

Static Analysis and Dataflow Analysis

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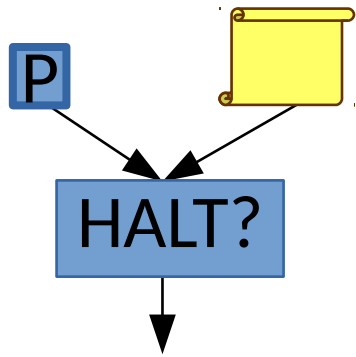
Static Analysis

Brief Review of Undecidability



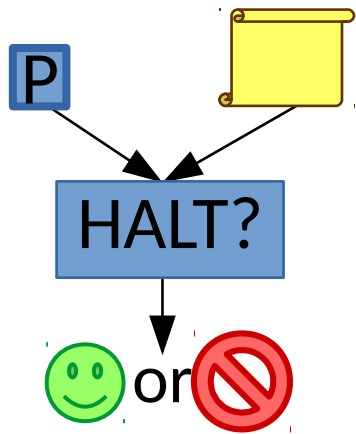
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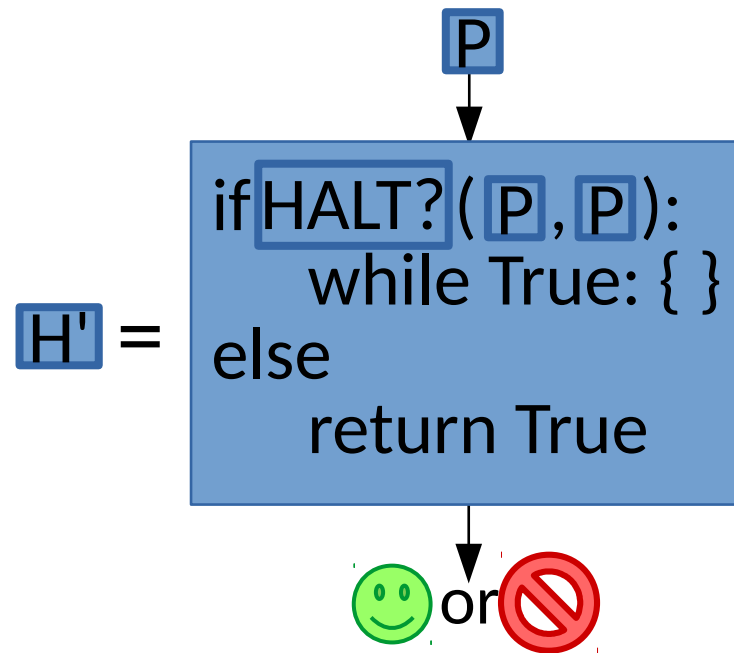
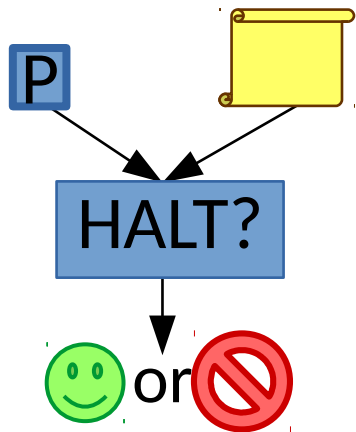
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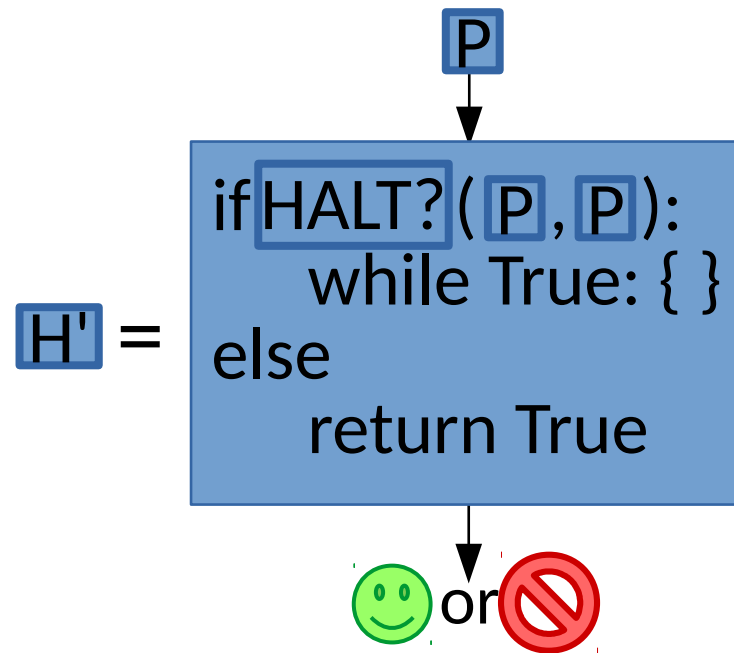
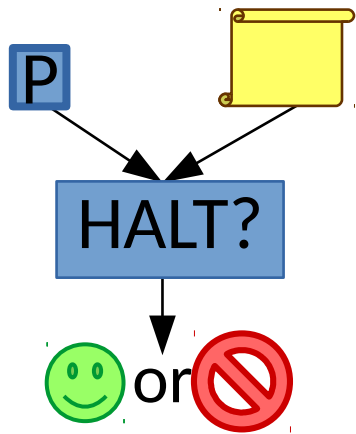
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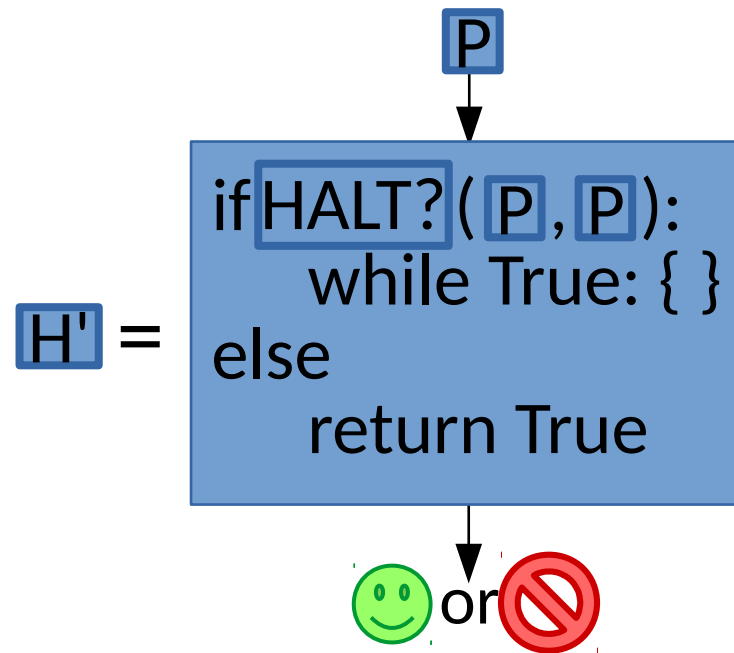
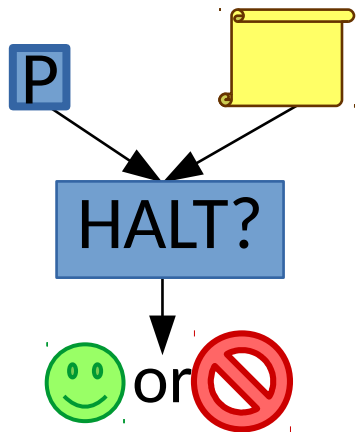
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It's a classic paradox!

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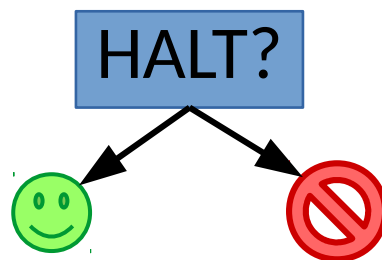
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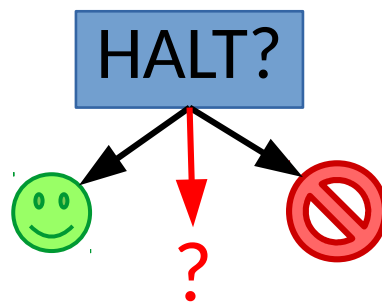
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Static Analysis

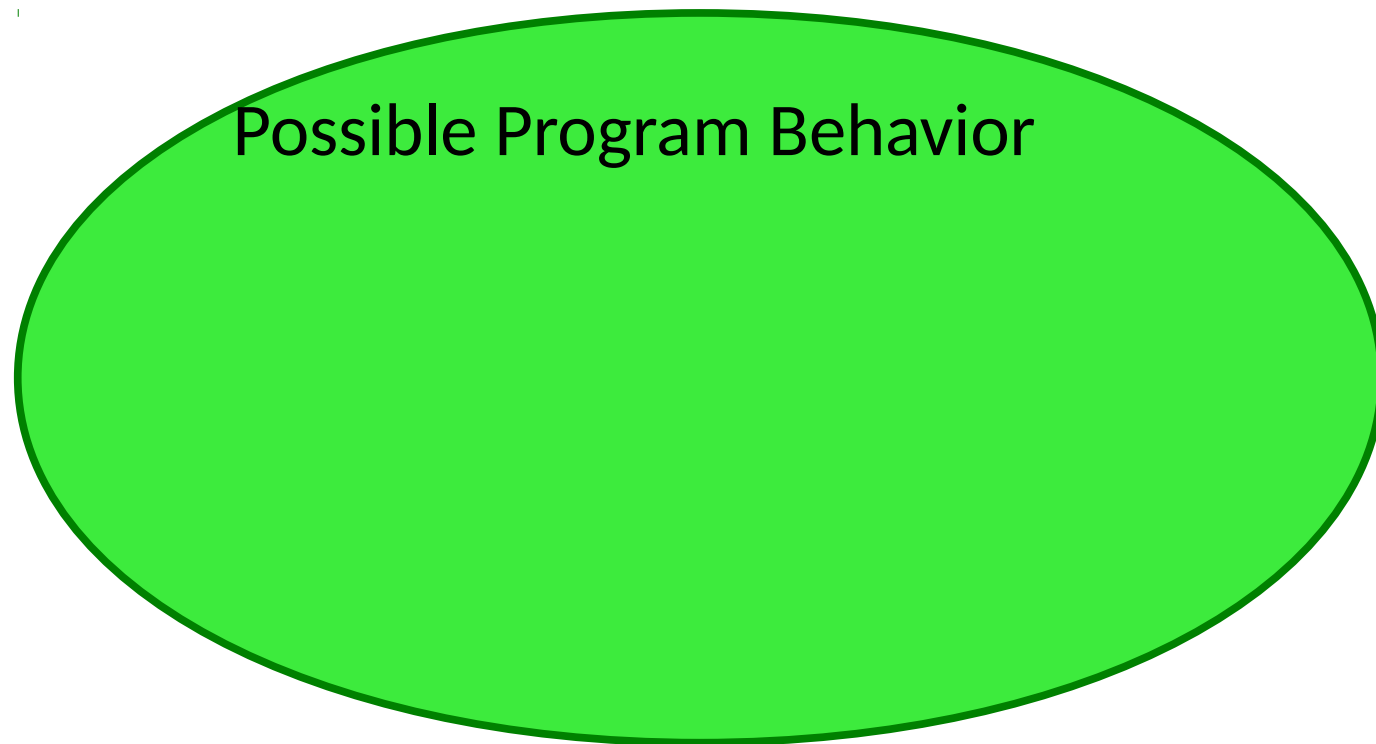
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Striking the right balance is key to a useful analysis

