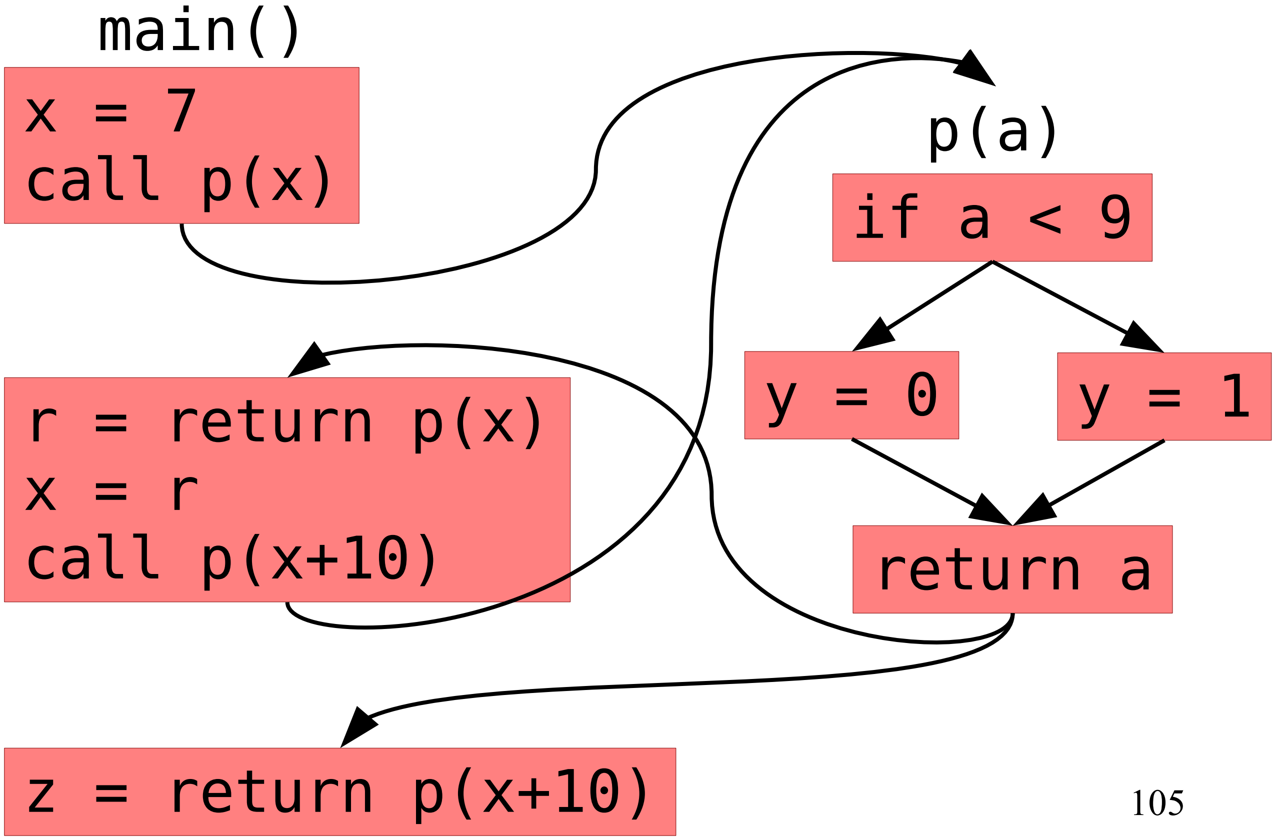

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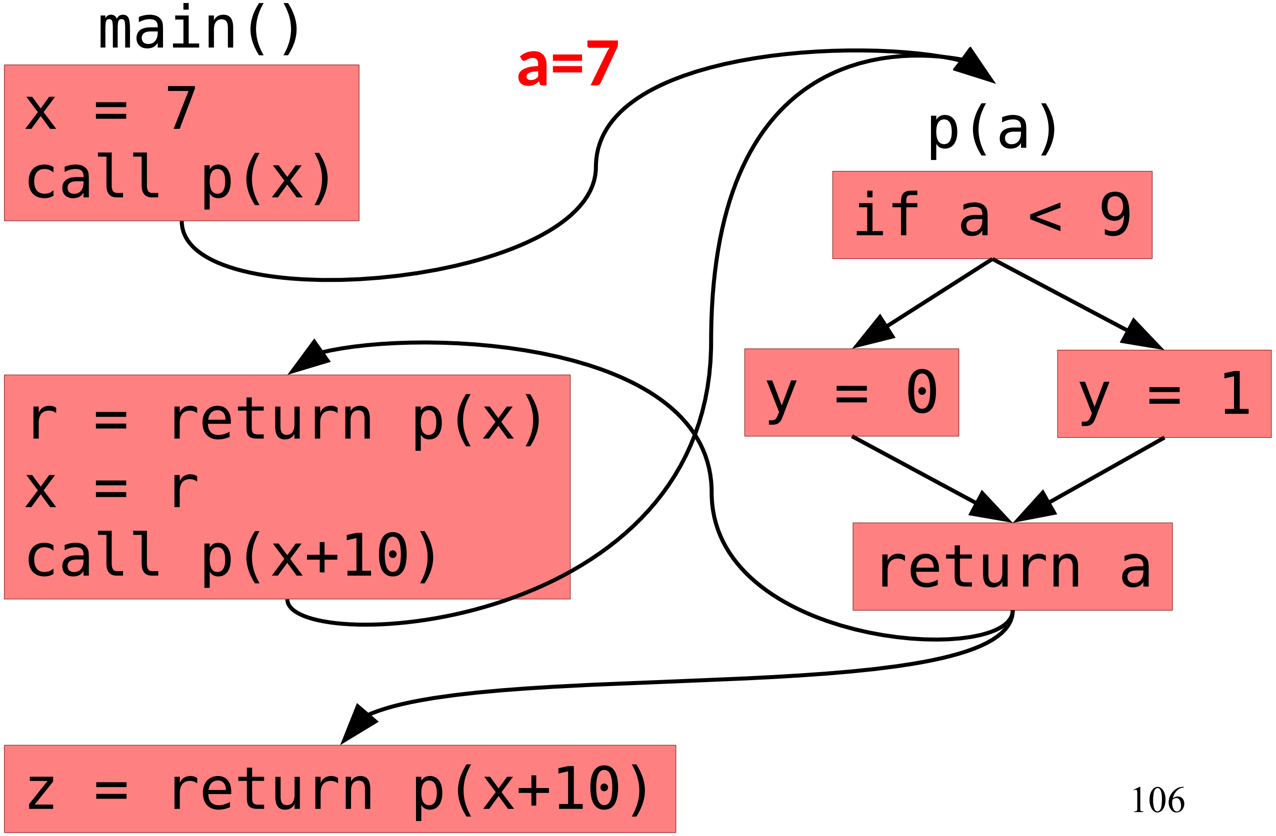

main()

```
x = 7
call p(x)
```

a=7

p(a)

```
if a < 9
```

```
y = 0
```

```
y = 1
```

```
return a
```

```
r = return p(x)
x = r
call p(x+10)
```

```
z = return p(x+10)
```

