Static Analysis
and
Dataflow Analysis
Static Analysis

Static analyses consider *all possible behaviors* of a program *without running* it.
Static Analysis

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- Look for a property of interest
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  - Do I dereference NULL pointers?
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- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
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- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
Static Analysis

Static analyses consider *all possible behaviors* of a program **without running** it.

- **Look for a property of interest**
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
Static Analysis

Static analyses consider *all possible behaviors* of a program *without running* it.

- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
  - Does my program terminate?
Static Analysis

Brief Review of Undecidability

HALT? "Does my program terminate?"
Static Analysis

Brief Review of Undecidability
Static Analysis

Brief Review of Undecidability
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```
if HALT?(P, P):
    while True: {}
else
    return True
```
Brief Review of Undecidability

```
if HALT?(P, P):
    while True: {}  
else
    return True
```
Static Analysis

Brief Review of Undecidability

```
if HALT?(P, P):
    while True: {}
else
    return True
```

It's a classic paradox!
Static analysis considers all possible behaviors of a program without running it.

- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
  - Does my program terminate?

But wait? Isn't that impossible?
Static Analysis

Static analyses consider *all possible behaviors* of a program *without running* it.

- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
  - Does my program terminate?

But wait? Isn't that impossible?

- Only if answers must be perfect.
Static Analysis

Overapproximate or underapproximate the problem, and try to solve this simpler version.
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- **Sound analyses**
  - Overapproximate
  - Guaranteed to find violations of property
  - May raise false alarms
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- **Complete analyses**
  - Underapproximate
  - Reported violations are real
  - May miss violations
Static Analysis

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  - Overapproximate
  - Guaranteed to find violations of property
  - May raise false alarms

- **Complete analyses**
  - Underapproximate
  - Reported violations are real
  - May miss violations

Striking the right balance is key to a useful analysis
Static Analysis

Modeled program behaviors

Possible Program Behavior
Static Analysis

Modeled program behaviors

Consider some behaviors possible when they are not.
Static Analysis

Modeled program behaviors

- Overapproximate
- Possible Program Behavior
- Underapproximate

Ignore some behaviors that are possible.
Static Analysis

Modeled program behaviors

Overapproximate

Possible Program Behavior

Underapproximate

One Execution
A Simple Example – Dataflow Analysis

Q: Is a particular number ever negative?
   – Might be an offset into invalid memory!

Approximate the program's behavior
A Simple Example – Dataflow Analysis

Q: Is a particular number ever negative?