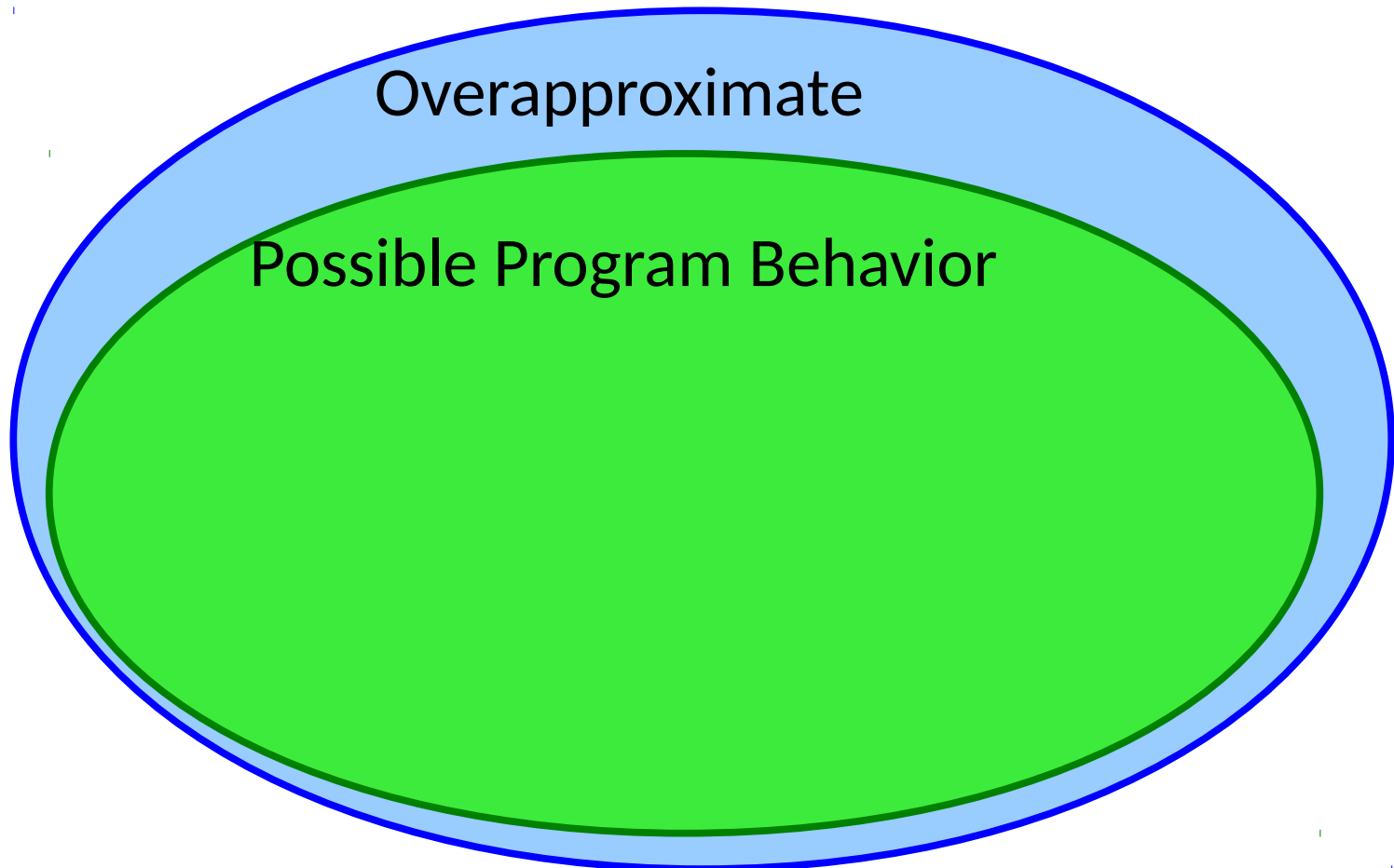
Approximation

Modeled program behaviors



Approximation

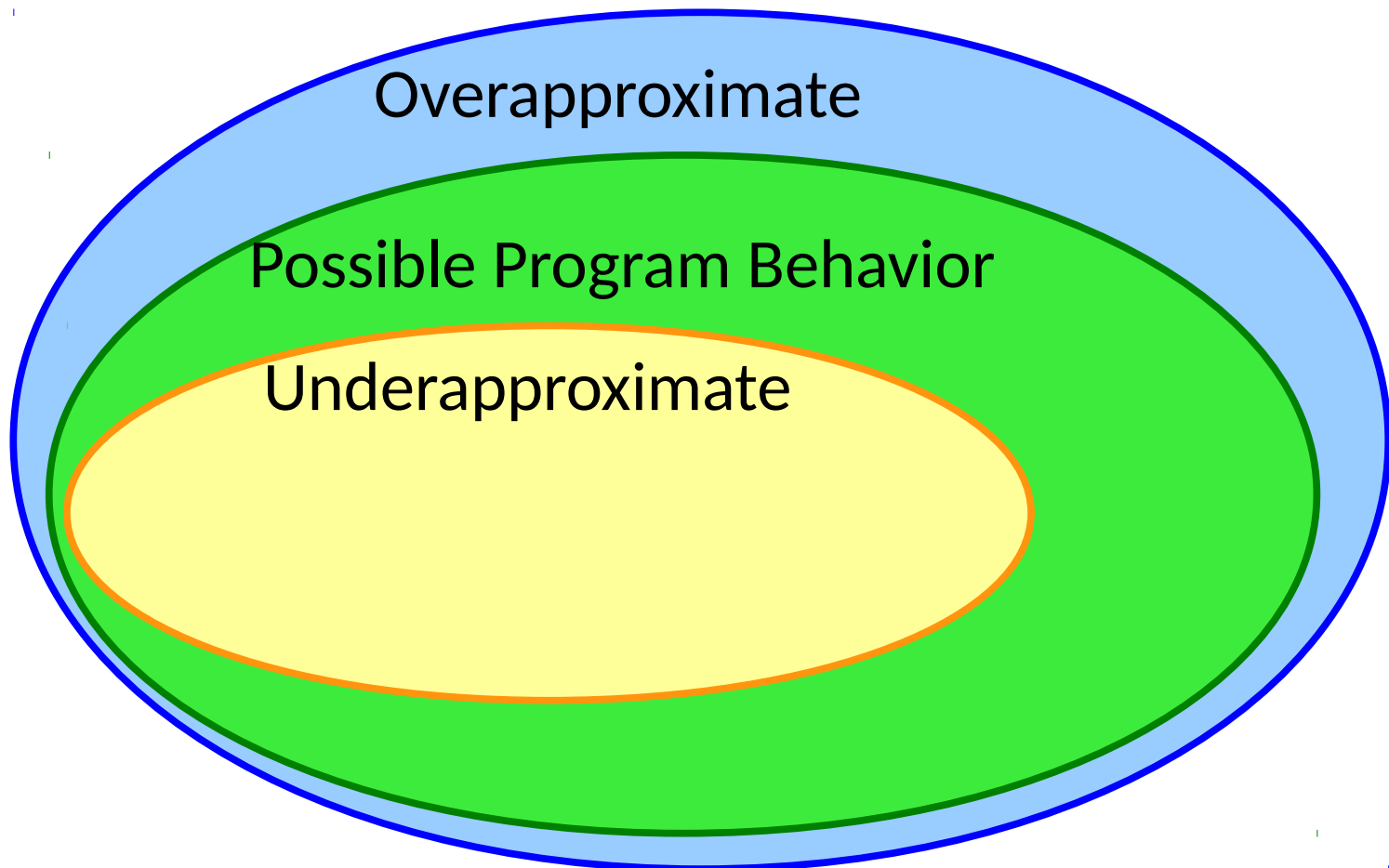
Modeled program behaviors



Consider some behaviors possible when they are not.

Approximation

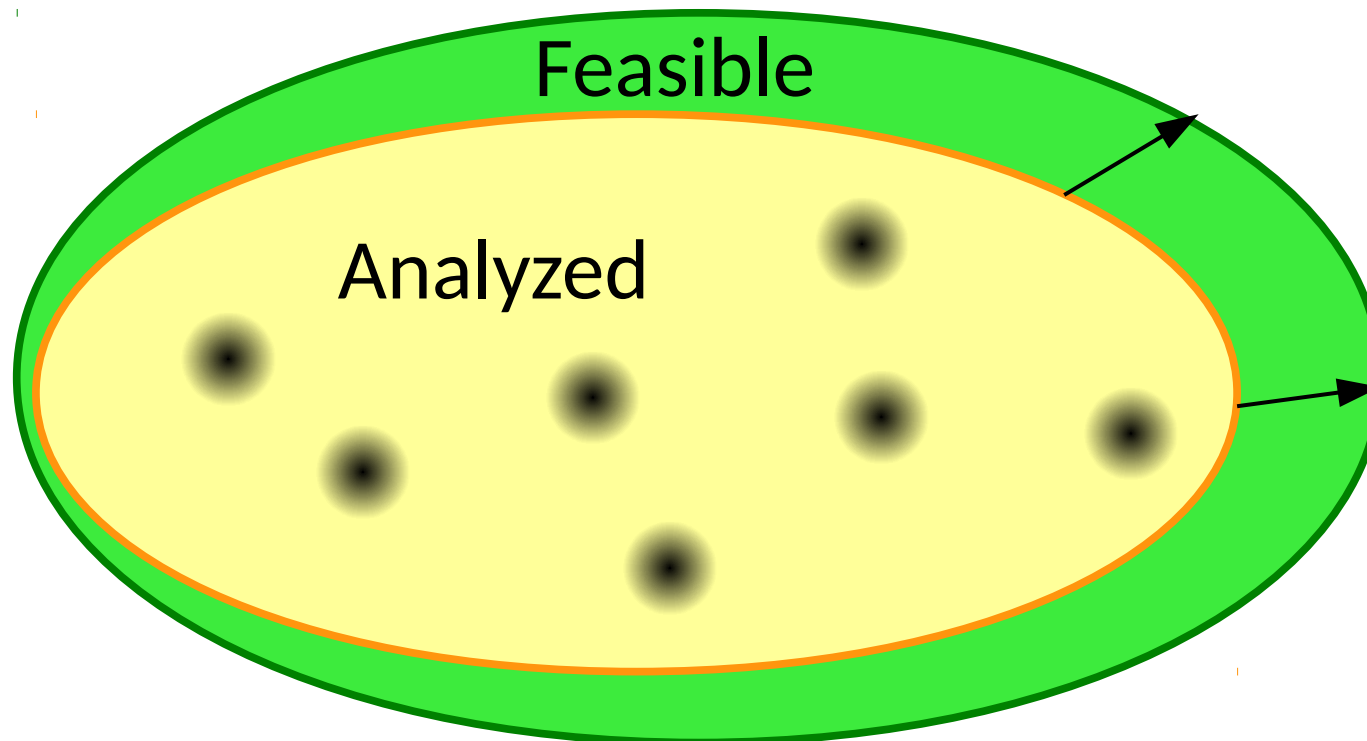
Modeled program behaviors



Ignore some behaviors that *are* possible.

Approximation

- Dynamic Analysis
 - Analyzed \subseteq Feasible
 - As # tests \uparrow , Analyzed \rightarrow Feasible

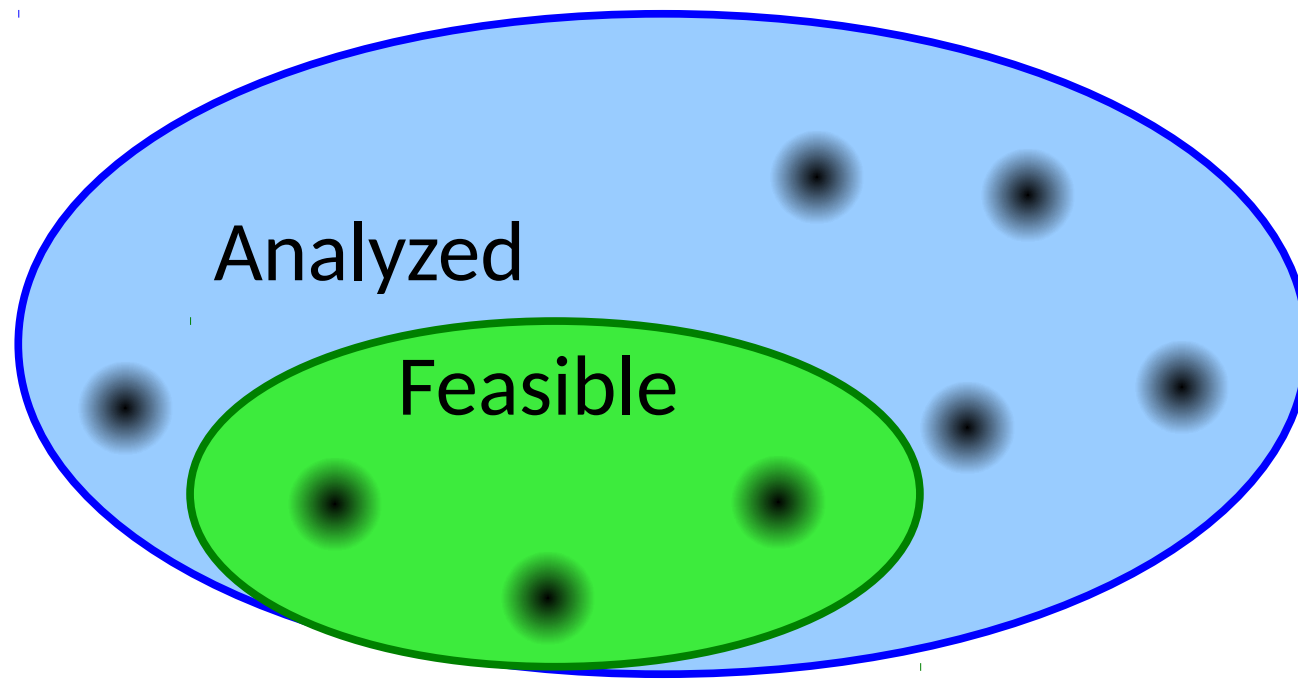


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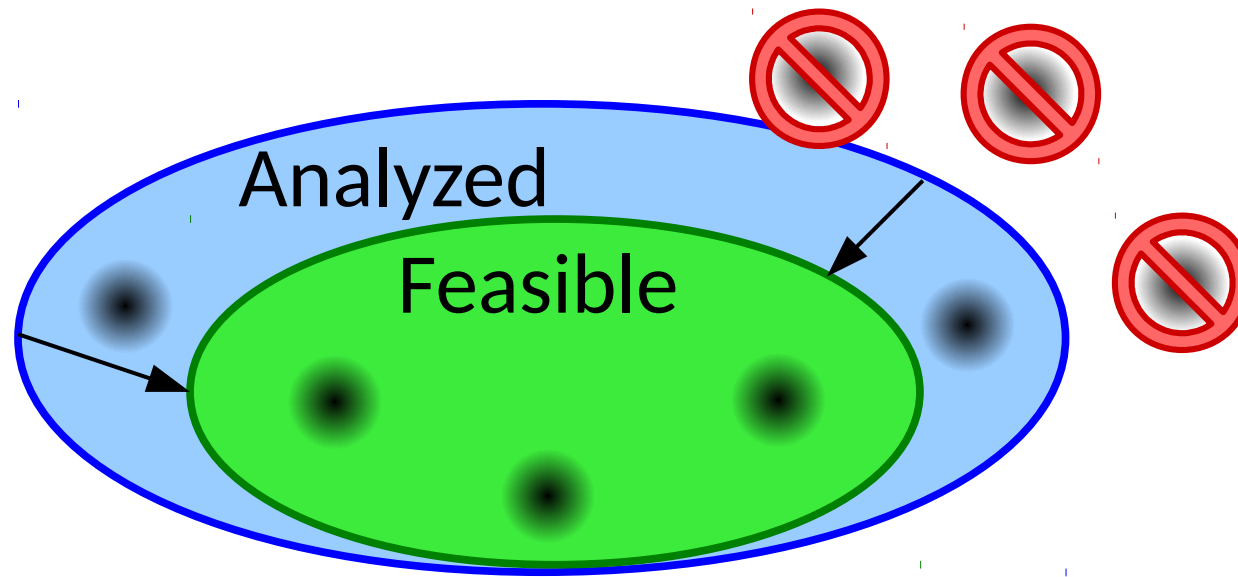
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 - As infeasible paths \downarrow , Analyzed \rightarrow Feasible
- The two areas complement each other
 - Static analysis can help generate useful tests
 - Dynamic analysis can help identify infeasibility

Abstract Interpretation

Q: Is a particular number ever negative?
– Might be an offset into invalid memory!

Approximate the program's behavior

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Approximate the program's behavior

- **Concrete** domain: integers
- **Abstract** domain: $\{-,0,+\} \cup \{\top,\perp\}$

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Q: Is a particular number ever negative?
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Approximate the program's behavior

- **Concrete** domain: integers
- **Abstract** domain: $\{-,0,+\} \cup \{\top,\perp\}$

concrete(x) = 5 \mapsto abstract(x) = +

concrete(y) = -3 \mapsto abstract(y) = -

concrete(z) = 0 \mapsto abstract(z) = 0

Combines sets of the concrete domain

Abstract Interpretation

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- **+** + **+** → **+**

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- ...

- **+** + **-** → **T** (unknown / might vary)

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Abstract Interpretation

- **Transfer Functions** show how to evaluate this approximated program:

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This type of approximation is called
abstract interpretation.