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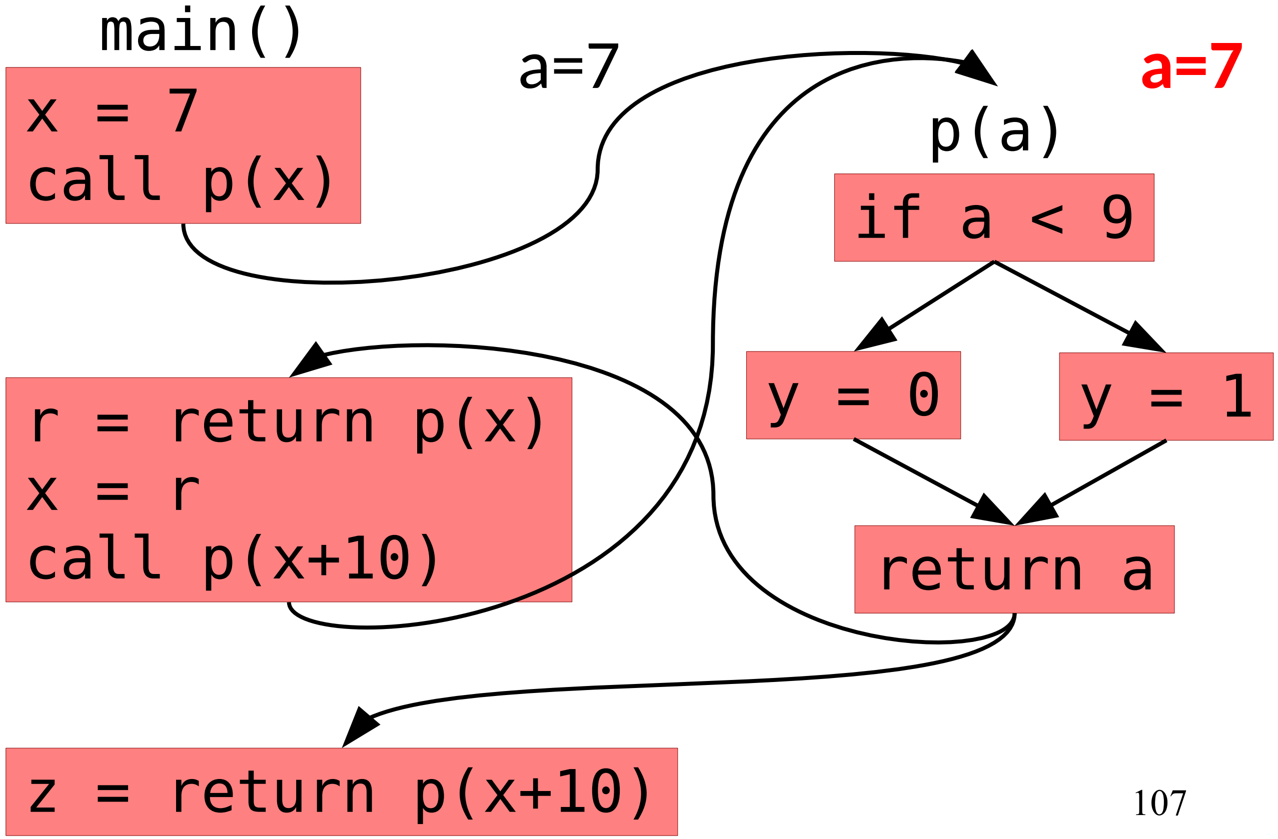

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Context Sensitivity

- Simplest Approach
  - Add edges between call sites & targets
  - Perform data flow on this larger graph