   – Might be an offset into invalid memory!

Approximate the program's behavior

- **Concrete** domain: integers
- **Abstract** domain: \{-,0,+\} \cup \{\top, \bot\}
A Simple Example – Dataflow Analysis

Q: Is a particular number ever negative?
   – Might be an offset into invalid memory!

Approximate the program's behavior

- **Concrete** domain: integers
- **Abstract** domain: \{-,0,+\} \cup \{\top,\bot\}

\[
\begin{align*}
\text{concrete}(x) &= 5 \implies \text{abstract}(x) = + \\
\text{concrete}(y) &= -3 \implies \text{abstract}(y) = - \\
\text{concrete}(z) &= 0 \implies \text{abstract}(z) = 0
\end{align*}
\]

Combines sets of the concrete domain
A Simple Example – Dataflow Analysis

- **Transfer Functions** show how to evaluate this approximated program:
A Simple Example – Dataflow Analysis

• **Transfer Functions** show how to evaluate this approximated program:
  
  - \( + + + \rightarrow + \)
  - \( - + - \rightarrow - \)
  - \( 0 + 0 \rightarrow 0 \)
  - \( 0 + - \rightarrow - \)
  - \( \ldots \)
  - \( + + - \rightarrow \top \) (unknown / might vary)
  - \( \ldots / 0 \rightarrow \bot \) (undefined)
A Simple Example – Dataflow Analysis

- **Transfer Functions** show how to evaluate this approximated program:
  - $+++ ightarrow +$
  - $-++ ightarrow -$
  - $0+0 ightarrow 0$
  - $0+- ightarrow -$
  - $...$
  - $++- ightarrow T$ (unknown / might vary)
  - $.../0 ightarrow ⊥$ (undefined)

This type of approximation is called *abstract interpretation*. 
A Simple Example – Dataflow Analysis

- **Transfer Functions** show how to evaluate this approximated program:
  - $+++ \rightarrow +$
  - $--+ \rightarrow -$
  - $0+0 \rightarrow 0$
  - $0+- \rightarrow -$
  - $...$
  - $++- \rightarrow \top$ (unknown / might vary)
  - $.../0 \rightarrow \bot$ (undefined)

- **Meet Operator** ($\sqcap$) combines results across program paths
A Simple Example – Dataflow Analysis

- **Transfer Functions** show how to evaluate this approximated program:
  - $+++ \rightarrow +$
  - $--+ \rightarrow -$
  - $0+0 \rightarrow 0$
  - $0+- \rightarrow -$
  - ...
  - $++- \rightarrow T$ (unknown / might vary)
  - $.../0 \rightarrow \perp$ (undefined)

- **Meet Operator** ($\prod$) combines results across program paths

- Can be subtle.
  - The above is not sound or complete. Why?
A Simple Example – Dataflow Analysis

- **Transfer Functions** show how to evaluate this approximated program:
  - \(+ + + \to +\)
  - \(- + - \to -\)
  - \(0 + 0 \to 0\)
  - \(0 + - \to -\)
  - \(\ldots\)
  - \(+ + - \to \top\) (unknown / might vary)
  - \(\ldots / 0 \to \bot\) (undefined)

- **Meet Operator** \(\prod\) combines results across program paths

- Can be subtle.
  - The above is not sound or complete. Why?
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.

1) sum = 0
2) i = 1

3) if i < N

4) i = i + 1
5) sum = sum + i

6) print(sum)
7) print(i)

\( \text{sum} \rightarrow \perp \)
\( \text{i} \rightarrow \perp \)
Now model the abstract program state and propagate through the CFG.

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) if \( i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) \( \text{print(sum)} \)
7) \( \text{print(i)} \)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
5) sum → 0
   i → +
6) print(sum)
7) print(i)
sum → ⊥
i → ⊥
sum → 0
i → +
```
Dataflow Analysis