Abstract Interpretation

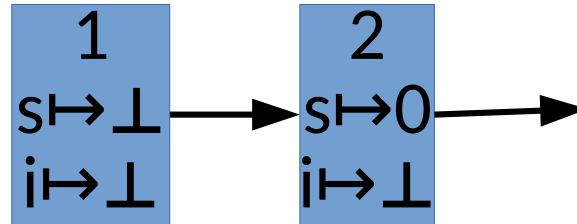
```
1) sum = 0  
2) i = 1
```

```
3) if i < N
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```
4) i = i + 1  
5) sum = sum + i
```

```
6) print(sum)  
7) print(i)
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Abstract Interpretation



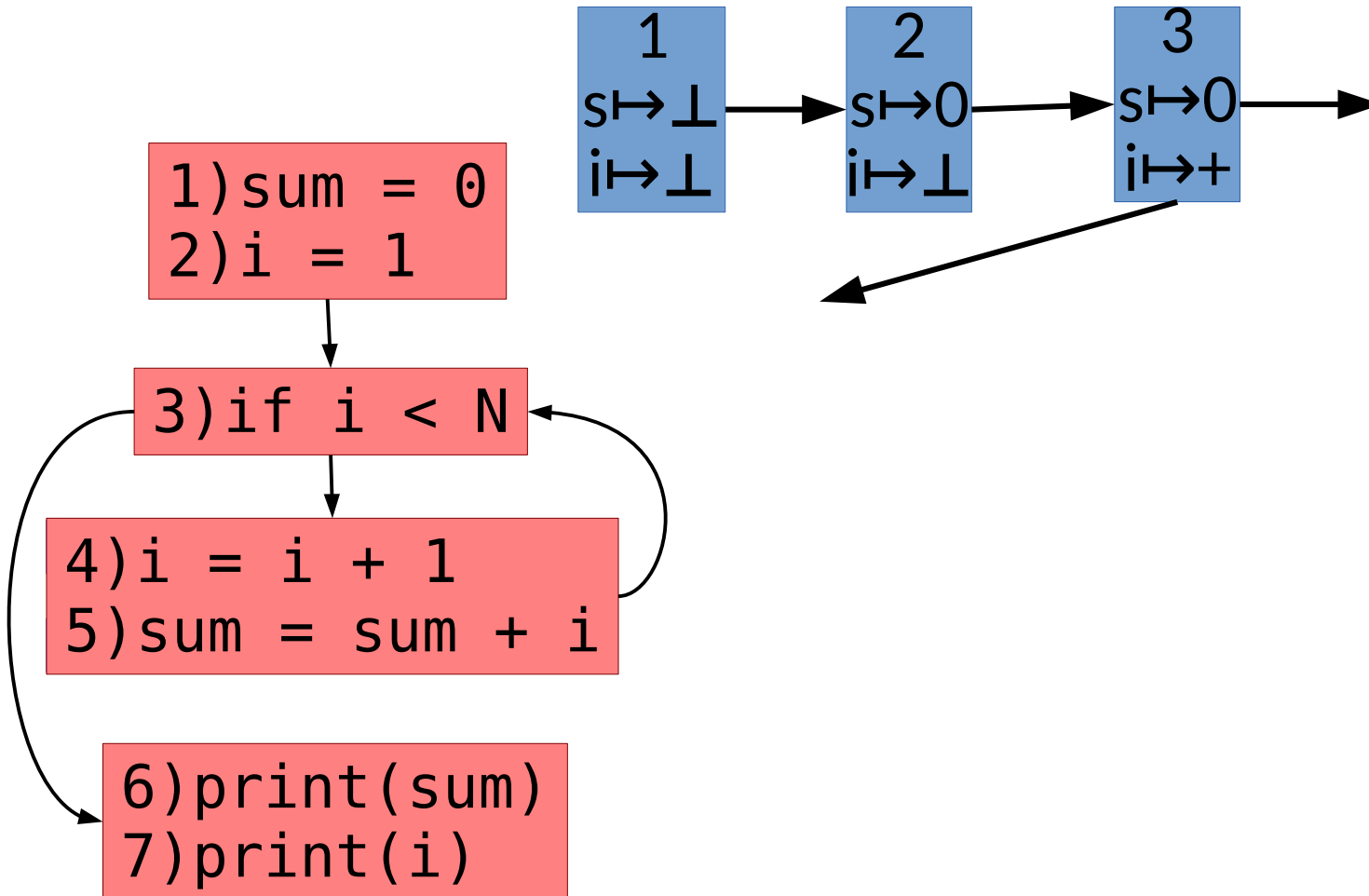
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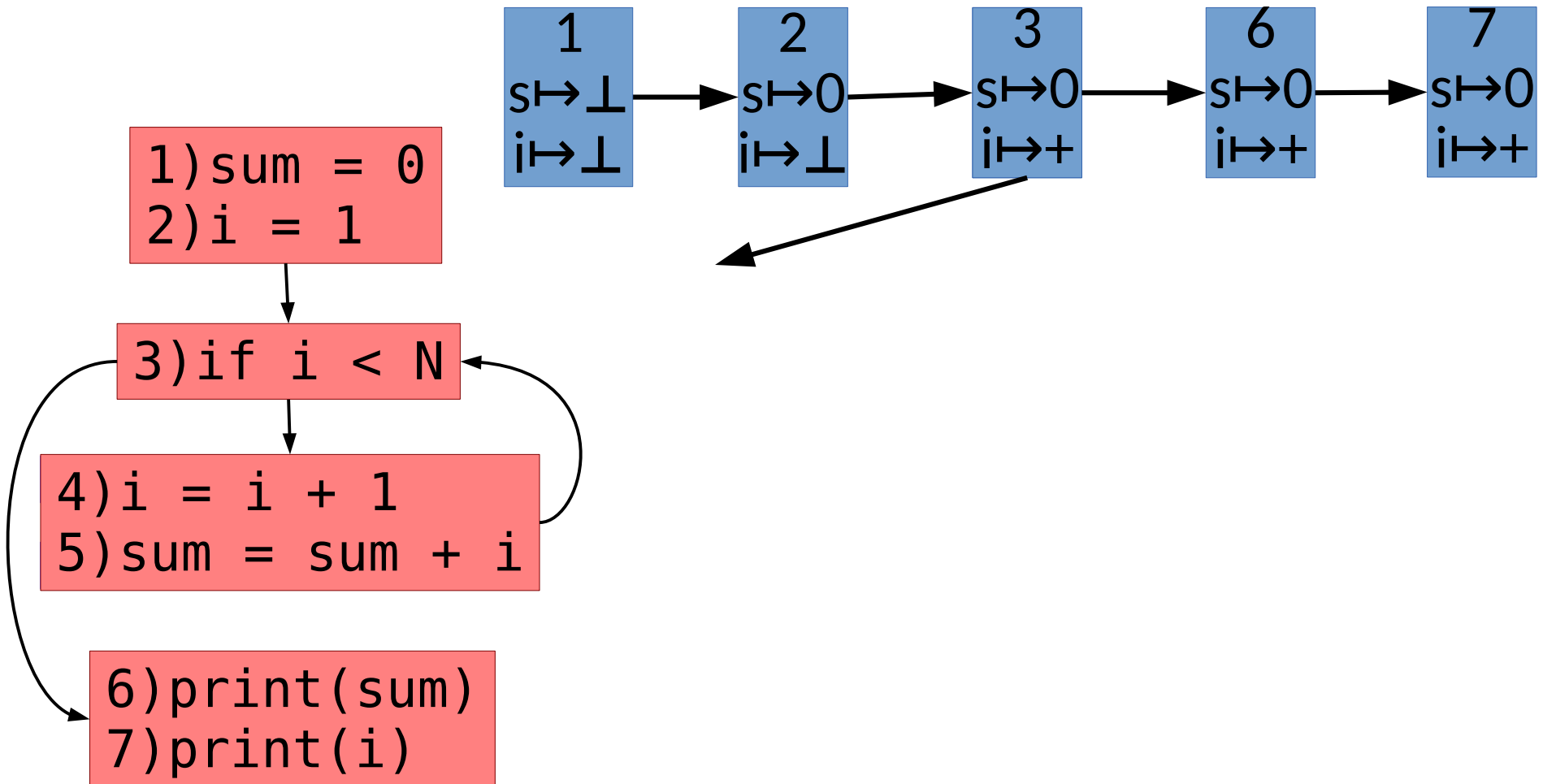
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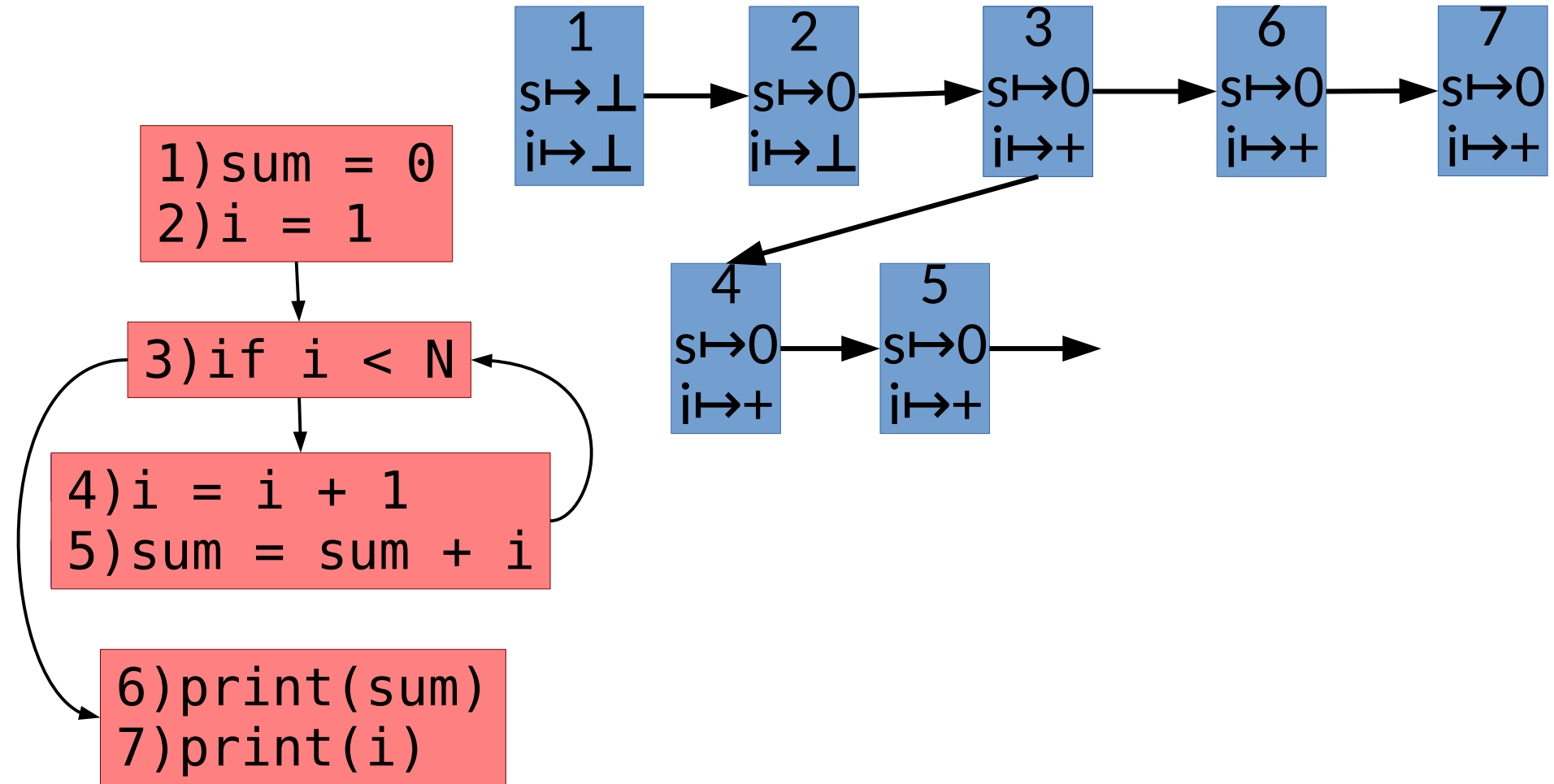
Abstract Interpretation



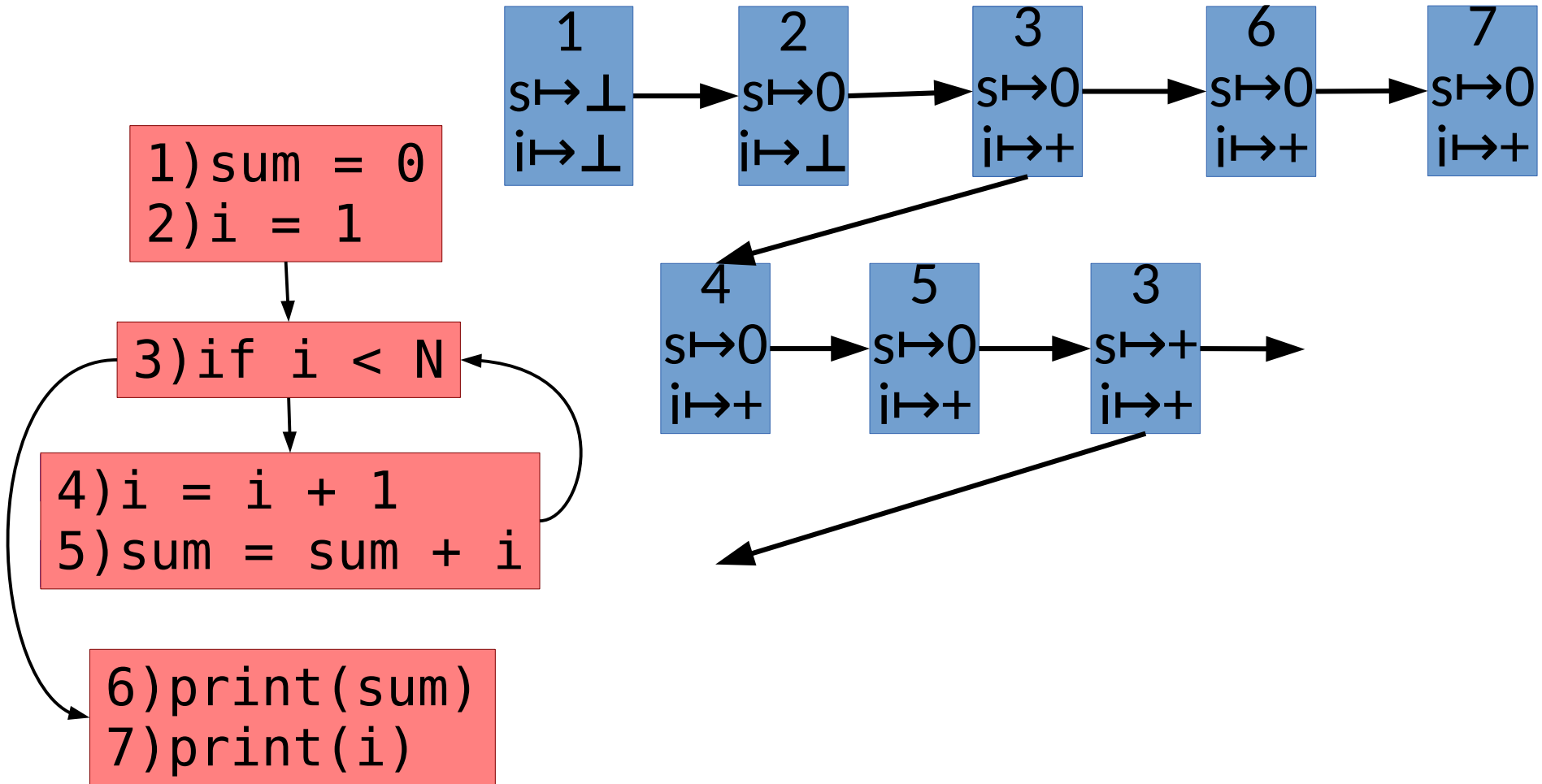
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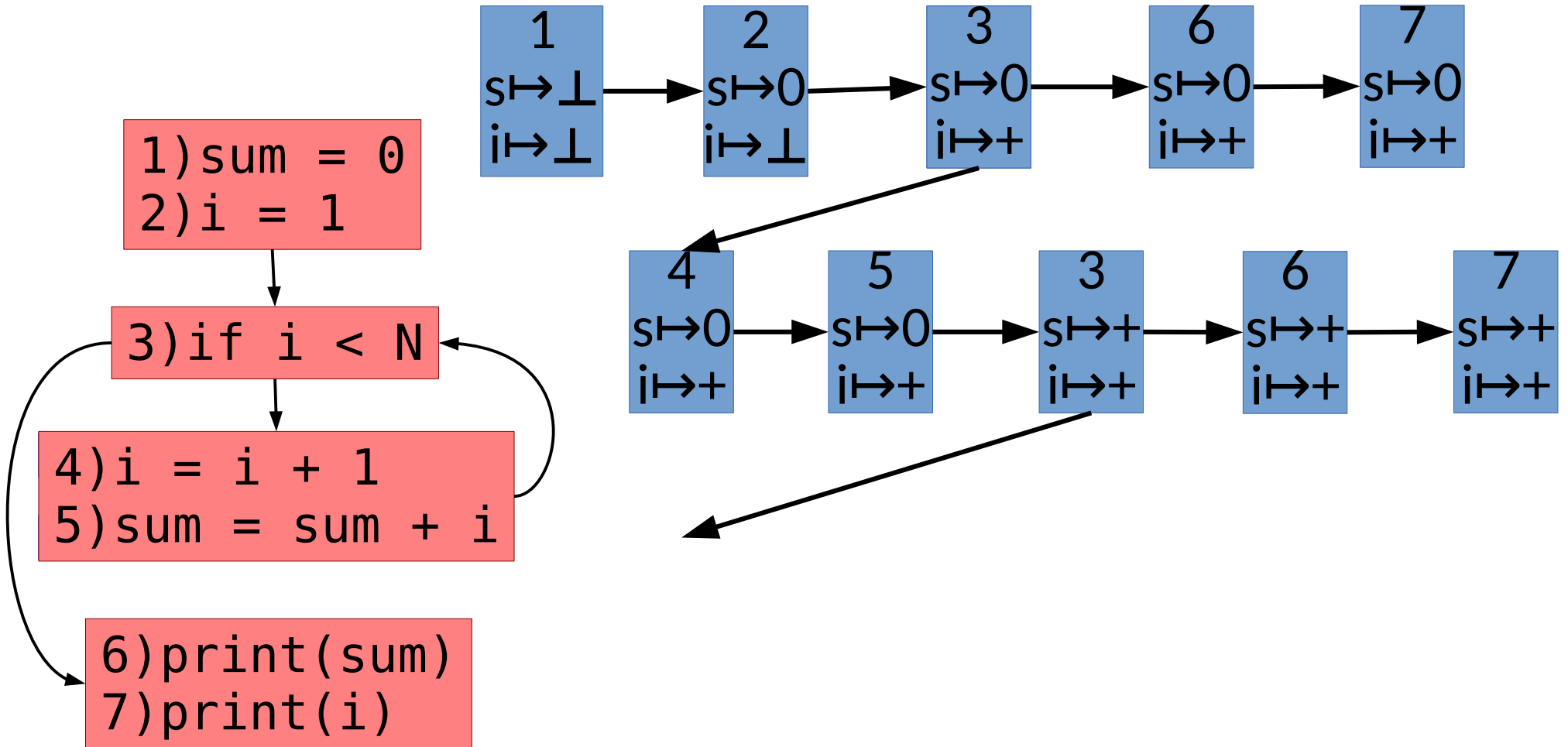