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def main():
  x = 7
  r = p(x)
  x = r
  z = p(x+10)
```

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def p(a):
  if a < 9:
    y = 0
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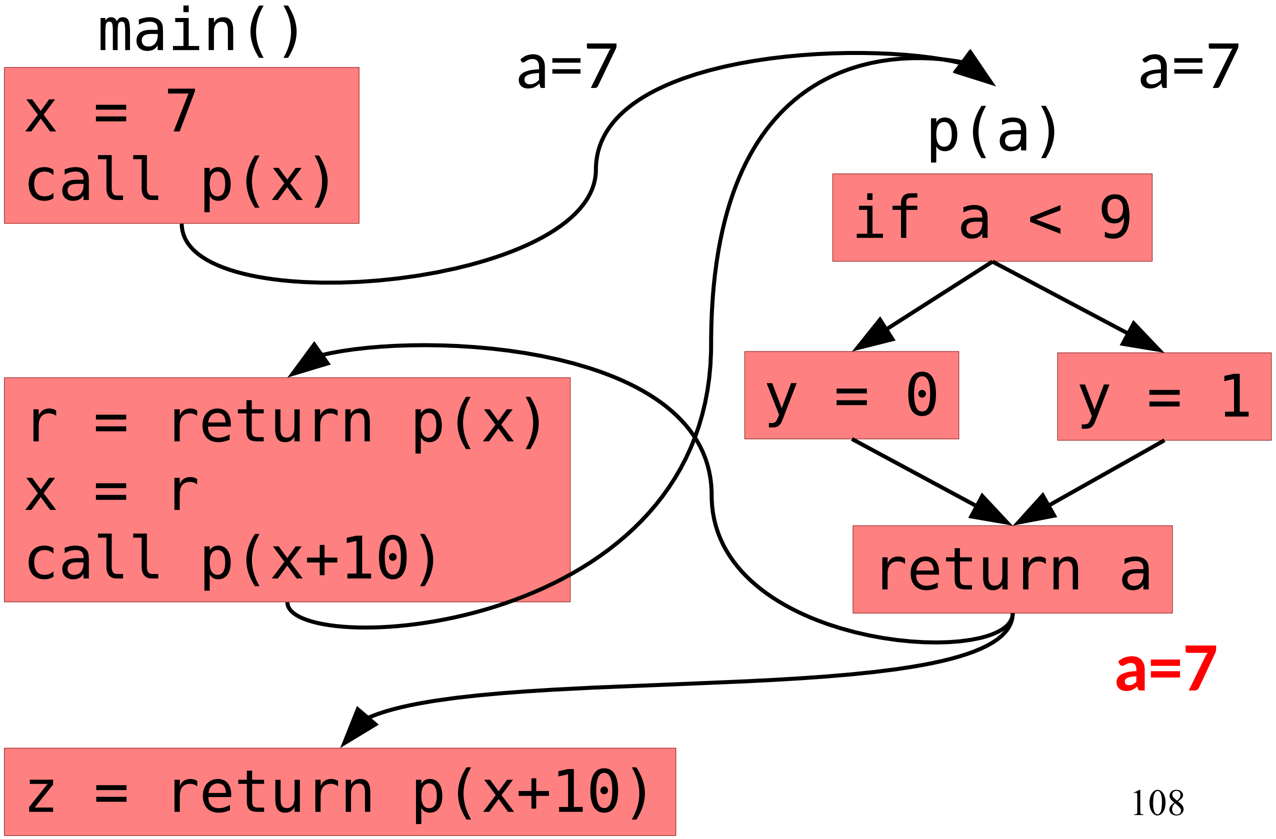

