Static Analysis
and
Dataflow Analysis
Static Analysis

Static analyses consider *all possible behaviors* of a program *without running* it.
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- Look for a property of interest
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  - Do I dereference NULL pointers?
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  - Do I dereference NULL pointers?
  - Do I leak memory?
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- Look for a property of interest
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
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Static analyses consider *all possible behaviors* of a program *without running* it.

- **Look for a property of interest**
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
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  - Is this file open?
  - Does my program terminate?
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But wait? Isn't that impossible?
Static Analysis

Brief Review of Undecidability

HALT? “Does my program terminate?”
Static Analysis

Brief Review of Undecidability
Static Analysis

Brief Review of Undecidability
Brief Review of Undecidability

```
if HALT?(P, P):
    while True: {}
else
    return True
```
Static Analysis

Brief Review of Undecidability

\[
\begin{align*}
\text{if } \text{HALT}\left( P, P \right): \\
\text{while True: } \{ \} \\
\text{else} \\
\text{return True}
\end{align*}
\]
Static Analysis

Brief Review of Undecidability

if \text{HALT?(P, P)}:
    \text{while True: \{ \}}
else
    \text{return True}

\text{\textcolor{green}{P}} \text{ or } \text{\textcolor{red}{\neg P}}

\text{\textcolor{green}{H'}} = \text{\textcolor{red}{\neg H'}}

\text{\textcolor{green}{It's a classic paradox!}}
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  - Do I violate a protocol specification?
  - Is this file open?
  - **Does my program terminate?**

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*Overapproximate* or *underapproximate* the problem, and try to solve this simpler version.
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- **Sound analyses**
  - Overapproximate
  - Guaranteed to find violations of property
  - May raise false alarms
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  - Underapproximate
  - Reported violations are real
  - May miss violations
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  - Guaranteed to find violations of property
  - May raise false alarms

- **Complete analyses**
  - Underapproximate
  - Reported violations are real
  - May miss violations

Striking the right balance is key to a useful analysis.
Approximation

Modeled program behaviors

Possible Program Behavior
Approximation

Modeled program behaviors

Overapproximate

Possible Program Behavior

Consider some behaviors possible when they are not.
Approximation

Modelled program behaviours

Overapproximate

Possible Program Behavior

Underapproximate

Ignore some behaviours that are possible.
Approximation

- Dynamic Analysis
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible
Approximation

- Dynamic Analysis
  - Analyzed \( \subseteq \) Feasible
  - As \#\ tests \( \uparrow \), Analyzed \( \rightarrow \) Feasible
- Static Analysis
  - Feasible \( \subseteq \) Analyzed
Approximation

- Dynamic Analysis
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible

- Static Analysis
  - Feasible $\subseteq$ Analyzed
Approximation

- **Dynamic Analysis**
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible

- **Static Analysis**
  - Feasible $\subseteq$ Analyzed
  - As infeasible paths $\downarrow$, Analyzed $\rightarrow$ Feasible
Approximation

• Dynamic Analysis
  – Analyzed $\subseteq$ Feasible
  – As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible

• Static Analysis
  – Feasible $\subseteq$ Analyzed
  – As infeasible paths $\downarrow$, Analyzed $\rightarrow$ Feasible

• The two areas complement each other
  – Static analysis can help generate useful tests
  – Dynamic analysis can help identify infeasibility
Abstract Interpretation

Q: Is a particular number ever negative?
   – Might be an offset into invalid memory!

Approximate the program's behavior
Abstract Interpretation

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  – Might be an offset into invalid memory!

Approximate the program's behavior

- **Concrete** domain: integers
- **Abstract** domain: \{-,0,+\} \cup \{
\top, \bot\}
Abstract Interpretation

Q: Is a particular number ever negative?
   – Might be an offset into invalid memory!