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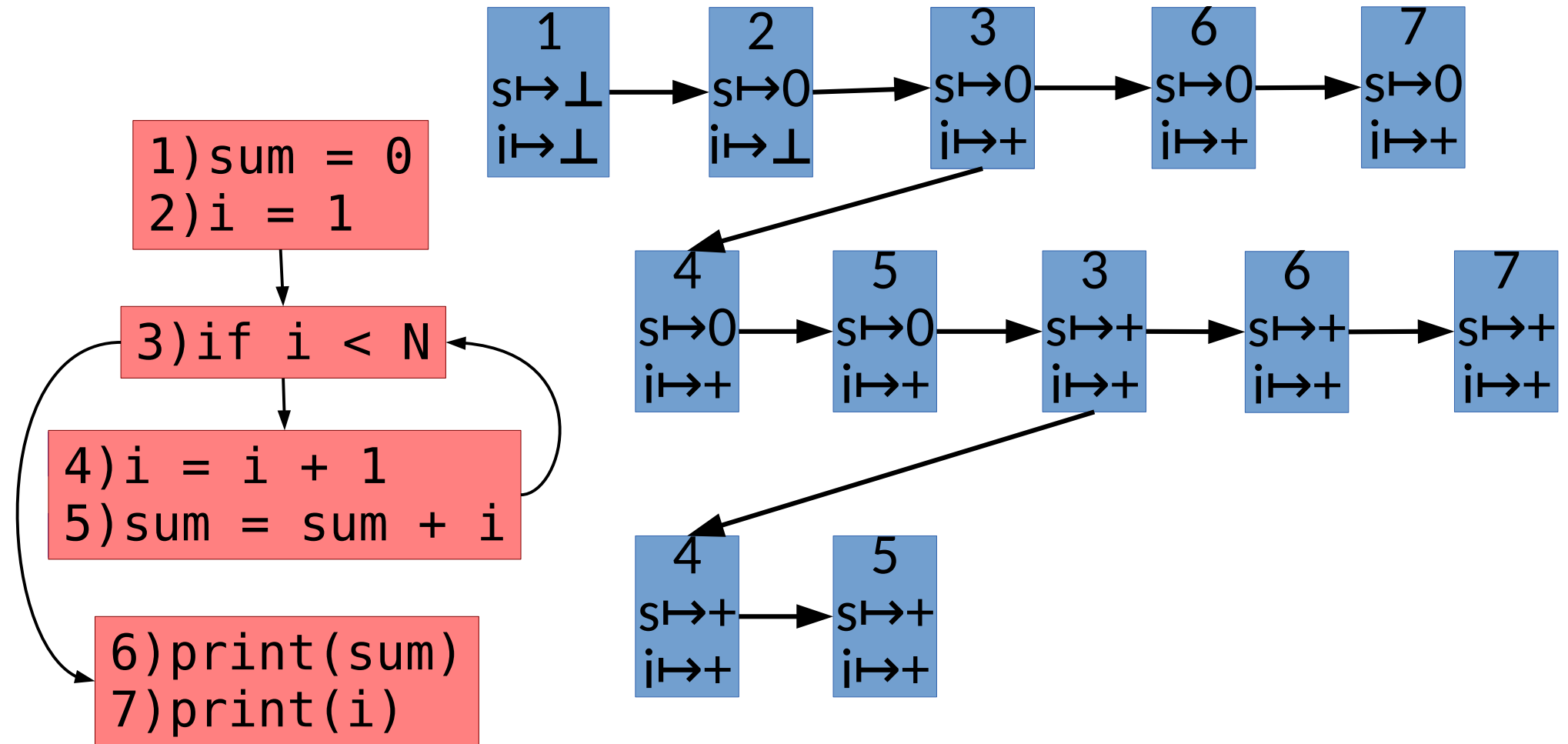
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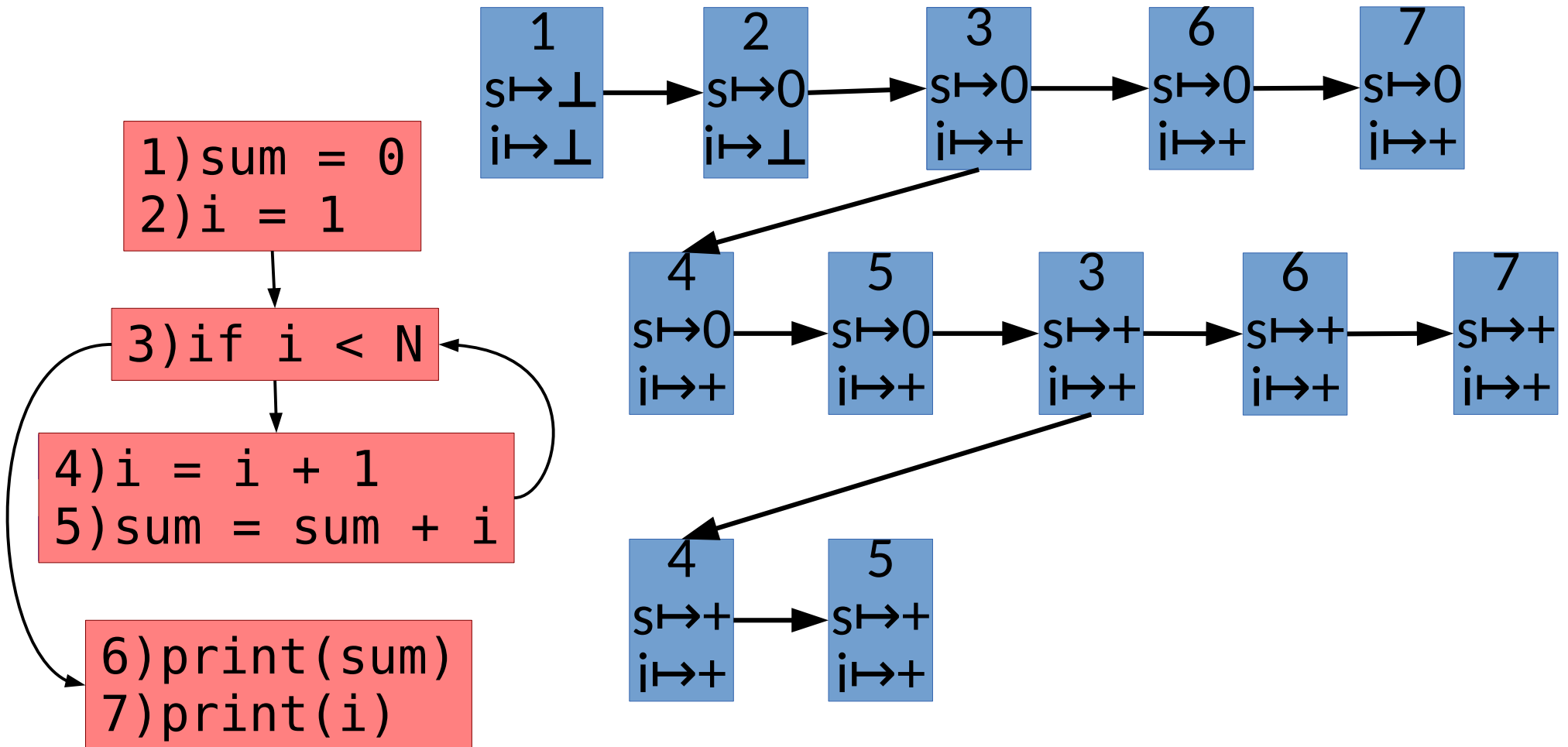
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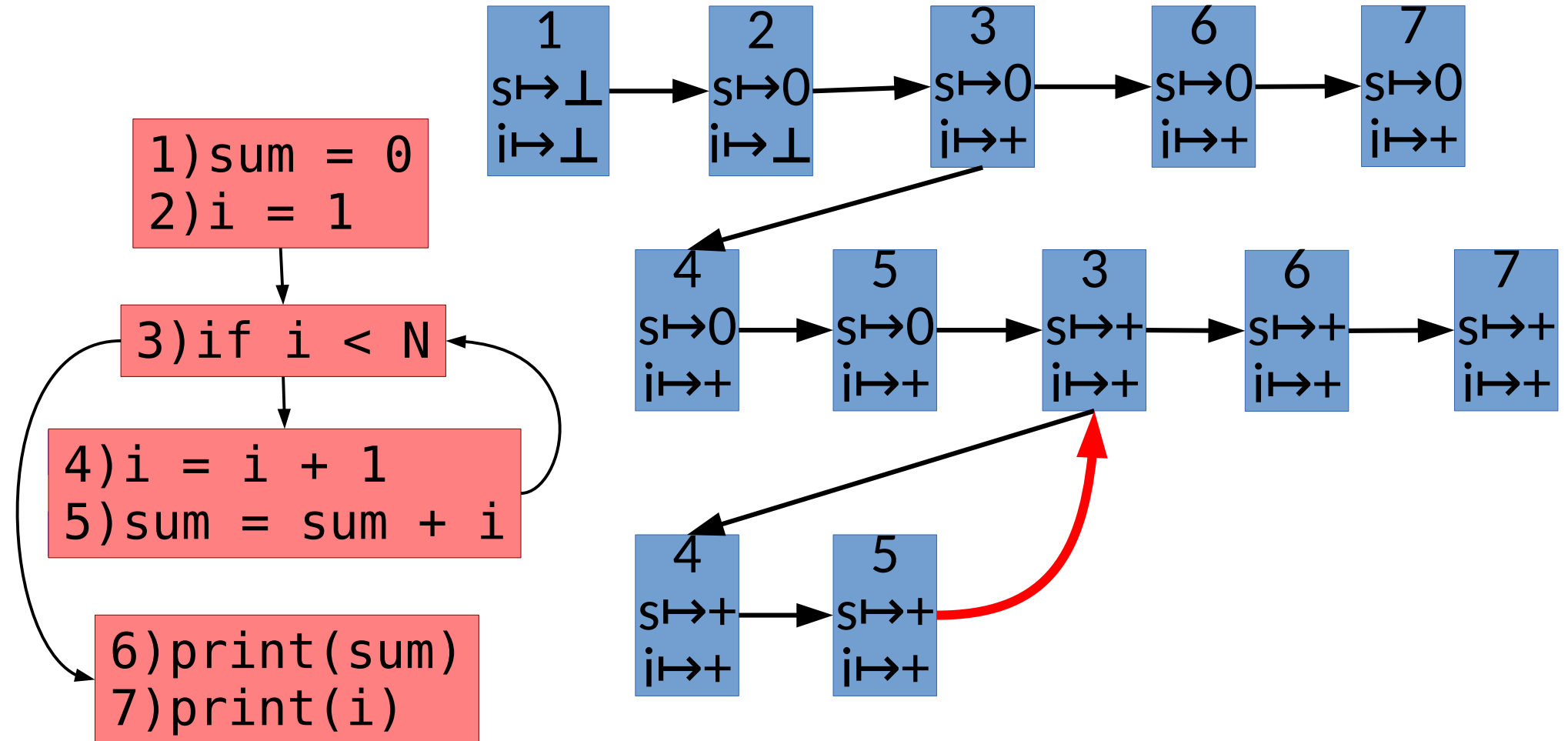


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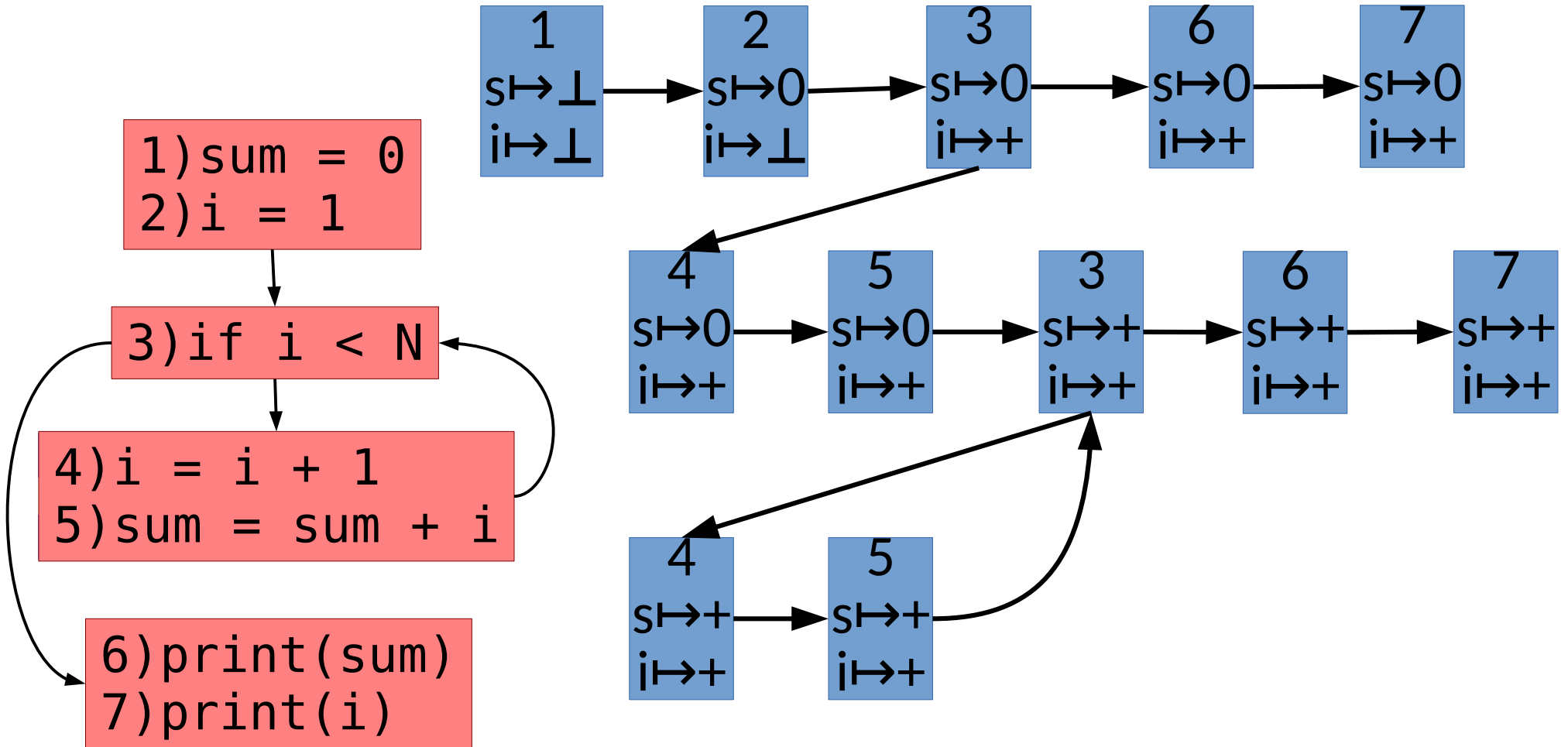


Does the process ever end?

Abstract Interpretation



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Can the final sum ever be negative?

Abstract Interpretation

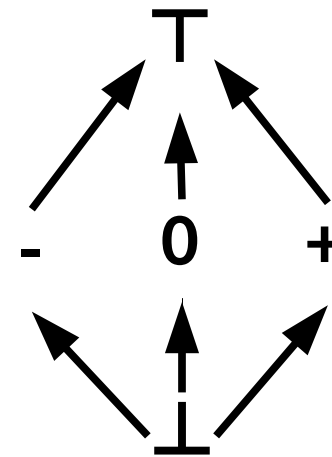
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- For basic analyses, use a **monotone framework**
Loosely: **<CFG, Transfer Function, Lattice Abstraction>**