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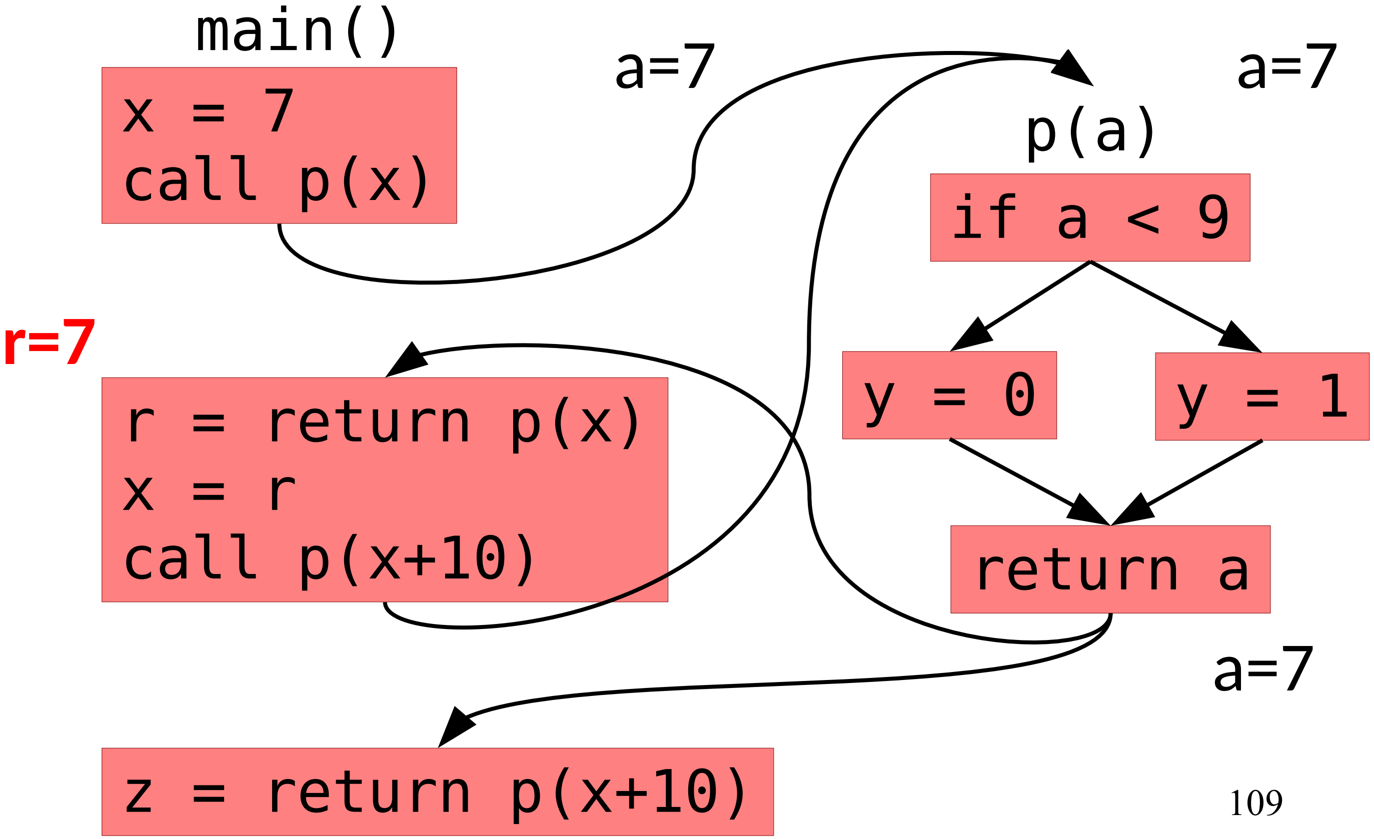

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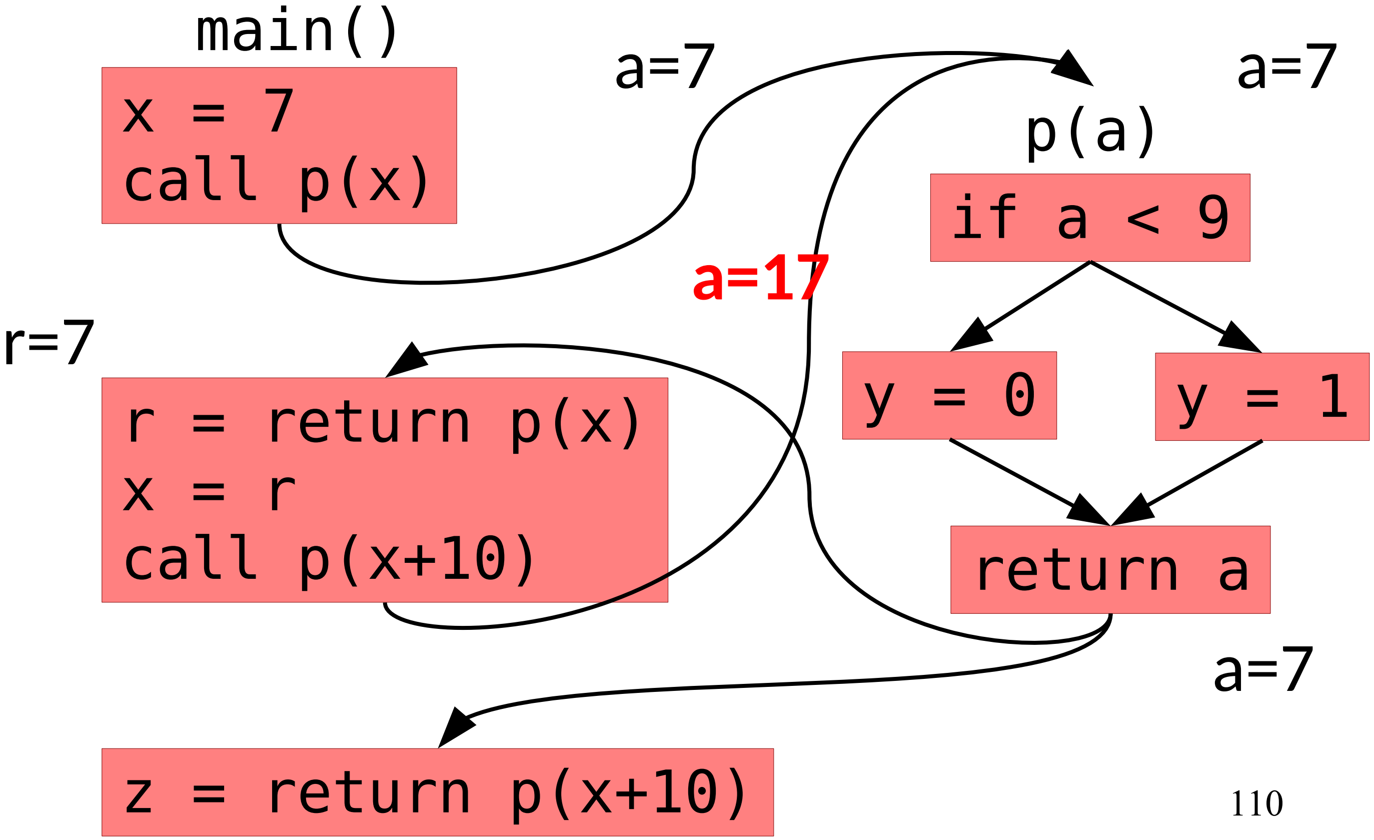

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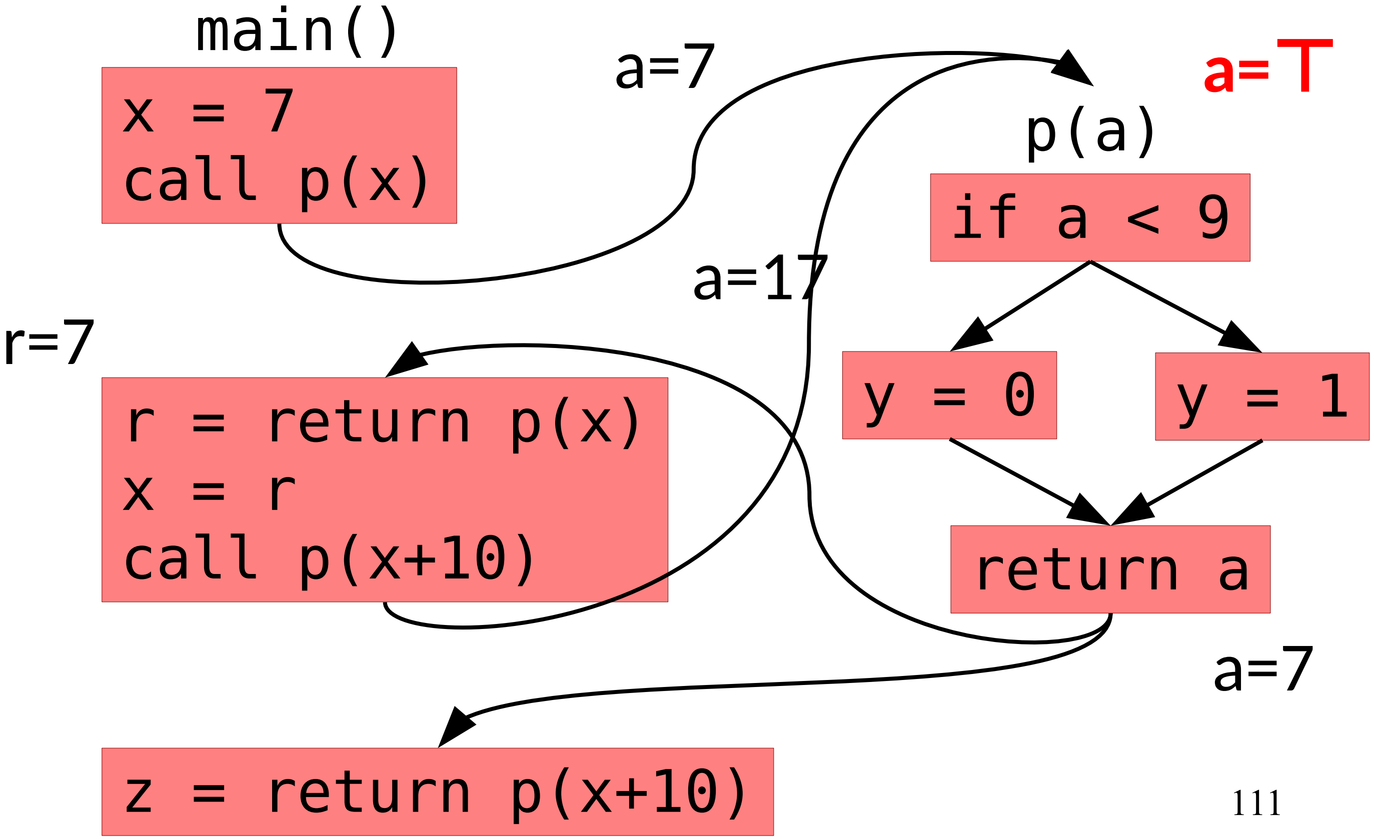

main()