Approximate the program's behavior

- **Concrete** domain: integers
- **Abstract** domain: \{-, 0, +\} \cup \{\top, \bot\}

\[
\begin{align*}
\text{concrete}(x) &= 5 \iff \text{abstract}(x) = + \\
\text{concrete}(y) &= -3 \iff \text{abstract}(y) = - \\
\text{concrete}(z) &= 0 \iff \text{abstract}(z) = 0
\end{align*}
\]

Combines sets of the concrete domain
Abstract Interpretation

- **Transfer Functions** show how to evaluate this approximated program:
Abstract Interpretation

- **Transfer Functions** show how to evaluate this approximated program:
  - $+++ \rightarrow +$
  - $--+ \rightarrow -$
  - $0+0 \rightarrow 0$
  - $0+- \rightarrow -$
  - ...$
  - $++- \rightarrow T$ (unknown / might vary)
  - $.../0 \rightarrow \perp$ (undefined)
Abstract Interpretation

- **Transfer Functions** show how to evaluate this approximated program:
  - \(+ + + \rightarrow +\)
  - \(- + - \rightarrow -\)
  - \(0 + 0 \rightarrow 0\)
  - \(0 + - \rightarrow -\)
  - ...
  - \(+ + - \rightarrow 1\) (unknown / might vary)
  - \(\ldots / 0 \rightarrow \bot\) (undefined)

This type of approximation is called *abstract interpretation*. 
Abstract Interpretation

1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
Abstract Interpretation

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Abstract Interpretation

1) $\text{sum} = 0$
2) $i = 1$

3) if $i < N$

4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$

6) print($\text{sum}$)
7) print($i$)

Diagram:

1) $\text{sum} \mapsto \bot$
   $i \mapsto \bot$

2) $\text{sum} \mapsto 0$
   $i \mapsto \bot$

3) $\text{sum} \mapsto 0$
   $i \mapsto +$

4) $i \mapsto +$

5) $\text{sum} \mapsto \bot$
   $i \mapsto \bot$

6) $\text{sum} \mapsto \bot$
   $i \mapsto \bot$
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Abstract Interpretation

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   4) i = i + 1
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6) print(sum)
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Abstract Interpretation

1) \text{sum} = 0
2) \text{i} = 1
3) \text{if } \text{i} < N
   \hspace{1cm} \text{4) } \text{i} = \text{i} + 1
   \hspace{1cm} \text{5) } \text{sum} = \text{sum} + \text{i}
6) \text{print(sum)}
7) \text{print(i)}

Does the process ever end?
Abstract Interpretation

1) $\text{sum} = 0$
2) $i = 1$
3) if $i < N$
4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$

6) print($\text{sum}$)
7) print($i$)

- $s \mapsto \bot$
- $i \mapsto \bot$
- $s \mapsto 0$
- $i \mapsto +$
- $s \mapsto 0$
- $i \mapsto +$
- $s \mapsto 0$
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- $s \mapsto 0$
- $i \mapsto +$
- $s \mapsto 0$
- $i \mapsto +$
Abstract Interpretation

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) if \( i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) print(\( \text{sum} \))
7) print(\( i \))

Can the final sum ever be negative?
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function

- For basic analyses, use a **monotone framework**
  Loosely: $<\text{CFG}, \text{Transfer Function}, \text{Lattice Abstraction}>$
Abstract Interpretation

• Guarantee termination by carefully choosing
  – The abstract domain
  – The transfer function

• For basic analyses, use a **monotone framework**
  – \{-,0,+\} \bigcup \{\top,\bot\}
  – They define a partial order
  – Abstract state can only move **up** lattice at a statement

\[
\begin{array}{ccc}
\top & & \bot \\
\downarrow & & \downarrow \\
0 & & + \\
\uparrow & & \uparrow \\
\downarrow & & \downarrow \\
- & & 0
\end{array}
\]
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
- For basic analyses, use a monotone framework
  - \{-,0,+\} \cup \{\top,\bot\}
  - They define a partial order
  - Abstract state can only move **up** lattice at a statement

Why does this specific example terminate?
Abstract Interpretation

• Guarantee termination by carefully choosing
  – The abstract domain
  – The transfer function
• For basic analyses, use a monotone framework
• But in theory a lattice need not be finite!
  (ranges/intervals, linear constraints, ...)

Abstract Interpretation

• Guarantee termination by carefully choosing
  – The abstract domain
  – The transfer function
• For basic analyses, use a monotone framework
• But in theory a lattice need not be finite!
  – Widening operators can still make it feasible
    (e.g., heuristically raise to $\top$)
Abstract Interpretation

- What properties should a good abstraction have?
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Concrete

\{1, 4\} \quad \{1, 5\}
Abstract Interpretation

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Concrete

\{1, 4\} \quad \{1, 5\}

\{1, 4, 5\}
Abstract Interpretation

- What properties should a good abstraction have?