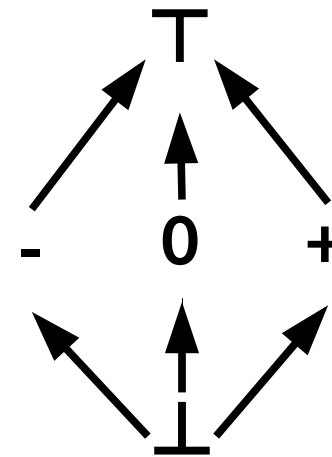
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Why does this specific example terminate?

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- But in theory a lattice need not be finite!
(ranges/intervals, linear constraints, ...)

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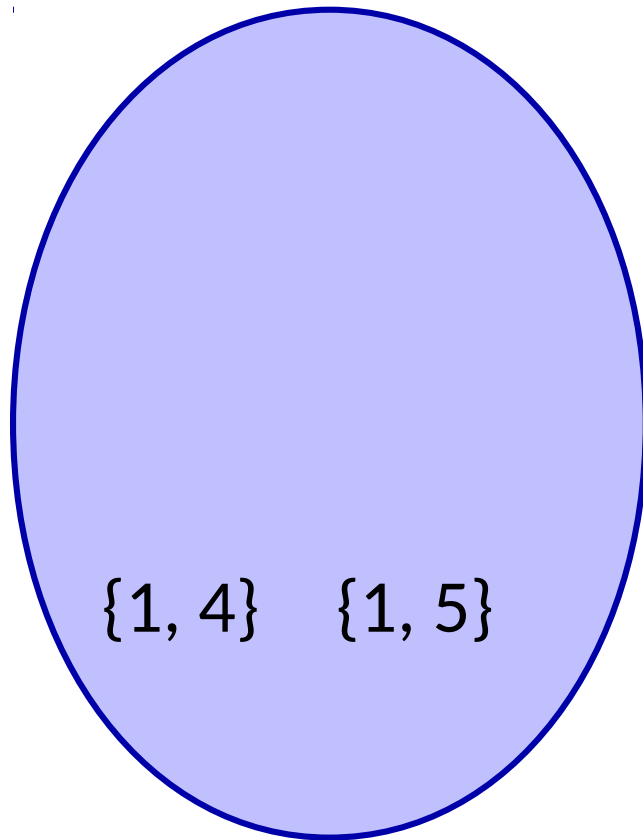
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 - Widening operators can still make it feasible (e.g., heuristically raise to \top)

Abstract Interpretation

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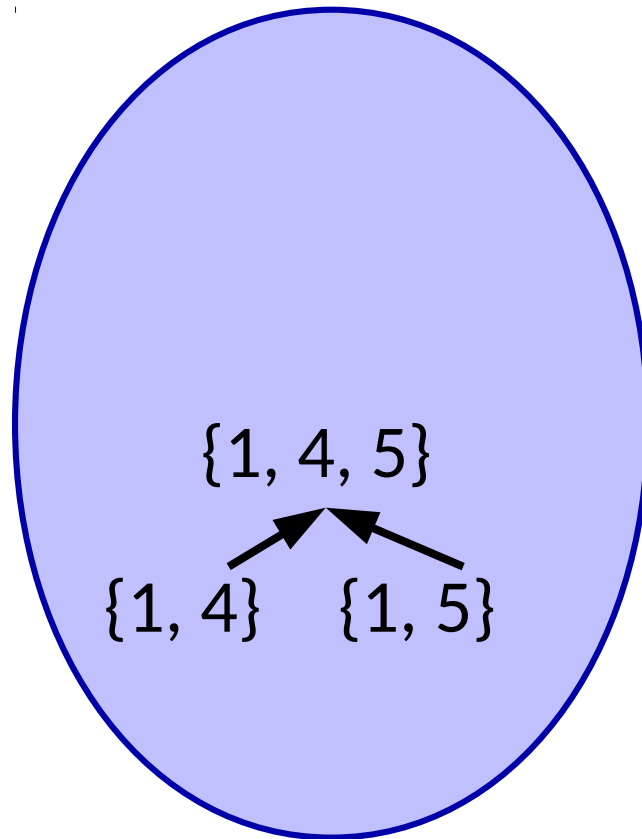
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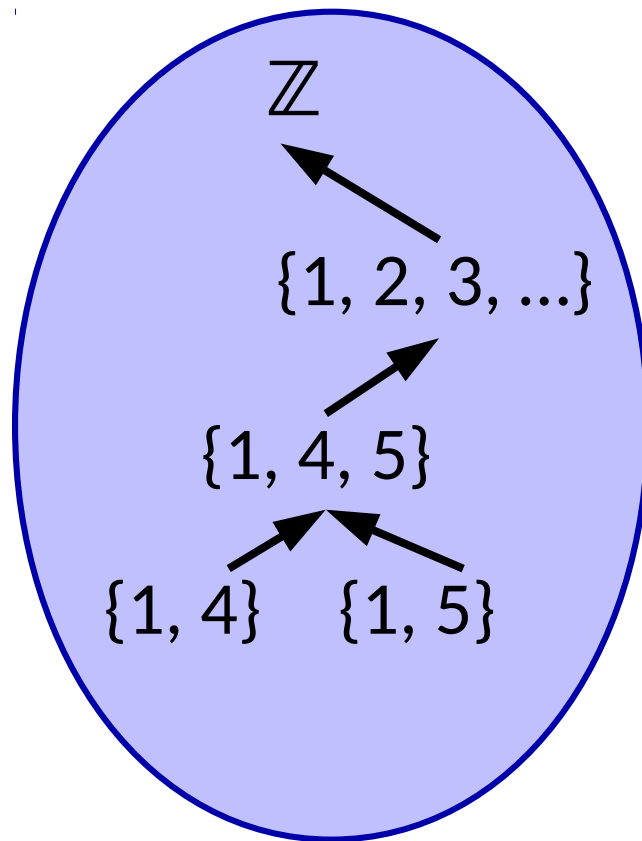