```
x = 7
call p(x)
```

a=7

p(a)

a=⊤

```
if a < 9
```

a=17

r=7

```
y = 0
```

```
y = 1
```

```
r = return p(x)
x = r
call p(x+10)
```

```
return a
```

a=7

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z = return p(x+10)
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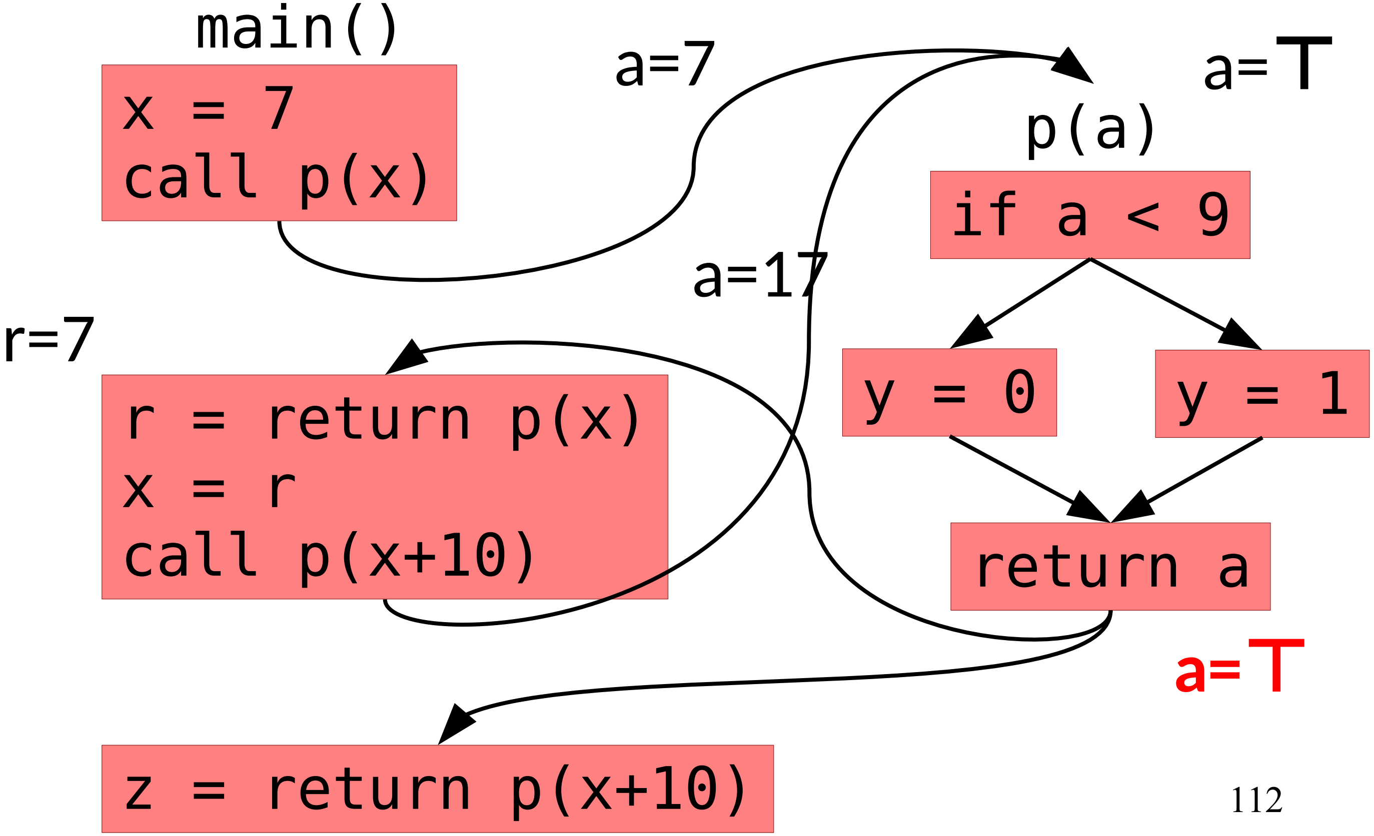

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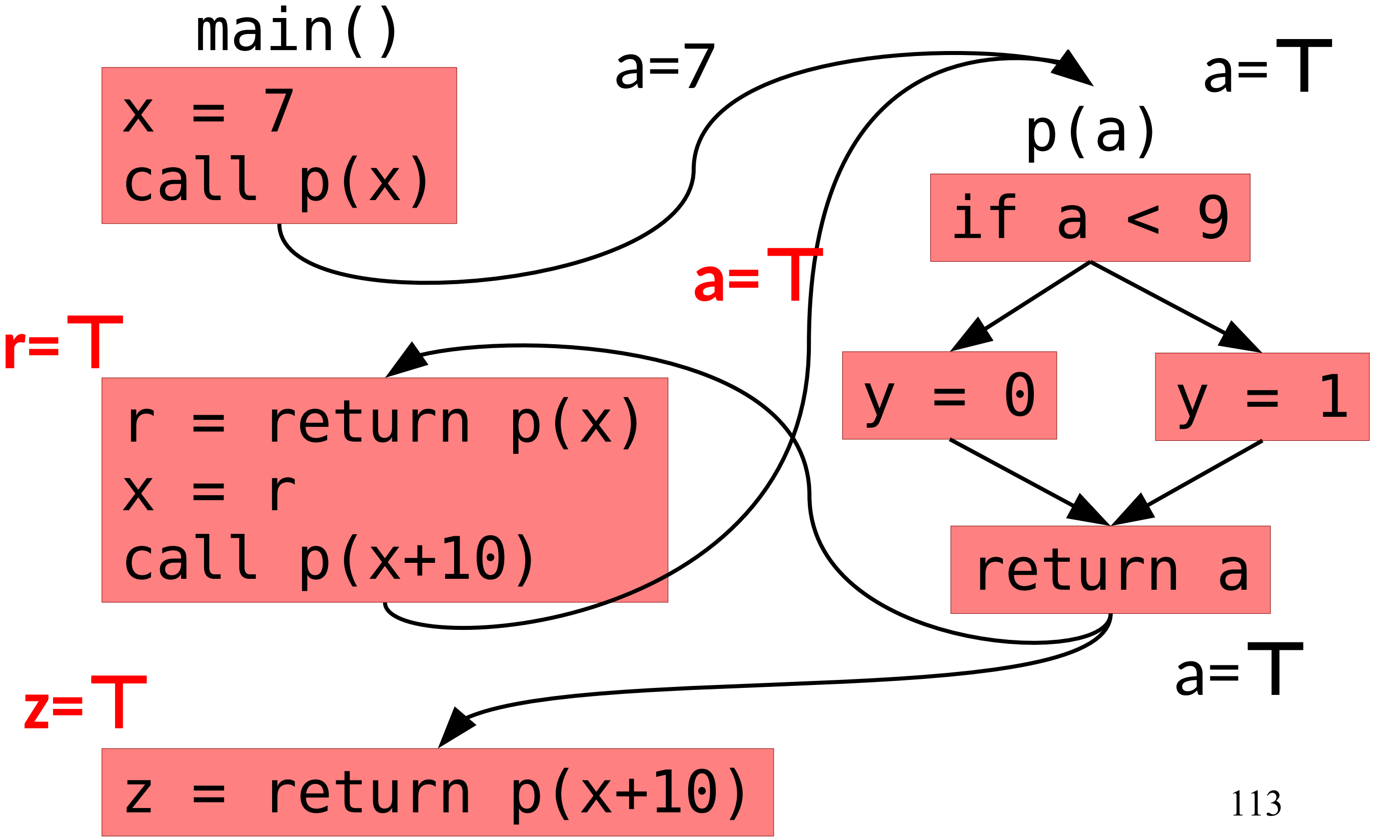

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return a

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r = return p(x)
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Example from Stephen Chong

Context Sensitivity

- Information from one call site can flow to a mismatched return site!

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- Information from one call site can flow to a mismatched return site!
- How could we address it?

Context Sensitivity

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  - Make a copy of the function at each call site
- What problems arise?
- What other strategies can we use?

Context Sensitivity

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  - Make one copy of each function per call site

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```
def main():
    a()
    a()
```

```
def a():
    b()
```

```
def b():
    pass
```