Concrete

\[ \mathbb{Z} \]

\{1, 2, 3, \ldots\}

\{1, 4, 5\}

\{1, 4\} \quad \{1, 5\}
Abstract Interpretation

- What properties should a good abstraction have?

Concrete

Abstract
Abstract Interpretation

- What properties should a good abstraction have?
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Abstract Interpretation

- What properties should a good abstraction have?

Concrete: $\mathbb{Z}$
- $\{1, 2, 3, \ldots\}$
- $\{1, 4, 5\}$
- $\{1, 4\}$
- $\{1, 5\}$

Abstract: $\mathbb{Z}$
- $\top$
- $\bot$
- $0$
- $+$
- $-$

$s \subseteq \gamma(\alpha(s))$
Abstract Interpretation

- What properties should a good abstraction have?

No concrete values were discarded by abstraction
Dataflow Analysis

• Dataflow analysis performs model checking of abstract interpretations
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- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator ($\sqcap$) combines results across program paths
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- Dataflow analysis performs model checking of abstract interpretations
- **Meet Operator** ($\sqcap$) combines results across program paths

\[ x_A \sqcap x_B = ? \]
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- **Meet Operator** ($\sqcap$) combines results across program paths

$$x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = ?$$
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- **Meet Operator** ($\sqcap$) combines results across program paths

$$x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = ?$$
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- **Meet Operator** ($\cap$) combines results across program paths

\[
x_A \cap x_B = \alpha(x_A) \sqcap \alpha(x_B) = ?
\]
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- **Meet Operator** ($\sqcap$) combines results across program paths

\[
x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = \top
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)

sum $\rightarrow \bot$

i $\rightarrow \bot$
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)
```

Diagram:
- `sum → 0`
- `i → +`
- `sum → ⊥`
- `i → ⊥`
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) if \( i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) print(sum)
7) print(i)

### CFG

- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
- \( \text{sum} \rightarrow \perp \)
- \( i \rightarrow \perp \)
- \( \text{sum} \rightarrow 0 \)
- \( i \rightarrow + \)
Dataflow Analysis

Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
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   4) i = i + 1
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1) sum = 0
2) i = 1
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Sum → ⊥

i → ⊥

Sum → 0

i → +

Sum → 0

i → +

Sum → 0

i → +
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) $\text{sum} = 0$
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4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$
6) print($\text{sum}$)
7) print($i$)

sum $\rightarrow$ ⊥
i $\rightarrow$ ⊥

sum $\rightarrow$ 0
i $\rightarrow$ +

sum $\rightarrow$ 0
i $\rightarrow$ +

sum $\rightarrow$ 0
i $\rightarrow$ +

sum $\rightarrow$ 0
i $\rightarrow$ +

sum $\rightarrow$ 0
i $\rightarrow$ +

sum $\rightarrow$ 0
i $\rightarrow$ +
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1) sum = 0
2) i = 1
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```

```
sum → ⊥  
i → ⊥

sum → 0  
i → +

sum → 0  
i → +

sum → 0  
i → +

sum → 0  
i → +

sum → 0  
i → +
```
Dataflow Analysis

Now model the abstract program state and propagate through the CFG.

\[
\begin{align*}
1) & \; \text{sum} = 0 \\
2) & \; i = 1 \\
3) & \; \text{if } i < N \\
4) & \; i = i + 1 \\
5) & \; \text{sum} = \text{sum} + i \\
6) & \; \text{print(sum)} \\
7) & \; \text{print}(i)
\end{align*}
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

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2) i = 1
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5) sum = sum + i
6) print(sum)
7) print(i)
```

Meet Operator

sum was 0, but what should it be now?
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.

The value *across all executions* is not -, 0, or +, so it is simply unknown/anything. (⊤)

1) \( \text{sum} = 0 \)  
2) \( i = 1 \)  
3) if \( i < N \)  
4) \( i = i + 1 \)  
5) \( \text{sum} = \text{sum} + i \)  
6) \( \text{print(sum)} \)  
7) \( \text{print}(i) \)

\( \text{sum} \rightarrow 0 \)  
\( i \rightarrow + \)  
\( \text{sum} \rightarrow 0 \)  
\( i \rightarrow + \)

\( \text{sum} \rightarrow 0 \)  
\( i \rightarrow + \)  
\( \text{sum} \rightarrow ? \)  
\( i \rightarrow + \)  
\( \text{sum} \rightarrow ? \)  
\( i \rightarrow + \)  
\( \text{sum} \rightarrow 0 \)  
\( i \rightarrow + \)
Dataflow Analysis

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- Now model the abstract program state and propagate through the CFG.