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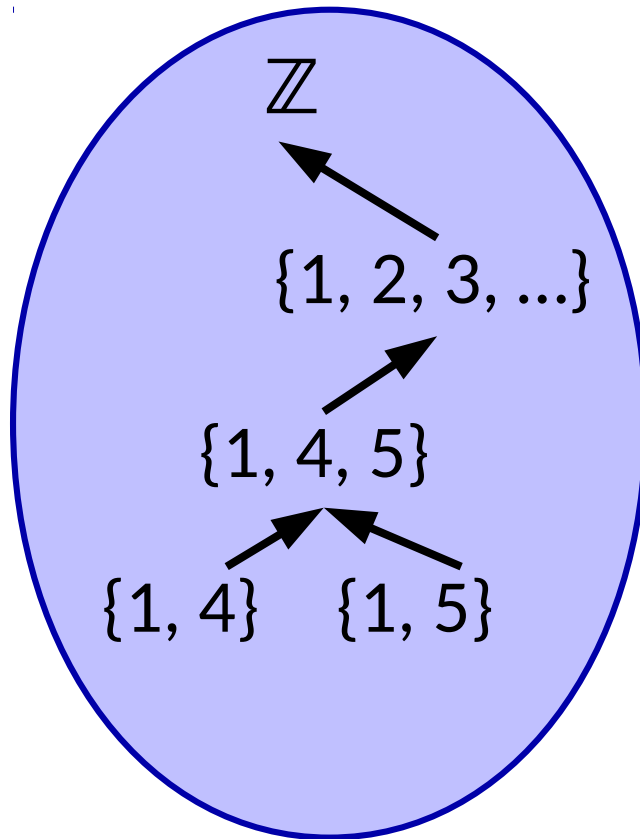
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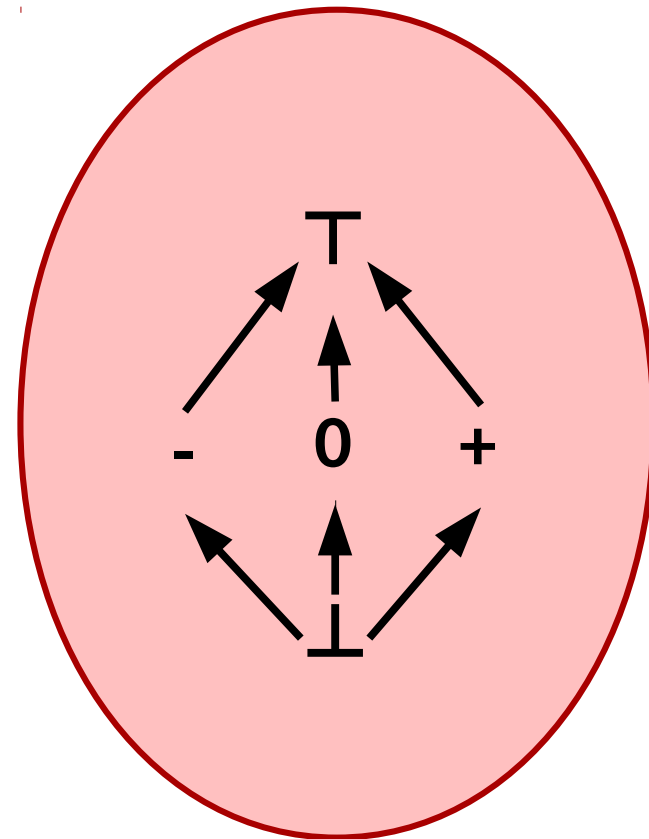
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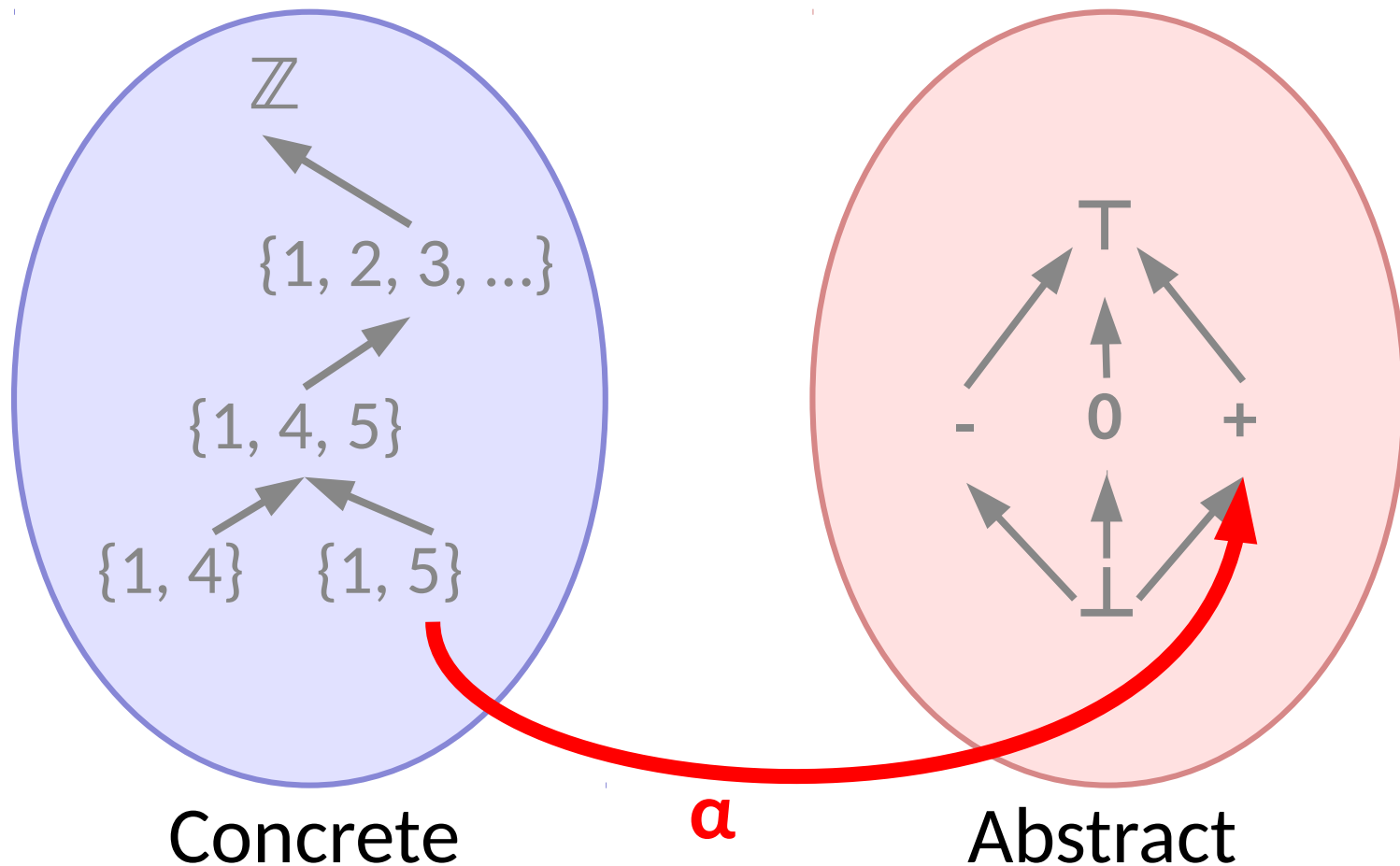
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Abstract

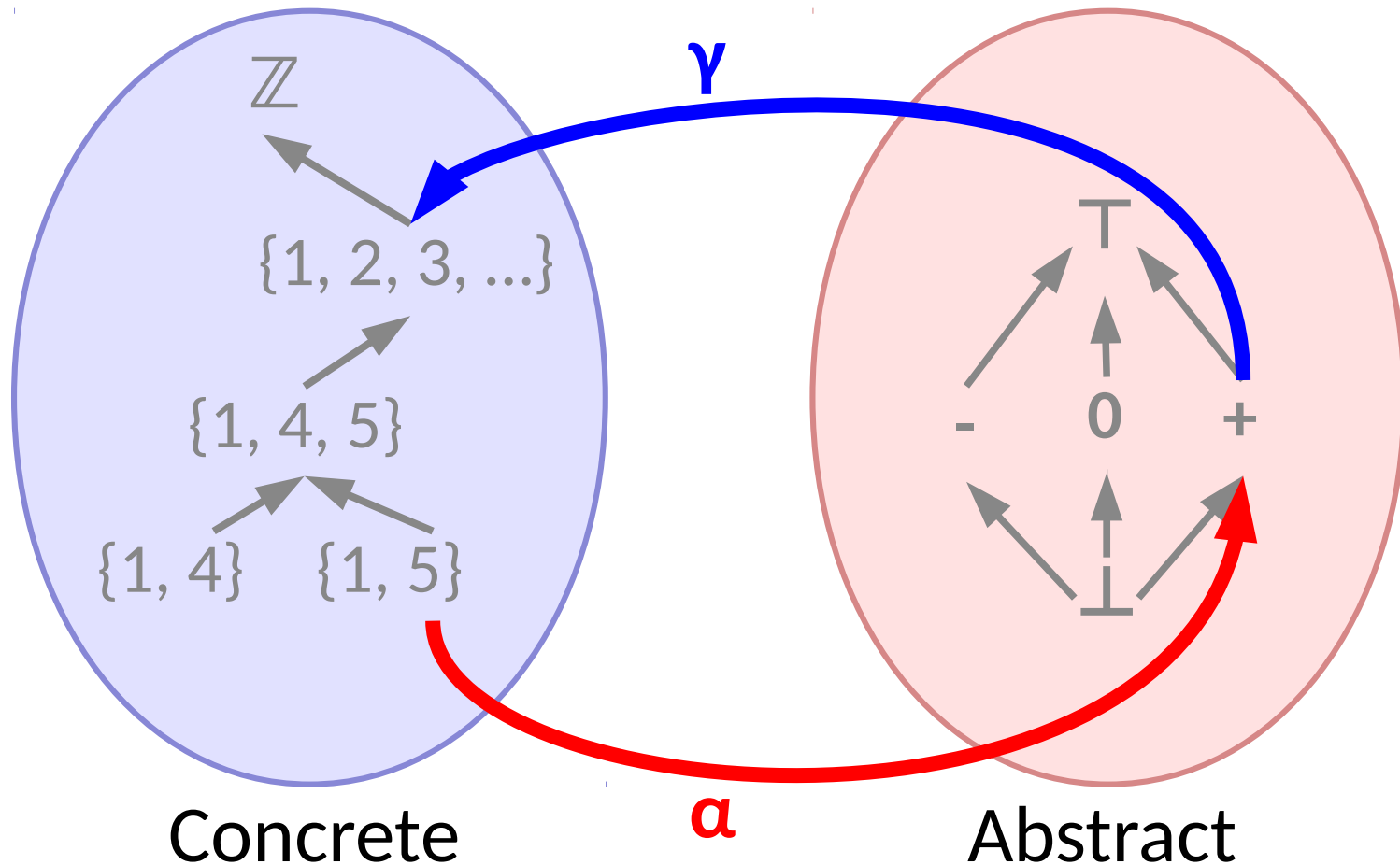
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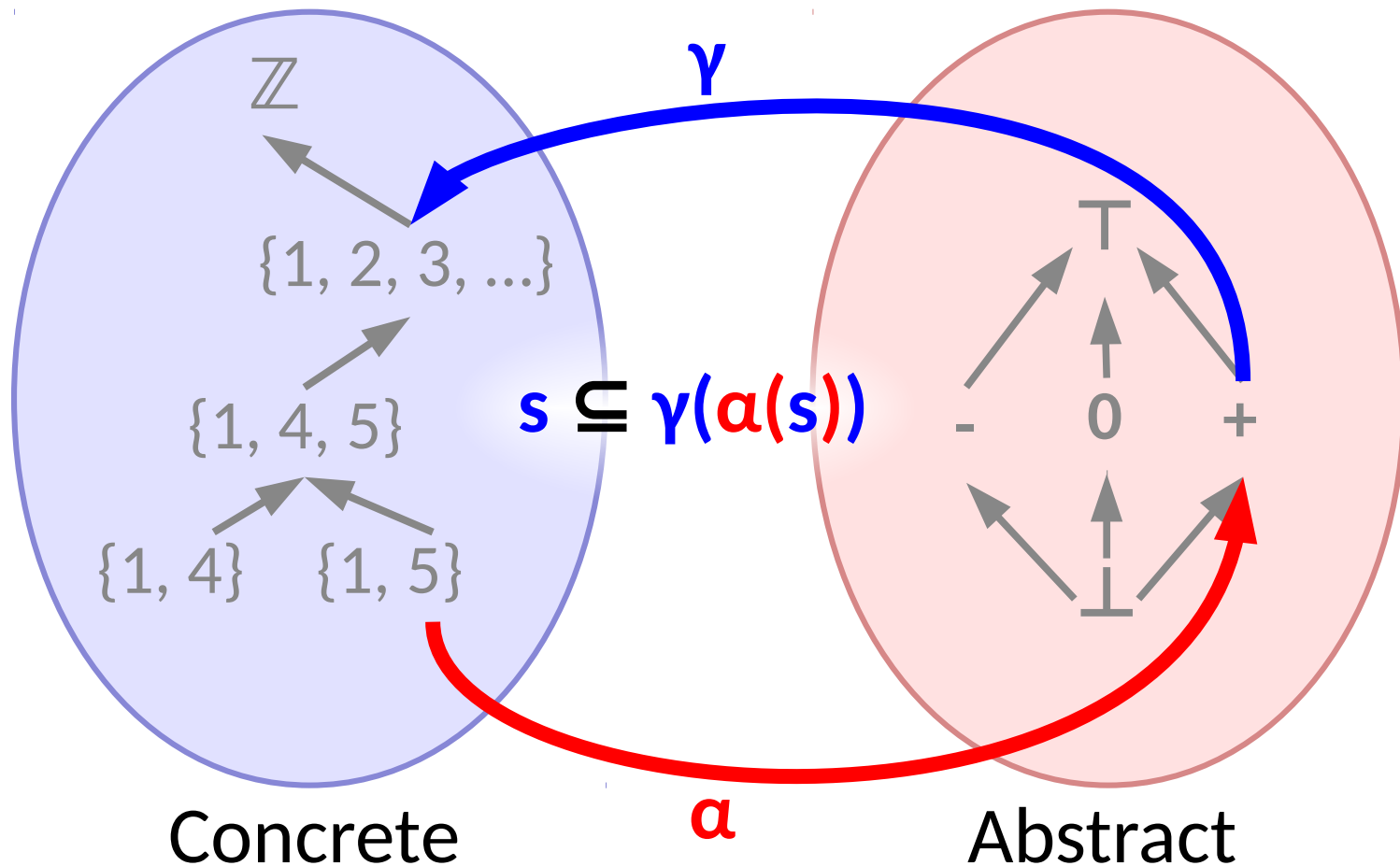
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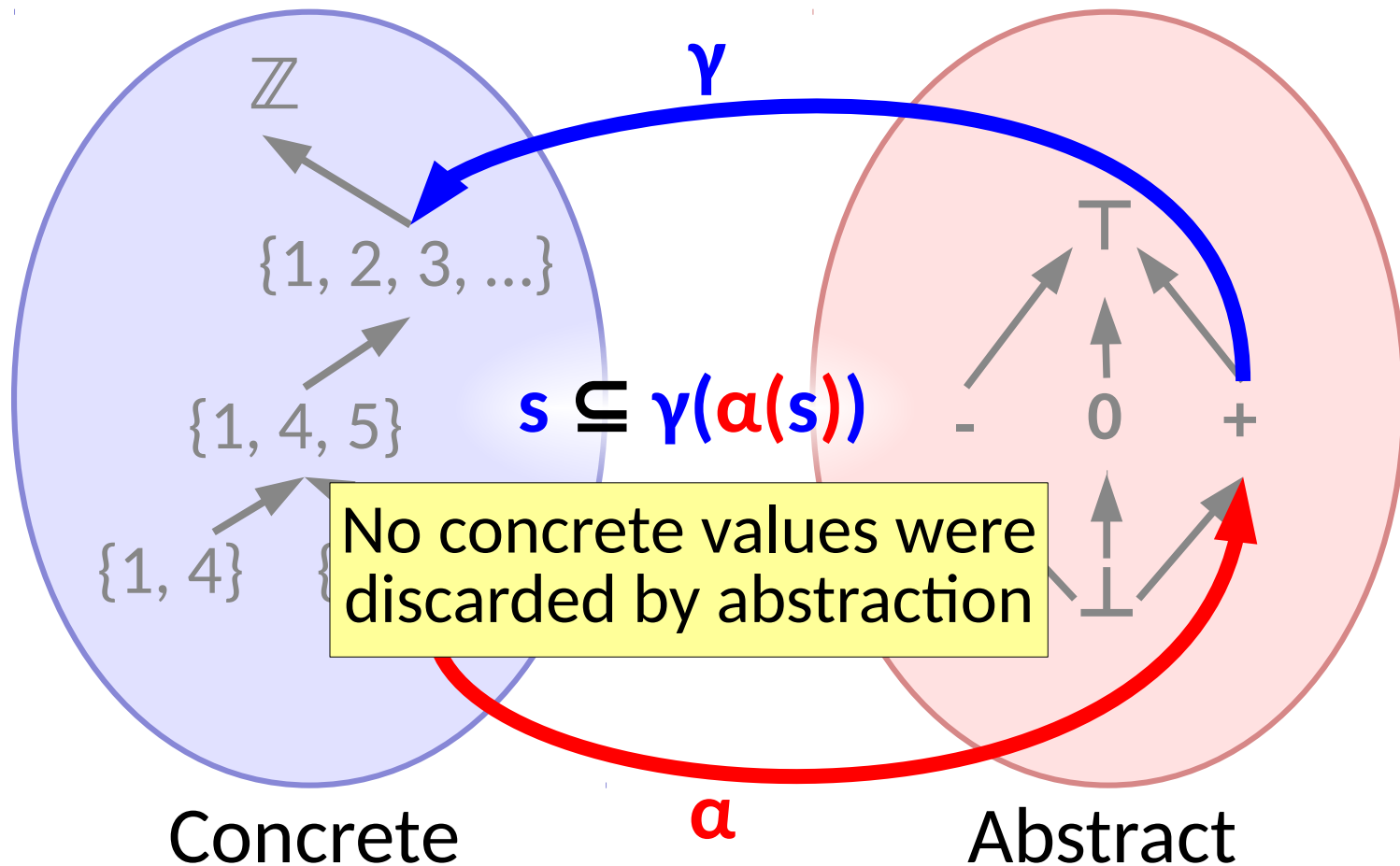
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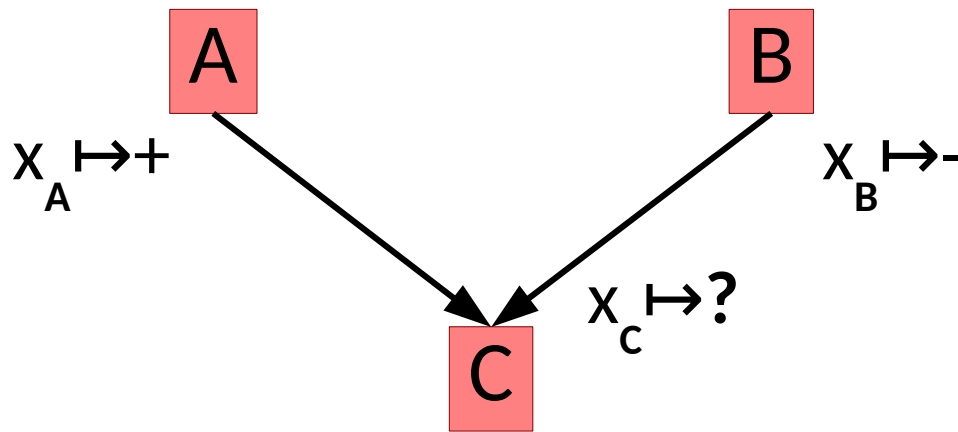