Now model the abstract program state and propagate through the CFG.

1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) $\text{sum} = 0$
2) $i = 1$
3) if $i < N$
   4) $i = i + 1$
   5) $\text{sum} = \text{sum} + i$
6) print($\text{sum}$)
7) print($i$)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)
```

```
sum → ⊥
i → ⊥
sum → 0
i → +
sum → 0
i → +
sum → 0
i → +
sum → 0
i → +
sum → 0
i → +
sum → 0
i → +
```
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
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6) print(sum)
7) print(i)
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
   6) print(sum)
   7) print(i)
```

- sum → ⊥
- i → ⊥

- sum → 0
- i → +

- sum → 0
- i → +

- sum → +
- i → +

- sum → 0
- i → +

- sum → 0
- i → +

- sum → 0
- i → +
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) \( \text{sum} = 0 \)
2) \( i = 1 \)

3) if \( i < N \)

4) \( i = i + 1 \)

5) \( \text{sum} = \text{sum} + i \)

6) \( \text{print(sum)} \)

7) \( \text{print}(i) \)

Meet Operator

\( \text{sum} \) was 0, but what should it be now?
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.

The value across all executions is not -, 0, or +, so it is simply unknown/anything. (∀)

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) if \( i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) \( \text{print} (\text{sum}) \)
7) \( \text{print} (i) \)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) if \( i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) print(sum)
7) print(i)

- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)

- \( \text{sum} \rightarrow \bot \)
- \( i \rightarrow \bot \)
- \( \text{sum} \rightarrow T \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) \textbf{sum} = 0
2) \textbf{i} = 1
3) \textbf{if} \textbf{i} < \textbf{N}
   \begin{align*}
   \textbf{sum} \rightarrow & \top \\
   \textbf{i} \rightarrow & \bot 
   \end{align*}
4) \textbf{i} = \textbf{i} + 1
5) \textbf{sum} = \textbf{sum} + \textbf{i}
   \begin{align*}
   \textbf{sum} \rightarrow & \top \\
   \textbf{i} \rightarrow & \bot \\
   \textbf{sum} \rightarrow & \bot \\
   \textbf{i} \rightarrow & \bot \\
   \end{align*}
6) \textbf{print}(\textbf{sum})
7) \textbf{print}(\textbf{i})

\textbf{sum} \rightarrow 0
\textbf{i} \rightarrow +
\textbf{sum} \rightarrow \top
\textbf{i} \rightarrow +
\textbf{sum} \rightarrow +
\textbf{i} \rightarrow +
\textbf{sum} \rightarrow 0
\textbf{i} \rightarrow +
\textbf{sum} \rightarrow 0
\textbf{i} \rightarrow +

\textbf{sum} \rightarrow \bot
\textbf{i} \rightarrow \bot
\textbf{sum} \rightarrow \top
\textbf{i} \rightarrow +
\textbf{sum} \rightarrow \bot
\textbf{i} \rightarrow \bot
\textbf{sum} \rightarrow \bot
\textbf{i} \rightarrow \bot
\textbf{sum} \rightarrow \bot
\textbf{i} \rightarrow \bot

Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

\[
\begin{align*}
1) & \text{sum} = 0 \\
2) & i = 1 \\
3) & \text{if } i < N \\
4) & i = i + 1 \\
5) & \text{sum} = \text{sum} + i \\
6) & \text{print(sum)} \\
7) & \text{print(i)}
\end{align*}
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

Diagram:

- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow \top \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow \top \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow \top \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow \bot \)
- \( i \rightarrow \bot \)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point
    (No more changes)
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.
  – Continue until we reach a fixed point
    (No more changes)
  – Proper ordering can improve the efficiency.
    (Topological Order, Strongly Connected Components)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point (No more changes)
  - Proper ordering can improve the efficiency.
    - (Topological Order, Strongly Connected Components)

Will it always terminate?
Dataflow Analysis

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
Dataflow Analysis

• Guarantee termination by carefully choosing
  – The abstract domain
  – The transfer function

• For basic analyses, use a monotone framework
  Loosely: \(<\text{CFG, Transfer Function, Lattice Abstraction}>\)
Dataflow Analysis

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function

- For basic analyses, use a monotone framework
  - \{-,0,+\} \cup \{\top,\bot\}
  - They define a partial order
  - Abstract state can only move up lattice at a statement
Dataflow Analysis

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
- For basic analyses, use a monotone framework
  - \{-,0,+\} \cup \{\top, \bot\}
  - They define a partial order
  - Abstract state can only move \textit{up} lattice at a statement

Why does this specific example terminate?
Dataflow Analysis

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
- For basic analyses, use a monotone framework
- But in theory a lattice need not be finite!
Dataflow Analysis

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
- For basic analyses, use a monotone framework
- But in theory a lattice need not be finite!
  - Widening operators can still make it feasible
    (e.g., heuristically raise to $\top$)
Dataflow Analysis

- Note: need to model program state before and after each statement
- Proper ordering & a work list algorithm improves the efficiency
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = \( \prod \) state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