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1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

Diagram:

```
sum → ⊥
i → ⊥
sum → T
i → +
sum → T
sum → 0
i → +
sum → T
i → +
sum → 0
i → +
sum → ⊤
i → +
sum → ⊤
i → +
sum → ⊤
i → +
sum → ⊤
i → +
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

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   4) i = i + 1
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Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.
  – Continue until we reach a fixed point
    (No more changes)
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.
  – Continue until we reach a fixed point
    (No more changes)
  – Proper ordering can improve the efficiency.
    (Topological Order, Strongly Connected Components)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point (No more changes)
  - Proper ordering can improve the efficiency.
    - (Topological Order, Strongly Connected Components)

Will it always terminate?
Dataflow Analysis

- Note: need to model program state before and after each statement
- Proper ordering & a work list algorithm improves the efficiency
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new
Worklist Algorithms

\[
\begin{align*}
\text{work} &= \text{nodes()} \\
\text{state}(n) &= \bot \quad \forall \ n \in \text{nodes()} \\
\text{while} \ \text{work} \neq \emptyset: \\
&\quad \text{unit} = \text{take}(\text{work}) \\
&\quad \text{old} = \text{state}(\text{unit}) \\
&\quad \text{before} = \bigwedge \text{state}(p) \\
&\quad \quad \quad \forall \ p \in \text{preds}(\text{unit}) \\
&\quad \text{new} = \text{transfer}(\text{before}, \text{unit}) \\
&\quad \text{if} \ \text{old} \neq \text{after}: \\
&\quad \quad \text{work} = \text{work} \cup \text{succs}(\text{unit}) \\
&\quad \text{state}(\text{unit}) = \text{new}
\end{align*}
\]

work: \boxed{1 2 3 4}

state: \{(\boxed{1} \leftrightarrow \bot), \quad (\boxed{3} \leftrightarrow \bot), \\
\quad (\boxed{2} \leftrightarrow \bot), \quad (\boxed{4} \leftrightarrow \bot)\}
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
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    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
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worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: 2 3 4
state: 
{(1 ↦ ⊥) (3 ↦ ⊥)}
{(2 ↦ ⊥) (4 ↦ ⊥)}
worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
              ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work: 2 3 4
state: { (1 ⊥)  (3 ⊥)  
          (2 ⊥)  (4 ⊥)  
         }
Worklist Algorithms

\[
\begin{align*}
\text{work} &= \text{nodes}() \\
\text{state}(n) &= \bot \quad \forall \ n \in \text{nodes}() \\
\text{while work} \neq \emptyset: \\
& \quad \text{unit} = \text{take}(\text{work}) \\
& \quad \text{old} = \text{state}(\text{unit}) \\
& \quad \text{before} = \prod_{p} \text{state}(p) \\
& \quad \forall \ p \in \text{preds}(\text{unit}) \\
& \quad \text{new} = \text{transfer}(\text{before}, \text{unit}) \\
& \quad \text{if old} \neq \text{after:} \\
& \qquad \text{work} = \text{work} \cup \text{succs}(\text{unit}) \\
& \quad \text{state}(\text{unit}) = \text{new}
\end{align*}
\]
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = \prod state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

state:

\{ (1 ↦ ⊥) \}
\{ (2 ↦ ⊥) \}
\{ (3 ↦ ⊥) \}
\{ (4 ↦ ⊥) \}
work = nodes()
state(n) = \bot \quad \forall \ n \in \text{nodes()}
while work ≠ \emptyset:
    unit = take(work)
    old = state(unit)
    before = \prod state(p) 
    \quad \forall \ p \in \text{preds(unit)}
    new = transfer(before, unit)
    if old ≠ after:
        work = work \cup \text{succs(unit)}
        state(unit) = new

2 was added back to the list

work: \{ (1 \rightarrow \text{sum} \rightarrow 0, \text{i} \rightarrow +),
          (2 \rightarrow \text{sum} \rightarrow 0, \text{i} \rightarrow +),
          (3 \rightarrow \text{sum} \rightarrow +, \text{i} \rightarrow +),
          (4 \rightarrow \bot) \}

state: \{ (1 \rightarrow \text{sum} \rightarrow 0, \text{i} \rightarrow +),
          (2 \rightarrow \text{sum} \rightarrow 0, \text{i} \rightarrow +),
          (3 \rightarrow \text{sum} \rightarrow +, \text{i} \rightarrow +),
          (4 \rightarrow \bot) \}
Worklist Algorithms

\[
\begin{align*}
\text{work} & = \text{nodes}() \\
\text{state}(n) & = \bot \quad \forall \ n \in \text{nodes}() \\
\text{while } \text{work} \neq \emptyset : \\
& \quad \text{unit} = \text{take}(\text{work}) \\
& \quad \text{old} = \text{state}(\text{unit}) \\
& \quad \text{before} = \prod \text{state}(p) \\
& \quad \forall \ p \in \text{preds}(\text{unit}) \\
& \quad \text{new} = \text{transfer}(\text{before}, \text{unit}) \\
& \quad \text{if } \text{old} \neq \text{after} : \\
& \quad \text{work} = \text{work} \cup \text{succs}(\text{unit}) \\
& \quad \text{state}(\text{unit}) = \text{new}
\end{align*}
\]
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

4, 3 were added back to the list
Worklist Algorithms

\[
\begin{align*}
\text{work} &= \text{nodes}() \\
\text{state}(n) &= \bot \quad \forall \ n \in \text{nodes}() \\
\text{while} \ \text{work} \neq \emptyset : \\
& \quad \text{unit} = \text{take}(\text{work}) \\
& \quad \text{old} = \text{state}(\text{unit}) \\
& \quad \text{before} = \prod \text{state}(p) \\
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& \quad \text{new} = \text{transfer}(\text{before}, \text{unit}) \\
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\end{align*}
\]
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work:

state: 
{(1) sum → 0, i → +}
{(2) sum → T, i → +}
{(3) sum → +, i → +}
{(4) sum → +, i → +}

No change
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
  unit = take(work)
  old = state(unit)
  before = ∏ state(p)
    ∀ p ∈ preds(unit)
  new = transfer(before, unit)
  if old ≠ after:
    work = work ∪ succs(unit)
  state(unit) = new

work:  