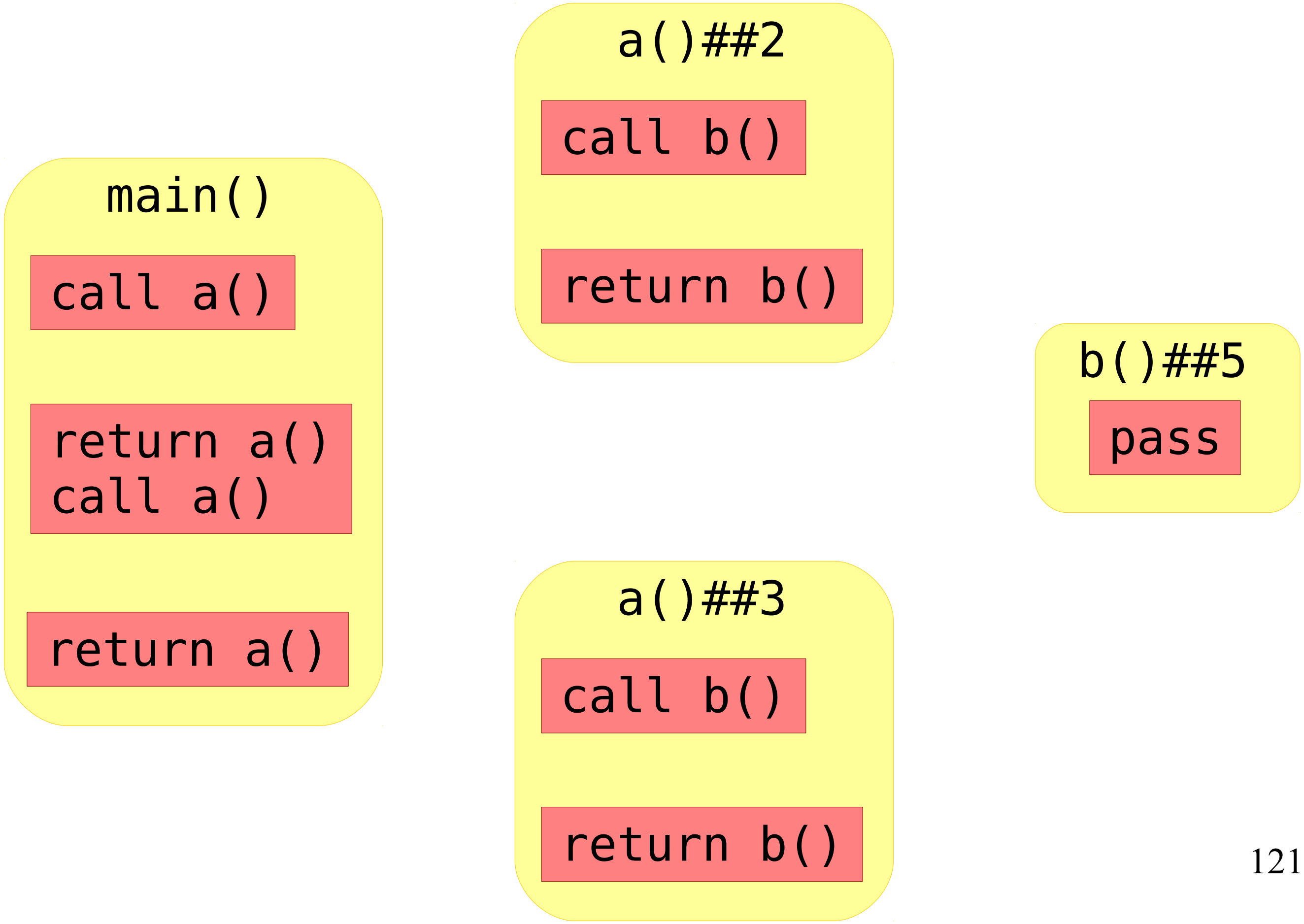
Context Sensitivity

- Solution 3: Make a Copy
  - Make one copy of each function per call site

```
1) def main():
2)     a()
3)     a()
```

```
4) def a():
5)     b()
```

```
6) def b():
7)     pass
```



main()

call a()

return a()
call a()

return a()

a()##2

call b()

return b()

a()##3

call b()

return b()