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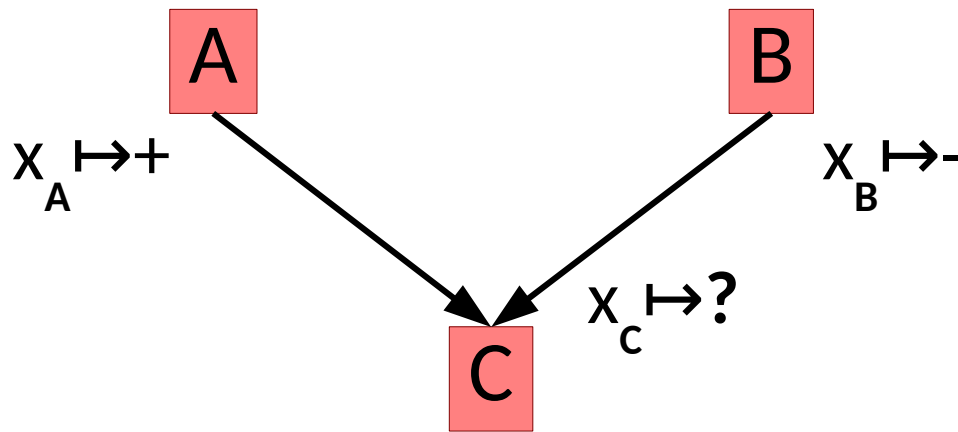
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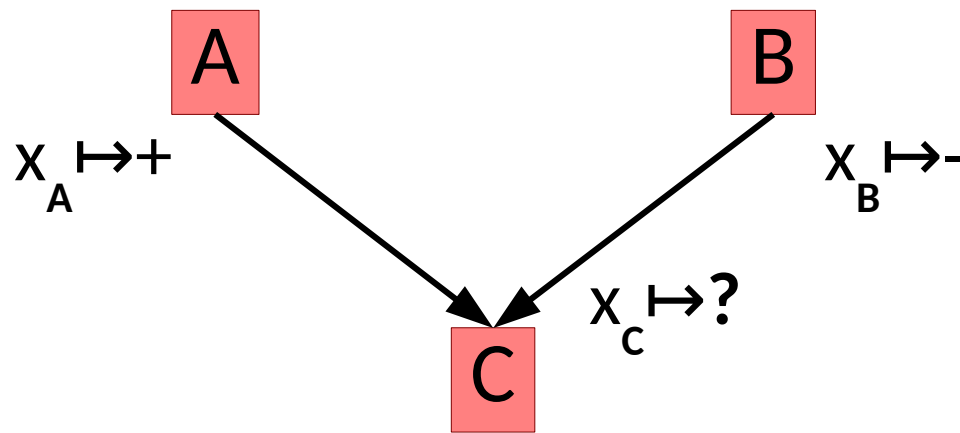
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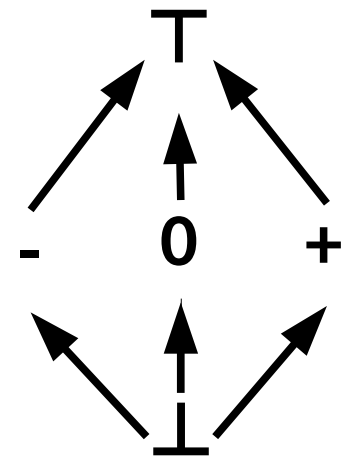
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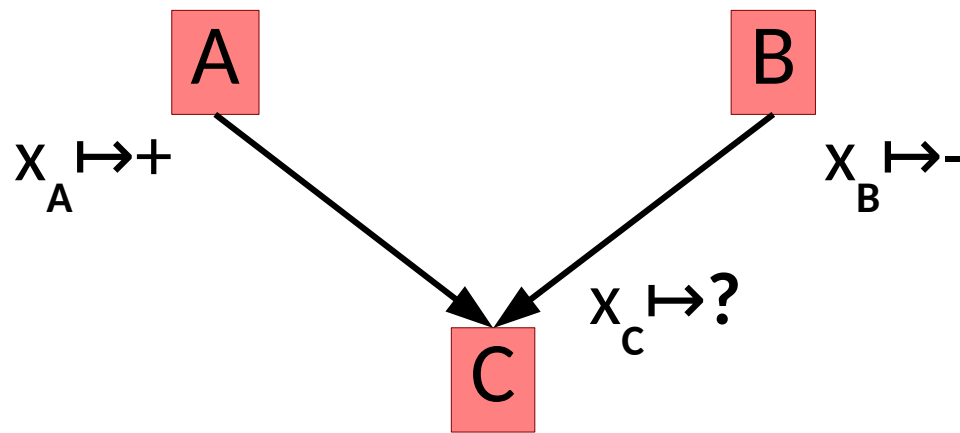


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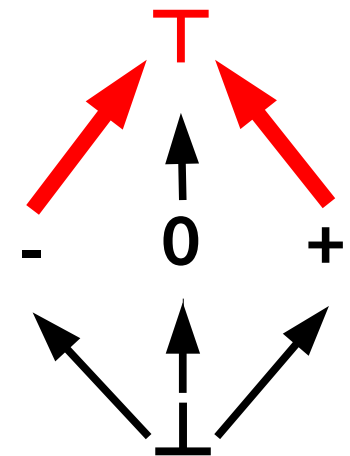


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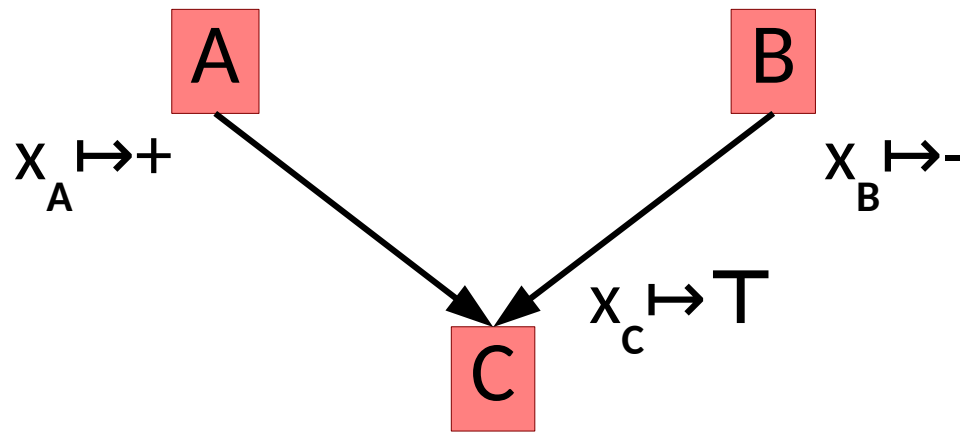


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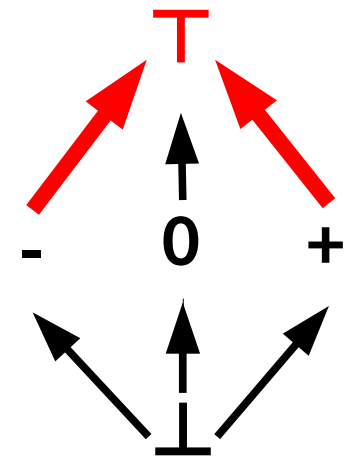


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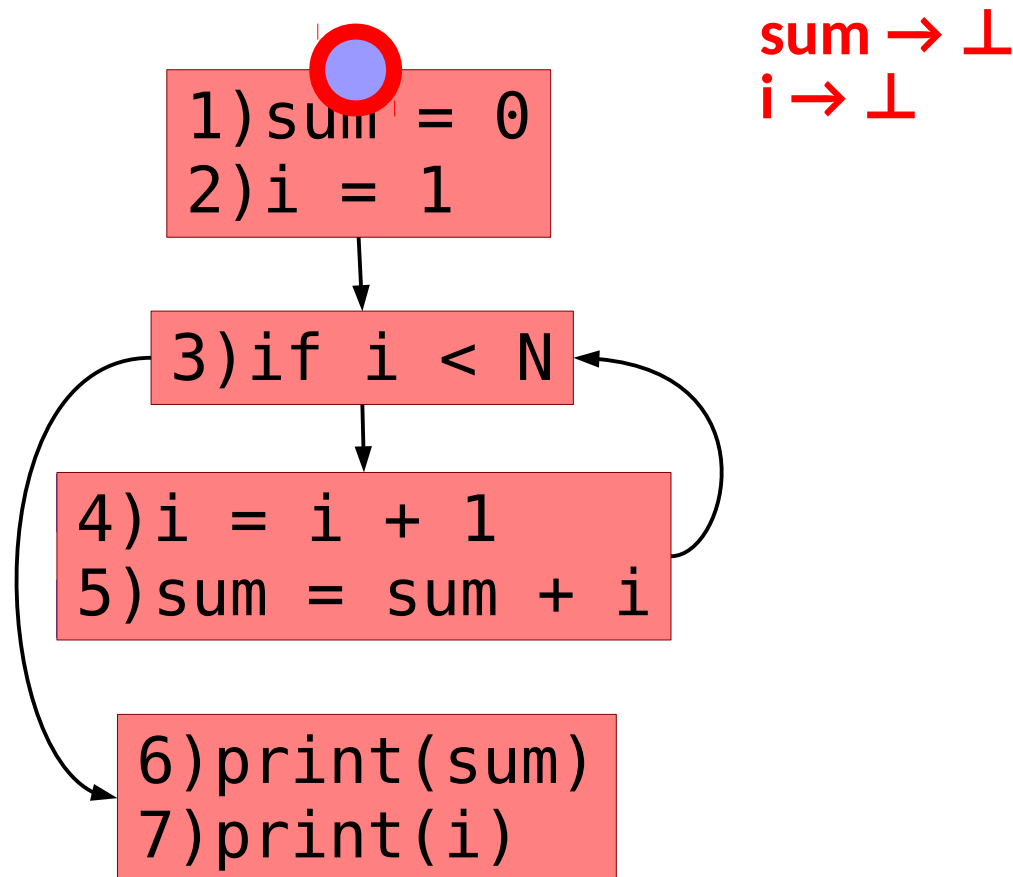