state:  
\{ (1 \mapsto \text{sum} \rightarrow 0 \text{ i} \rightarrow +), 
  (2 \mapsto \text{sum} \rightarrow \top \text{ i} \rightarrow +), 
  (3 \mapsto \text{sum} \rightarrow + \text{ i} \rightarrow +), 
  (4 \mapsto \text{sum} \rightarrow \top \text{ i} \rightarrow +) \}
Effect of Approximation

- There are several possible sources of imprecision
Effect of Approximation

- There are several possible sources of imprecision
Effect of Approximation

- There are several possible sources of imprecision:

  1. $x = 2$
  2. $y = 1$
  3. $x = -2$
  4. $y = -1$
  5. $c = x \times y$

  $x \rightarrow +, y \rightarrow +$
  $x \rightarrow -, y \rightarrow -$

  $c \rightarrow ?$
Effect of Approximation

• There are several possible sources of imprecision
• 2 Key sources are
  – Control flow
    • Many different paths are summarized together
Effect of Approximation

- There are several possible sources of imprecision
- 2 Key sources are
  - Control flow
    - Many different paths are summarized together
  - Abstraction
    - Deliberately throwing away information
    - Granularity of program state affects correlations across variables
Effect of Approximation

• We compute results with maximal fixed points (MFP) in the lattice
**Effect of Approximation**

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
  
  For one path \( p \): \( f_p(\bot) = f_n(f_{n-1}(\ldots f_1(f_0(\bot)))) \)
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice

- Ideal solution is a Meet Over all Paths (MOP)

  - For one path $p$: $f_p(\perp) = f_n(f_{n-1}(\ldots f_1(f_0(\perp))))$
  - For all paths $p$: $\bigwedge_p f_p(\perp)$
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
Effect of Approximation

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  - Sometimes. But sometime solutions are perfect.
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
  - Sometimes. But sometime solutions are perfect.
  - When f() is distributive, MFP=MOP
    \[ f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z) \]
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
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- Are they different?
  - Sometimes. But sometime solutions are perfect.
  - When \( f() \) is distributive, MFP=MOP
    \[
    f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z)
    \]
  - This applies to an important class of problems called bitvector frameworks.
When the property concerns subsets of a finite set, the abstract domain & lattice are easy:

- Concrete: \( D = \{a, b, c, d, \ldots \} \)
- Abstract: \( \mathcal{P}(D) = \{ \emptyset, \{a\}, \{b\}, \ldots, \{a, b\}, \{a, c\}, \ldots \} \)
- Lattice: Defined by subset relation:
When the property concerns subsets of a finite set, the abstract domain & lattice are easy:

- Concrete: $D = \{a, b, c, d, \ldots \}$
- Abstract: $\mathcal{P}(D) = \{\emptyset, \{a\}, \{b\}, \ldots, \{a, b\}, \{a, c\}, \ldots\}$
- Lattice: Defined by subset relation:

$$
\begin{array}{c}
\emptyset \\
\{a\} \\
\{b\} \\
\{c\} \\
\{a,b\} \\
\{a,c\} \\
\{b,c\} \\
\{b,d\} \\
\{a,b,c\} \\
\{a,b,d\} \\
\end{array}
$$

What would the meet operator be?
Bitvector Frameworks

• Why is this convenient?
  – Hint: bitvector frameworks
Bitvector Frameworks

• Why is this convenient?
  – Hint: *bitvector* frameworks
  – $X = \{a, b\}$, $Y = \{c, d\} \rightarrow X \sqcup Y = \{a, b\} \cup \{c, d\} = \{a, b, c, d\}$
  – We can implement the abstract state using efficient bitvectors!
Effect of Approximation

• If approximation yields imprecise results, why do we do it?
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- **Abstract domain**: e.g. \{-,0,+\} \cup \{\top,\bot\}
- **Transfer functions**: \(- + + \rightarrow \top\)
- **Bounded domain lattice height**:
- **Concern for false + & -**
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- **Abstract domain:** e.g. $\{-,0,+\} \cup \{\top,\bot\}$
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Implementation:

- **Computing using work lists**
- **Speeding up by sorting CFG nodes**
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- Abstract domain: e.g. \{-,0,+\} \cup \{\top,\bot\}
- Transfer functions: \(- + + \rightarrow \top\)
- Bounded domain lattice height:
- Concern for false + & -

Implementation:

- Computing using work lists
- Speeding up by sorting CFG nodes

Let's see an example
File Policy Analysis

**Goal:** Identify potential misuses of open/closed files
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- Files may be open or closed
File Policy Analysis

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- Files may be open or closed
- Many operations may only occur on open files
  e.g. read, write, print, flush, close, ...
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What should our design actually be?
- Abstract domain?
- Transfer functions?
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**File Policy Analysis**

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e.g. read, write, print, flush, close, ...

What should our design actually be?
- Abstract domain?
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[DEMO]
Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care
Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care

- *Flow insensitive* analyses aggregate into a global state
  - Better scalability
  - Less precision
  - “Does this function modify global variable X?”
Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
  - “If bar() is called by foo(), then it is exception free.”
  - Can enable more precise *interprocedural* analyses
Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
  - “If bar() is called by foo(), then it is exception free.”
  - Can enable more precise interprocedural analyses

Can you imagine how to solve this? What problems might arise?
Context Sensitivity

• Recall that we can extract a call graph
  – Just as you are doing in your first project!