b()##5

pass

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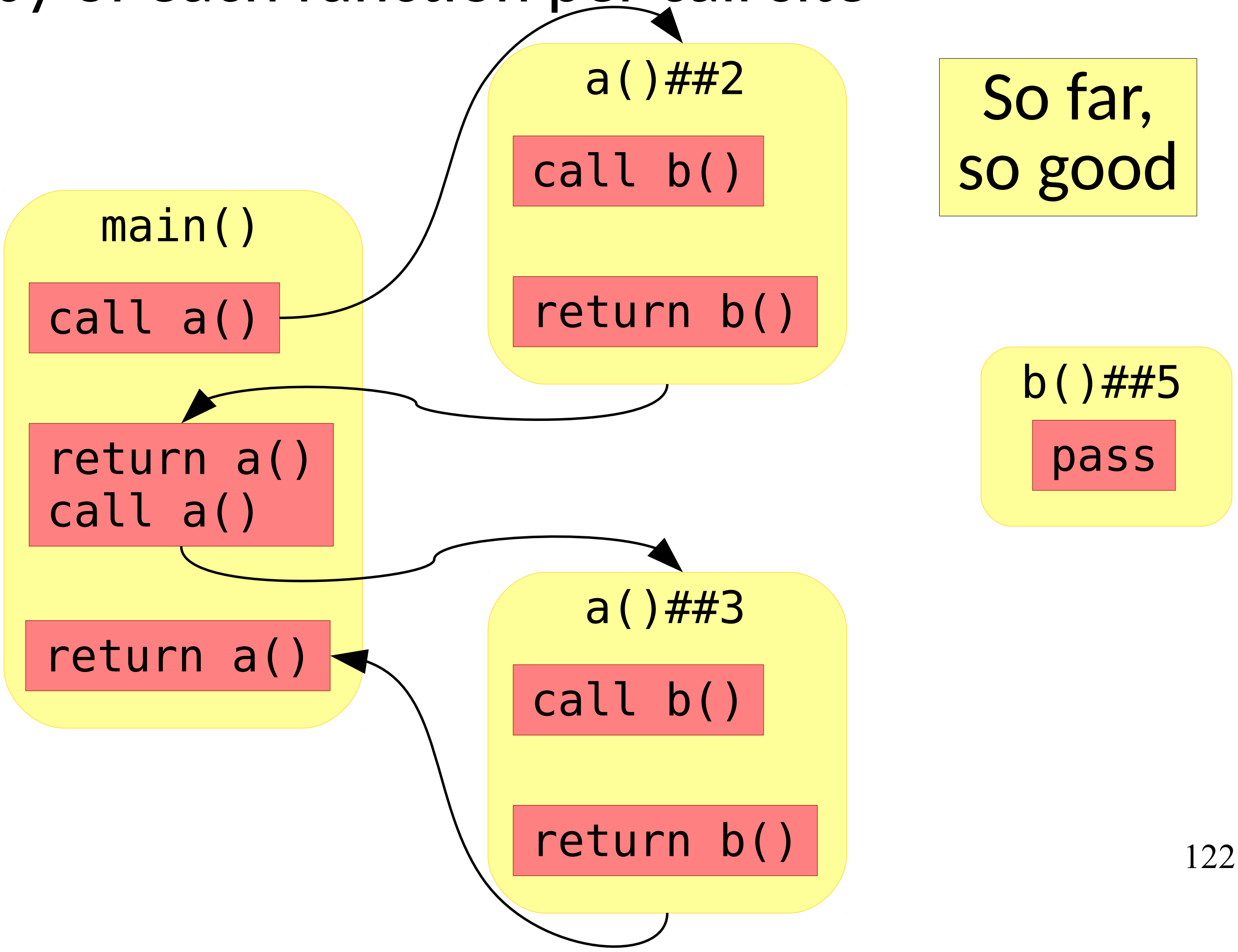

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So far, so good

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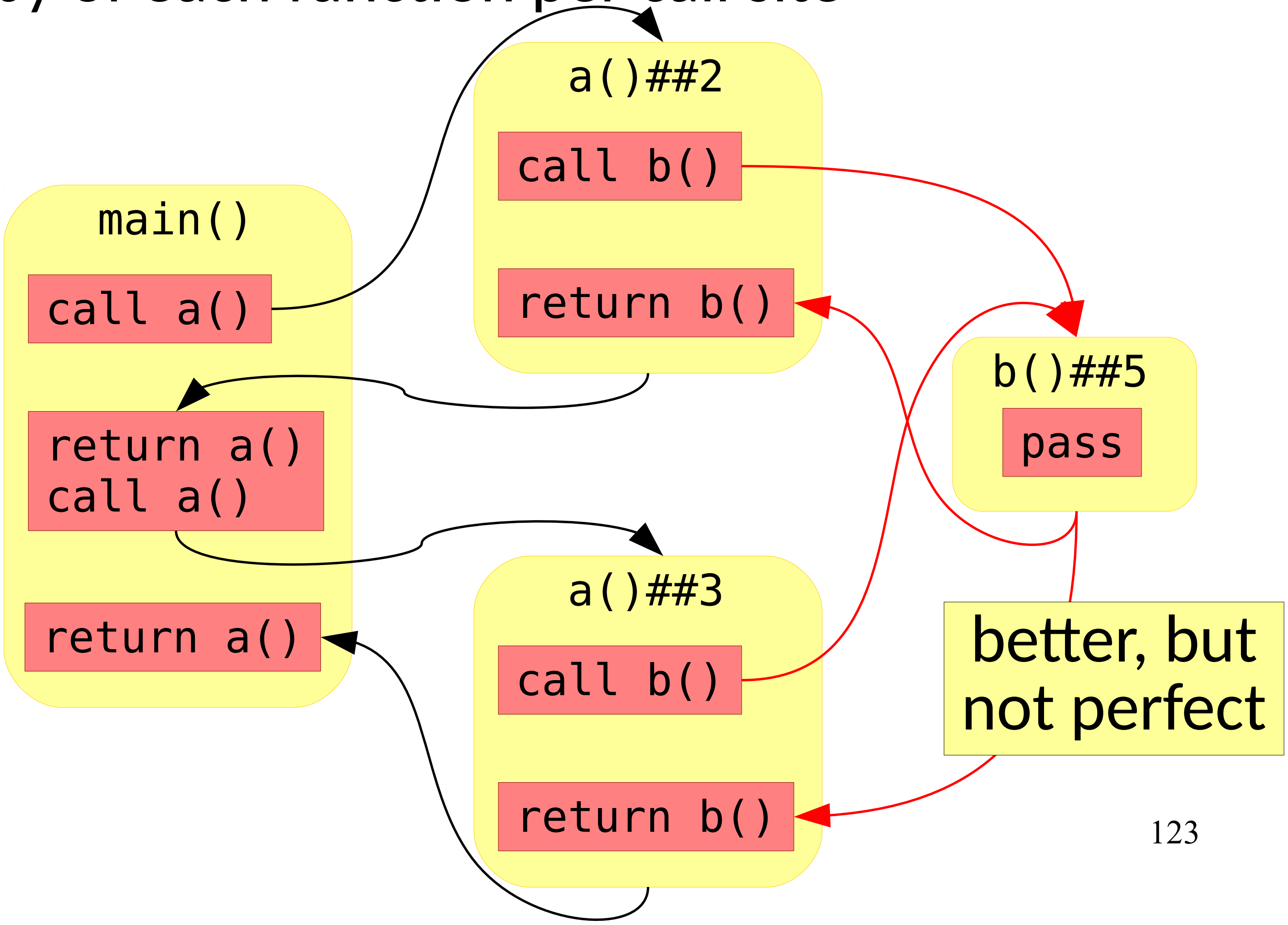