Worklist Algorithms
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
        ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: 1 2 3 4
state: { (1 → ⊥) (3 → ⊥) 
          (2 → ⊥) (4 → ⊥) }
Worklist Algorithms

work = nodes()  
state(n) = ⊥ ∀ n ∈ nodes()  
while work ≠ ∅:
    unit = take(work)  
    old = state(unit)  
    before = \prod state(p) \quad ∀ p ∈ preds(unit)  
    new = transfer(before, unit)  
    if old ≠ after:
        work = work U succs(unit)  
        state(unit) = new

work: [2, 3, 4]  
state: \{(1 → ⊥), (2 → ⊥)\}  
        \{(3 → ⊥), (4 → ⊥)\}
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
   unit = take(work)
   old = state(unit)
   before = ∏ state(p)
   ∀ p ∈ preds(unit)
   new = transfer(before, unit)
   if old ≠ after:
      work = work ∪ succs(unit)
   state(unit) = new
worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ⋂ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: 2 3 4
state: {(1 ↦ ⊥), (2 ↦ ⊥), (3 ↦ ⊥), (4 ↦ ⊥)}
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work U succs(unit)
    state(unit) = new

work: 2 3 4
state: {(1 ↦ sum → 0 i → +)} (3 ↦ ⊥) (4 ↦ ⊥)
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
   unit = take(work)
   old = state(unit)
   before = ∏ state(p)
       ∀ p ∈ preds(unit)
   new = transfer(before, unit)
   if old ≠ after:
       work = work ∪ succs(unit)
   state(unit) = new

work: (3) (4)
state: (1) (3 → ⊥)
       (2) (3 → ⊥)
           (sum → 0)
           (i → +)

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = \prod state(p)
     ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

unit = 3
old = ⊥
new = \{ sum → +, i → + \}

work: \{ (1 → sum → 0, i → +), (2 → sum → 0, i → +), (3 → sum → +, i → +), (4 → ⊥) \}

2 was added back to the list
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work U succs(unit)
    state(unit) = new

4,3 were added back to the list
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ⋂ state(p)
        ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

unit =
old =
new =

work:

state:
{((1)  
    sum → 0
    i → +  
),
  ((2)  
    sum → T
    i → +  
),
  ((3)  
    sum → +
    i → +  
),
  ((4)  
    sum → T
    i → +  
)}
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work U succs(unit)
    state(unit) = new

No change
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
        ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

unit = 4
old = ⊥
new = 4

Done!

work:

state:
Effect of Approximation

- There are several possible sources of imprecision
Effect of Approximation

- There are several possible sources of imprecision

1) $x = 2$
2) $y = 1$
3) $x = 2$
4) $y = 1$
5) $c = x \times y$
Effect of Approximation

- There are several possible sources of imprecision

\[
\begin{align*}
1) & \ x = 2 \\
2) & \ y = 1 \\
3) & \ x = -2 \\
4) & \ y = -1 \\
5) & \ c = x * y
\end{align*}
\]

\[
\begin{align*}
x & \rightarrow +, \ y \rightarrow + \\
x & \rightarrow -, \ y \rightarrow - \\
c & \rightarrow ?
\end{align*}
\]
Effect of Approximation

- There are several possible sources of imprecision
- 2 Key sources are
  - Control flow
    - Many different paths are summarized together
Effect of Approximation

There are several possible sources of imprecision

2 Key sources are

- Control flow
  - Many different paths are summarized together

- Abstraction
  - Deliberately throwing away information
  - Granularity of program state affects correlations across variables
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
  
  For one path \( p \):  
  \[
  f_p(\bot) = f_n(f_{n-1}(...f_1(f_0(\bot))))
  \]
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice

- Ideal solution is a Meet Over all Paths (MOP)
  
  For one path $p$: $f_p(\bot) = f_n(f_{n-1}(\ldots f_1(f_0(\bot))))$

  For all paths $p$: $\prod_{p} f_p(\bot)$
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
Effect of Approximation

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- Are they different?
  - Sometimes. But sometime solutions are perfect.
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
  - Sometimes. But sometime solutions are perfect.
  - When $f()$ is distributive, MFP=MOP
    \[ f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z) \]
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
  - Sometimes. But sometime solutions are perfect.
  - When \( f() \) is distributive, MFP=MOP
    \[
    f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z)
    \]
  - This applies to an important class of problems called bitvector frameworks.
Effect of Approximation