```python
def a():
    b()
    ...
    b()
def b():
    ...
    c()
def c():
    ...
```

The behavior of c() could be affected by each “...”

Modeling them can make analysis more precise.
Context Sensitivity

- Simplest Approach
  - Add edges between call sites & targets
  - Perform data flow on this larger graph

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
```

Example from Stephen Chong
Context Sensitivity

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    return a
```

```
def main():
    x = 7
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main()
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```
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```
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Example from Stephen Chong
**Context Sensitivity**

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main()
```

Example from Stephen Chong
Context Sensitivity

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```

Example from Stephen Chong
Context Sensitivity

• Information from one call site can flow to a mismatched return site!
Context Sensitivity

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• How could we address it?
Context Sensitivity

• Solution 2: Inlining
  – Make a copy of the function at each call site
Context Sensitivity

• Solution 2: Inlining
  – Make a copy of the function at each call site

• What problems arise?
Context Sensitivity

- Solution 2: Inlining
  - Make a copy of the function at each call site
- What problems arise?
- What other strategies can we use?
Context Sensitivity

• Solution 3: Make a Copy
  – Make one copy of each function per call site
Context Sensitivity

• Solution 3: Make a Copy
  – Make one copy of each function per call site

1) def main():
2)   a()
3)   a()
4) def a():
5)   b()
6) def b():
7)   pass
Context Sensitivity

• Solution 3: Make a Copy
  – Make one copy of each function per call site

1) def main():
2)   a()
3)   a()

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6) def b():
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Solution 3: Make a Copy

- Make one copy of each function per call site