better, but not perfect

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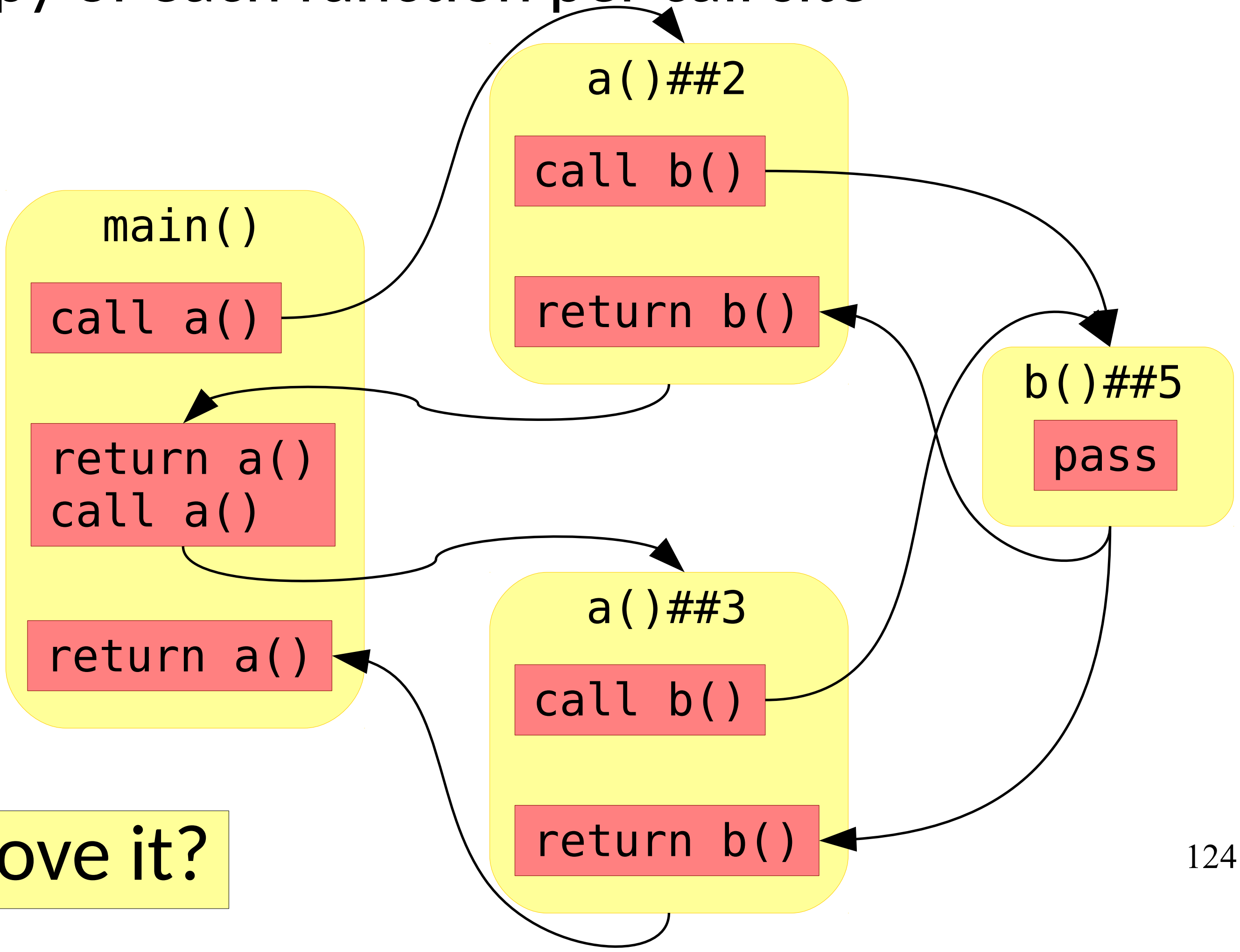

How can we improve it?

Context Sensitivity

Generalized:

- Make a bounded number of copies

Context Sensitivity

Generalized:

- Make a bounded number of copies
- Choose a key/feature that determines which copy to use
  - Bounded calling context/call stack (*call site sensitivity*)
  - Allocation sites of objects (*object sensitivity*)

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Otherwise, process foo(in) in C and update S with (in$\sqcap$S.in).

If the result changes, reprocess all callers of (foo,C)

Context Sensitivity

- In some cases, context sensitive analysis can be reduced to special forms of graph reachability.

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- Sensitivity {Path? Flow? Context?}

The configuration is ultimately driven by the property/problem of interest

Static Analysis

- We've already seen a few static analyses:
  - Call graph construction
  - Points-to graph construction (What are MAY/MUST?)
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  - Call graph construction
  - Points-to graph construction (What are MAY/MUST?)
  - Static slicing
- The choices for approximation are why these analyses are imprecise.

Other (Traditionally) Static Approaches

- Type based analyses
- Bounded state exploration
- Symbolic execution
- Model checking

Many of these have been integrated into *dynamic* analyses, as we shall see over the semester.

Static Analysis Summary

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- Can answer queries like:
  - Must my program always …?
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- Dataflow analysis is one common form of static analysis