● If approximation yields imprecise results, why do we do it?
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- **Abstract domain**: e.g. \{-,0,+\} ∪ \{⊤,⊥\}
- **Transfer functions**: - + + → \(\top\)
- **Bounded domain lattice height**:
- **Concern for false + & -**
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- Abstract domain: e.g. \{-, 0, +\} \cup \{\top, \bot\}
- Transfer functions: - + + \rightarrow \top
- Bounded domain lattice height:
- Concern for false + & -

Implementation:

- Computing using work lists
- Speeding up by sorting CFG nodes
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Let's see an example
File Policy Analysis

**Goal**: Identify potential misuses of open/closed files
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What should our design actually be?
- Abstract domain?
- Transfer functions?
- Lattice?
When the property concerns subsets of a finite set, the abstract domain & lattice are easy:

- Concrete: \( D = \{a, b, c, d, \ldots \} \)
- Abstract: \( \mathcal{P}(D) = \{\{\}, \{a\}, \{b\}, \ldots, \{a, b\}, \{a, c\}, \ldots\} \)
- Lattice: Defined by subset relation:
When the property concerns subsets of a finite set, the abstract domain & lattice are easy:

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- Lattice: Defined by subset relation:

What would the meet operator be?
Bitvector Frameworks

- Why is this convenient?
  - Hint: *bitvector* frameworks
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  - $X=\{a, b\}$, $Y=\{c, d\} \rightarrow X \uplus Y = \{a, b\} \cup \{c, d\} = \{a, b, c, d\}$
  - We can implement the abstract state using efficient bitvectors!
Bitvector Frameworks

- Why is this convenient?
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Let's see how we might implement the file policy framework in LLVM...

[DEMO]
Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care
Flow Insensitive Analysis

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  - Modeling state at each statement is expensive
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- *Flow insensitive* analyses aggregate into a global state
  - Better scalability
  - Less precision
  - “Does this function modify global variable X?”
Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
  - “If bar() is called by foo(), then it is exception free.”
  - Can enable more precise interprocedural analyses
Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
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  - Can enable more precise *interprocedural* analyses

Can you imagine how to solve this? What problems might arise?
Context Sensitivity

- Recall that we can extract a call graph
  - Just as you are doing in your first project!

```python
def a():
    b()
    ...
    b()
def b():
    ...
    c()
def c():
    ...
```

The behavior of `c()` could be affected by each “…”

Modeling them can make analysis more precise.
Context Sensitivity

• Simplest Approach
  – Add edges between call sites & targets
  – Perform data flow on this larger graph

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
```

Example from Stephen Chong
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main()
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    x = r
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if a < 9
    r = return p(x)
    x = r
    call p(x+10)
else:
    y = 0
    call p(x+10)
    return a
z = return p(x+10)
Context Sensitivity

- Information from one call site can flow to a mismatched return site!
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- How could we address it?
Context Sensitivity

• Solution 2: Inlining
  – Make a copy of the function at each call site
Context Sensitivity

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- What problems arise?
Context Sensitivity

- **Solution 2: Inlining**
  - Make a copy of the function at each call site

- **What problems arise?**

- **What other strategies can we use?**
Context Sensitivity

- Solution 3: Make a Copy
  - Make one copy of each function per call site
Context Sensitivity

- Solution 3: Make a Copy
  - Make one copy of each function per call site

```python
1) def main():
2)   a()
3)   a()

4) def a():
5)   b()

6) def b():
7)   pass
```
Context Sensitivity

- Solution 3: Make a Copy
  - Make one copy of each function per call site

```
1) def main():
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4) def a():
5)   b()
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```

```
main()
call a()
return b()
call b()
return b()
```

```
a()##2

call b()
return b()
```

```
b()##5

call b()
return b()
```

```
a()##3

call b()
return b()
```
Context Sensitivity