```python
1) def main():
2)   a()
3)   a()

4) def a():
5)   b()

6) def b():
7)   pass

So far, so good

```
### Context Sensitivity

- **Solution 3: Make a Copy**
  - Make one copy of each function per call site

```python
1) def main():
2)   a()
3)   a()
4) def a():
5)   b()
6) def b():
7)   pass
8) call a()
9) return a()
10) call a()
11) return a()
12) main()
13) call b()
14) return b()
15) a()##2
16) call b()
17) return b()
18) a()##3
19) call b()
20) return b()
21) b()##5
22) pass
23) better, but not perfect
```
Context Sensitivity

• Solution 3: Make a Copy
  – Make one copy of each function per call site

```
1) def main():
2)   a()
3)   a()

4) def a():
5)   b()

6) def b():
7)   pass
```

How can we improve it?
Context Sensitivity

Generalized:

- Make a bounded number of copies
Context Sensitivity

Generalized:

- Make a bounded number of copies
- Choose a key/feature that determines which copy to use
  - Bounded calling context/call stack (*call site sensitivity*)
  - Allocation sites of objects (*object sensitivity*)
Context Sensitivity

- Solution 4: Make a *logical* copy
Context Sensitivity

• Solution 4: Make a *logical* copy
  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
Context Sensitivity

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  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
  – Compute results (called *procedure summaries*) for each logical copy of a function.
Context Sensitivity

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  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
  – Compute results (called *procedure summaries*) for each logical copy of a function.
  – Modify the treatment of calls slightly:

On `foo(in)` with context `C`:
Context Sensitivity

- Solution 4: Make a *logical* copy
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis.
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:
    
    On `foo(in)` with context `C`:
    
    If `(foo,C)` doesn't have a summary, process `foo(in)` in `C` and save the result to `S`. 
Context Sensitivity

• Solution 4: Make a *logical* copy
  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
  – Compute results (called *procedure summaries*) for each logical copy of a function.
  – Modify the treatment of calls slightly:

    On `foo(in)` with context `C`:
    * If `(foo,C)` doesn't have a summary, process `foo(in)` in `C` and save the result to `S`.
    * If the summary `S` already approximates `foo(in)`, use `S`
Context Sensitivity

- Solution 4: Make a *logical* copy
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:
    On `foo(in)` with context C:
    - If `(foo,C)` doesn't have a summary, process `foo(in)` in C and save the result to S.
    - If the summary S already approximates `foo(in)`, use S
    - Otherwise, process `foo(in)` in C and update S with `(in\sqcap S.in)`.
Context Sensitivity

• Solution 4: Make a *logical* copy
  
  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
  
  – Compute results (called *procedure summaries*) for each logical copy of a function.
  
  – Modify the treatment of calls slightly:

    On `foo(in)` with context `C`:
    
    If `(foo,C)` doesn't have a summary, process `foo(in)` in `C` and save the result to `S`.
    
    If the summary `S` already approximates `foo(in)`, use `S`.
    
    Otherwise, process `foo(in)` in `C` and update `S` with `(in \sqsubseteq S.in)`.
    
    If the result changes, reprocess all callers of `(foo,C)`
Context Sensitivity - IFDS

• In some cases, context sensitive analysis can be reduced to special forms of graph reachability.
In some cases, context sensitive analysis can be reduced to special forms of graph reachability.

- Set of dataflow facts $D$ is finite
- Transfer functions are distributive $[f(x \sqcap y) = f(x) \sqcap f(y)]$
- Domain and range of transfer functions is $\mathcal{P}(D)$
- Lattice ordering is set containment
Context Sensitivity - IFDS

• In some cases, context sensitive analysis can be reduced to special forms of graph reachability.
  – Set of dataflow facts D is finite
  – Transfer functions are distributive \[f(x \sqcap y) = f(x) \sqcap f(y)\]
  – Domain and range of transfer functions is \(\mathcal{P}(D)\)
  – Lattice ordering is set containment

(Interprocedural Finite Distributive Subsets)
Context Sensitivity - IFDS

• Consider an undefined variable analysis...

a = 7
b = a
c = d
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

\[ a = 7 \]
\[ b = a \]
\[ c = d \]
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

\[
\begin{align*}
a &= 7 \\
b &= a \\
c &= d
\end{align*}
\]
Context Sensitivity - IFDS

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```
a = 7
b = a
c = d
```
Context Sensitivity - IFDS

• Consider an undefined variable analysis...

a = 7
b = a
c = d

a is defined, so make it unreachable
Context Sensitivity - IFDS

• Consider an undefined variable analysis...

\[
\begin{align*}
a &= 7 \\
b &= a \\
c &= d
\end{align*}
\]

\[
\text{c is unchanged, so propagate its reachability}
\]
• Consider an undefined variable analysis...

```
a = 7
b = a
c = d
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

\[ a = 7 \]
\[ b = a \]
\[ c = d \]
Context Sensitivity - IFDS

• Consider an undefined variable analysis...

\[
\begin{align*}
a &= 7 \\
b &= a \\
c &= d
\end{align*}
\]

\(c\) and \(d\) are reachable here. They are undefined at this point.
Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
```

```
x = 7
    call p(x)
    r = return p(x)
    x = r
    call p(x+10)
z = return p(x+10)
p(a)
    if a < 9
        y = 0
        call p(x+10)
    else:
        y = 1
        return a
```
Consider an undefined variable analysis...

def main():
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def p(a):
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    else:
        y = 1
    return a

if __name__ == '__main__':
    main()
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
    x = 7
    call p(x)
    r = return p(x)
    x = r
    call p(x+10)
    z = return p(x+10)
```

```
string: ( \_ \_ )
```
Consider an undefined variable analysis...

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r = return p(x)
call p(x)
x = 7
call p(x)

y = 0
y = 1
return a

z = return p(x+10)
```

String: \( (1)_2 \)   unreachable
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

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string: (₁)₁
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        y = 1

    r = return p(x)
    x = r
call p(x+10)

    y = 0
    y = 1

    return a

z = return p(x+10)
```

string: $\begin{pmatrix}
    1 & 1 \\
    2 & 2
\end{pmatrix}$
Context Sensitivity - IFDS

• Consider an undefined variable analysis...

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```

string: $(1)_{1}(2)_{2}$  reachable
Context Sensitivity - IFDS

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```python
def main():
    x = 7
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    else:
        y = 1
    x = 7
    call p(x)
```

- A fact f holds before a node if f is **CFL-Reachable** in a language of matched parentheses
Context Sensitivity - IFDS

[Reps, POPL 1995]
Context Sensitivity - IFDS

- Does constant propagation fit our definition of IFDS?
Context Sensitivity - IFDS

• Does constant propagation fit our definition of IFDS?
• Can you think of ways that it could be made to fit into IFDS?
Dataflow Configurations

Can be configured in many ways:
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- Forward / Backward (e.g. reaching vs liveness)
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- Sensitivity \{Path? Flow? Context?\}
Dataflow Configurations

Can be configured in many ways:

- Forward / Backward (e.g. reaching vs liveness)
- May / Must (∪ vs ∩ in lattice when paths ∏)
- Sensitivity {Path? Flow? Context?}

The configuration is ultimately driven by the property/problem of interest
Static Analysis

• We've already seen a few static analyses:
  – Call graph construction
  – Points-to graph construction (What are MAY/MUST?)
  – Static slicing
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  – Call graph construction
  – Points-to graph construction (What are MAY/MUST?)
  – Static slicing

• The choices for approximation are why these analyses are imprecise.
Other (Traditionally) Static Approaches

- Type based analyses
- Bounded state exploration
- Symbolic execution
- Model checking

Many of these have been integrated into dynamic analyses, as we shall see over the semester.
Static Analysis Summary

- Considers all possible executions
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- Approximates program behavior to fight undecidability
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- Can answer queries like:
  - **Must** my program always ...?
  - **May** my program ever ...?
Static Analysis Summary

- Considers all possible executions
- Approximates program behavior to fight undecidability
- Can answer queries like:
  - Must my program always ...?
  - May my program ever ...?
- Dataflow analysis is one common form of static analysis