**Solution 3: Make a Copy**

- Make one copy of each function per call site

1) def main():
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So far, so good
Context Sensitivity

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  - Make one copy of each function per call site

1) def main():
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Better, but not perfect
Solution 3: Make a Copy

- Make one copy of each function per call site

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How can we improve it?
Context Sensitivity

Generalized:

- Make a bounded number of copies
Context Sensitivity

Generalized:

- Make a bounded number of copies
- Choose a key/feature that determines which copy to use
  - Bounded calling context/call stack (*call site sensitivity*)
  - Allocation sites of objects (*object sensitivity*)
Context Sensitivity

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Context Sensitivity

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  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
Context Sensitivity

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  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
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  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:
    On `foo(in)` with context `C`:
Context Sensitivity

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    On $\texttt{foo(in)}$ with context $C$:
    
    If $(\texttt{foo},C)$ doesn't have a summary, process $\texttt{foo(in)}$ in $C$ and save the result to $S$. 
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- Modify the treatment of calls slightly:

  On `foo(in)` with context C:
  
  If `(foo,C)` doesn't have a summary, process `foo(in)` in C and save the result to S.
  
  If the summary S already approximates `foo(in)`, use S.
Context Sensitivity

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  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
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    On `foo(in)` with context `C`:
    
    If `(foo,C)` doesn't have a summary, process `foo(in)` in `C` and save the result to `S`.
    
    If the summary `S` already approximates `foo(in)`, use `S`.
    
    Otherwise, process `foo(in)` in `C` and update `S` with `(in ∩ S.in)`. 
Context Sensitivity

- Solution 4: Make a *logical* copy
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis.
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:
    - On $\text{foo}(\text{in})$ with context $C$:
      - If $(\text{foo}, C)$ doesn't have a summary, process $\text{foo}(\text{in})$ in $C$ and save the result to $S$.
      - If the summary $S$ already approximates $\text{foo}(\text{in})$, use $S$.
      - Otherwise, process $\text{foo}(\text{in})$ in $C$ and update $S$ with $(\text{in} \cap S.\text{in})$.
    - If the result changes, reprocess all callers of $(\text{foo}, C)$.
Context Sensitivity

- In some cases, context sensitive analysis can be reduced to special forms of graph reachability.
Dataflow Configurations

Can be configured in many ways:
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- Forward / Backward (e.g. reaching vs liveness)
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- Forward / Backward (e.g. reaching vs liveness)
- May / Must ($\cup$ vs $\cap$ in lattice when paths $\prod$)
- Sensitivity {Path? Flow? Context?}
Dataflow Configurations

Can be configured in many ways:

- Forward / Backward (e.g. reaching vs liveness)
- May / Must (\(\cup\) vs \(\cap\) in lattice when paths \(\prod\))
- Sensitivity \{Path? Flow? Context?\}

The configuration is ultimately driven by the property/problem of interest
Static Analysis

- We've already seen a few static analyses:
  - Call graph construction
  - Points-to graph construction (What are MAY/MUST?)
  - Static slicing
Static Analysis

- We've already seen a few static analyses:
  - Call graph construction
  - Points-to graph construction (What are MAY/MUST?)
  - Static slicing
- The choices for approximation are why these analyses are imprecise.
Other (Traditionally) Static Approaches

- Type based analyses
- Bounded state exploration
- Symbolic execution
- Model checking

Many of these have been integrated into *dynamic* analyses, as we shall see over the semester.
Static Analysis Summary

- Considers all possible executions
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- Approximates program behavior to fight undecidability
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- Can answer queries like:
  - **Must** my program always ...?
  - **May** my program ever ...?
Static Analysis Summary

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- Approximates program behavior to fight undecidability
- Can answer queries like:
  - **Must** my program always ...?
  - **May** my program ever ...?
- Dataflow analysis is one common form of static analysis