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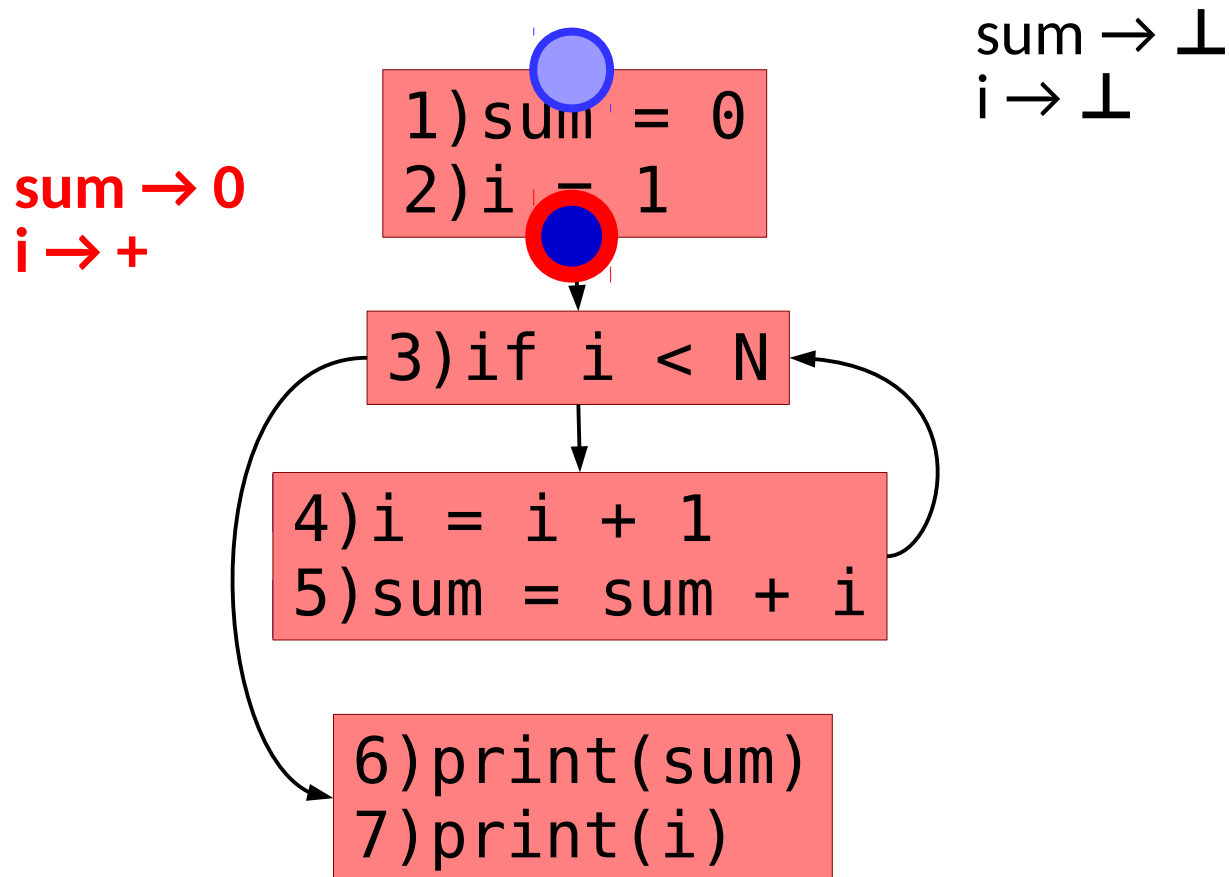
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- Now model the abstract program state and propagate through the CFG.



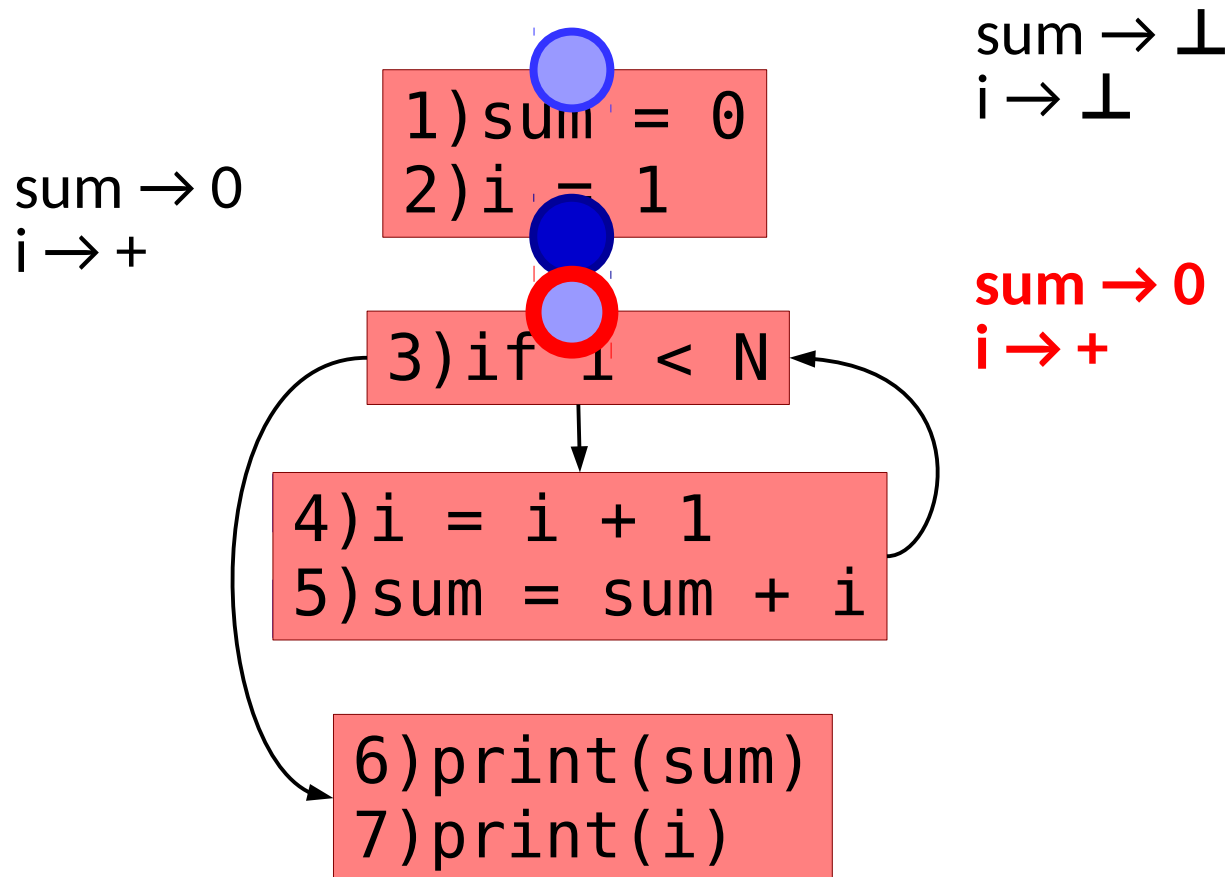
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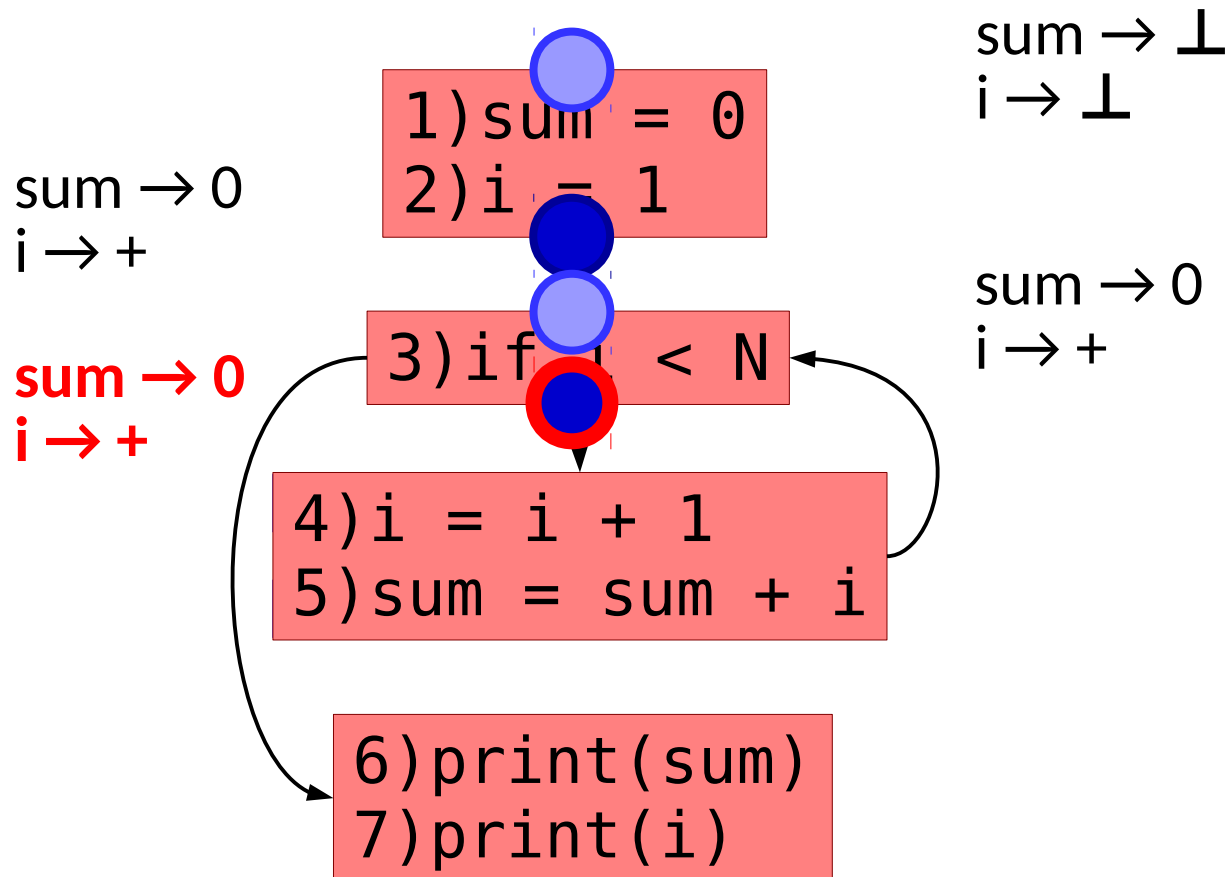
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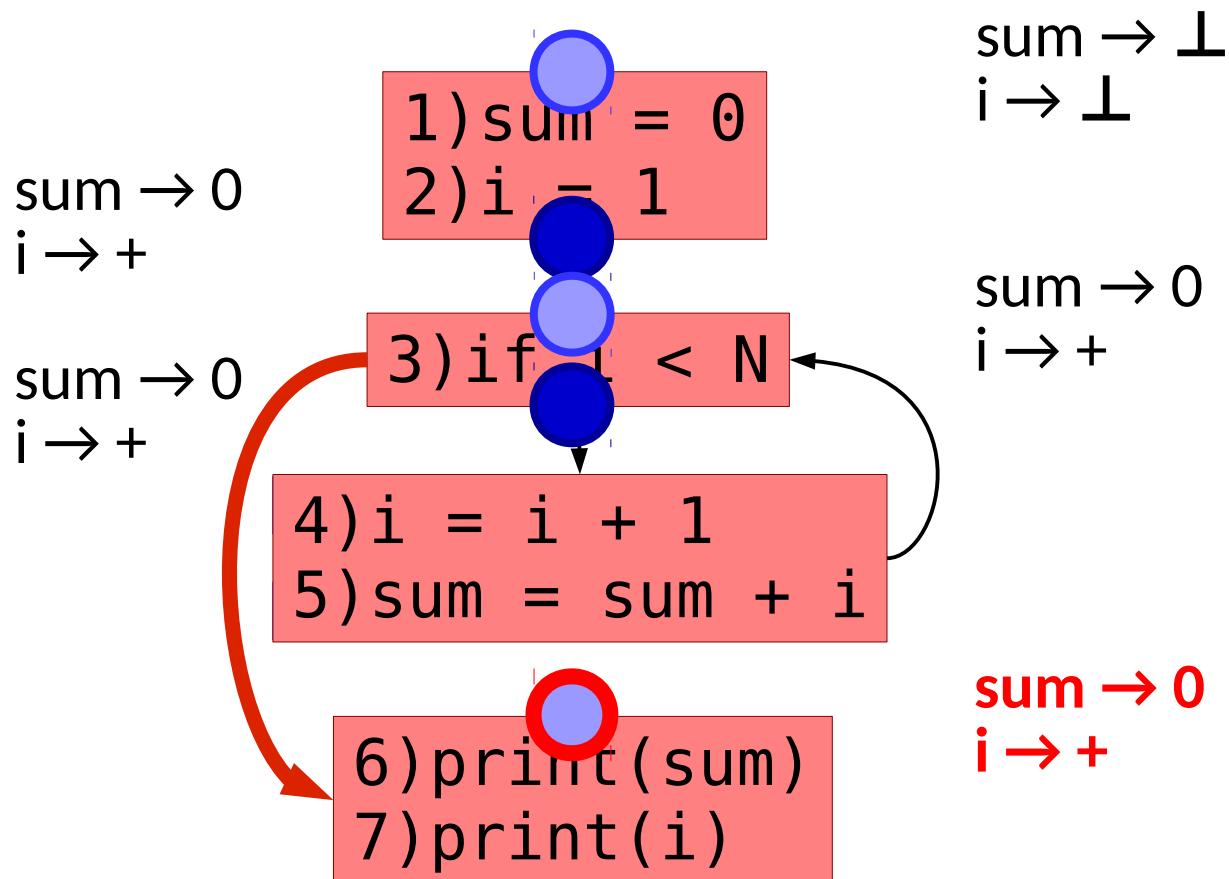
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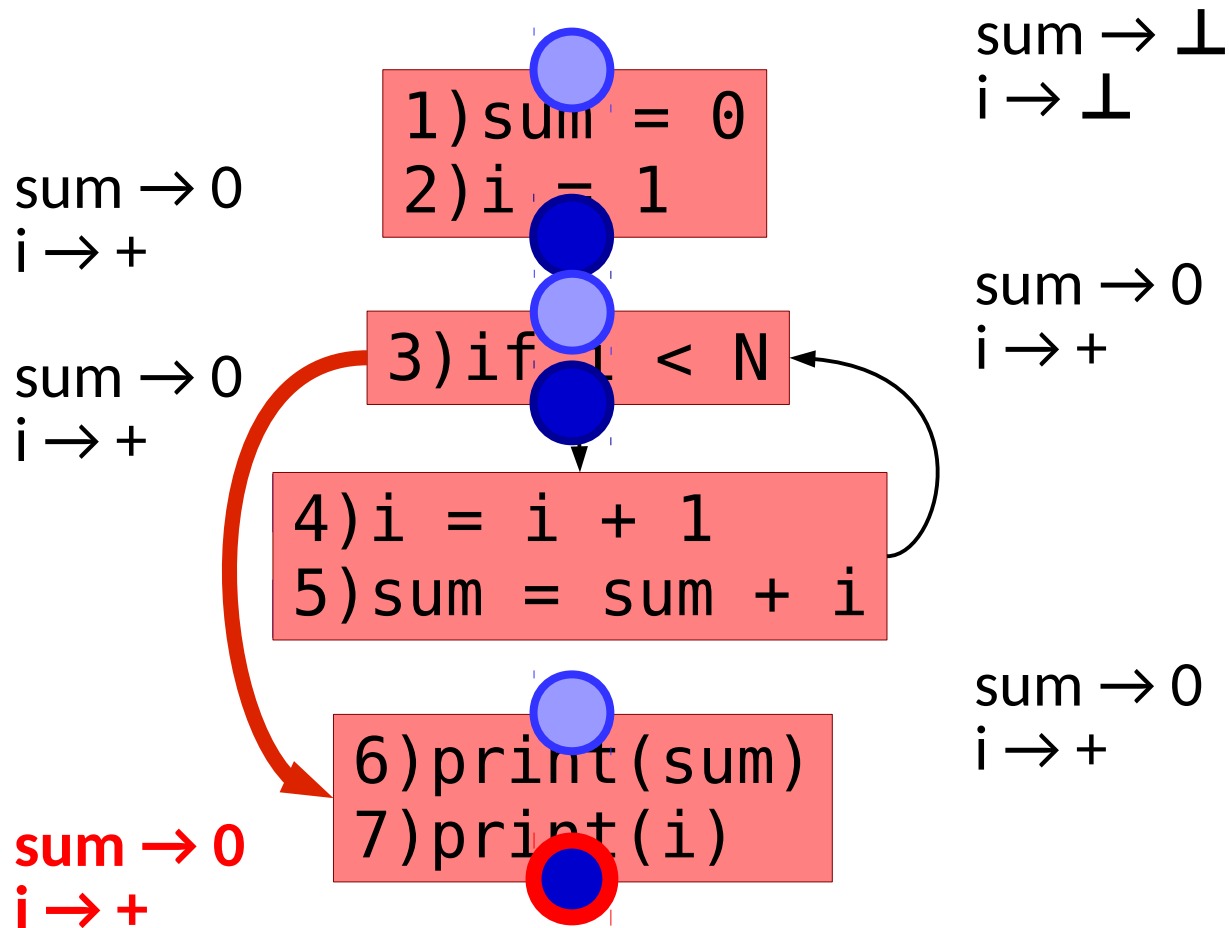
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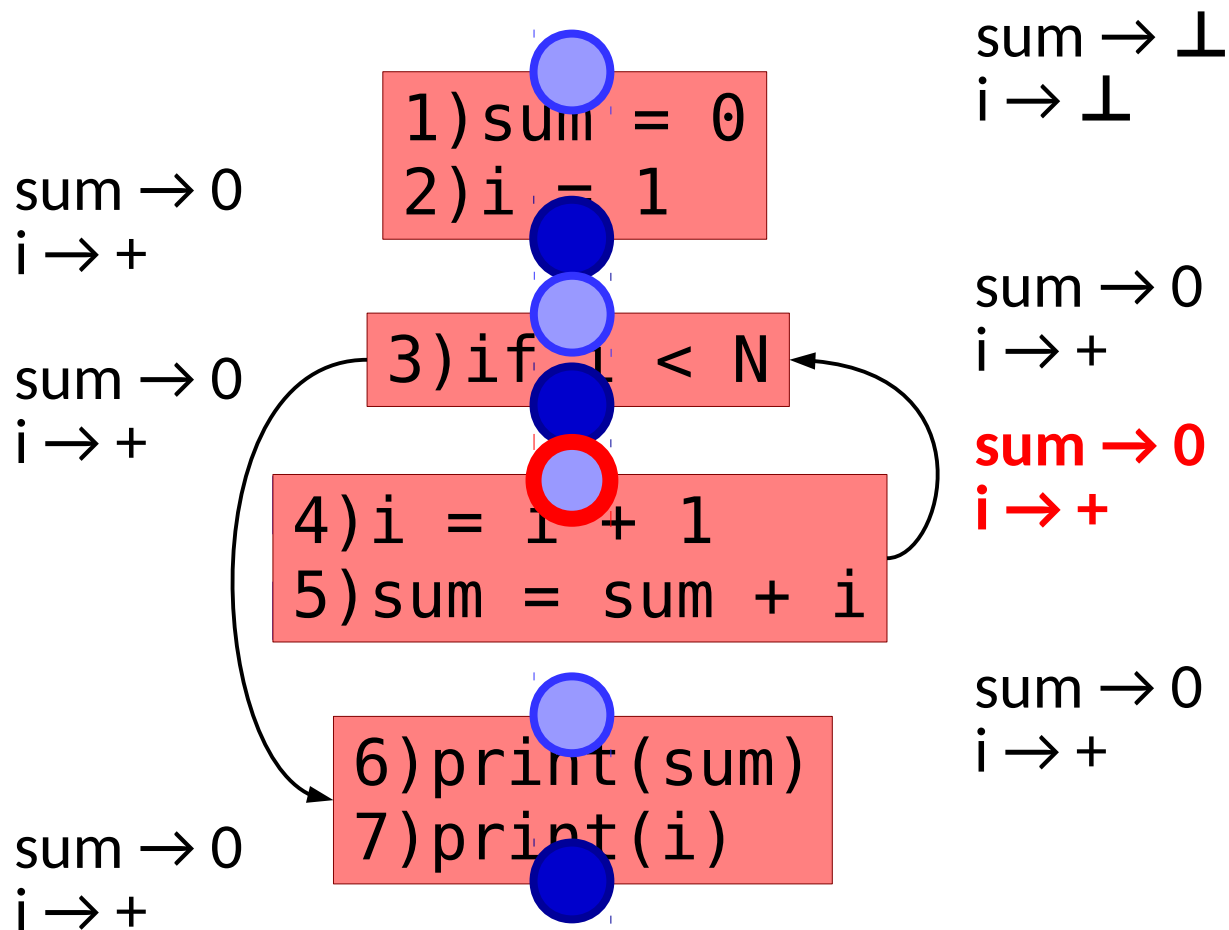
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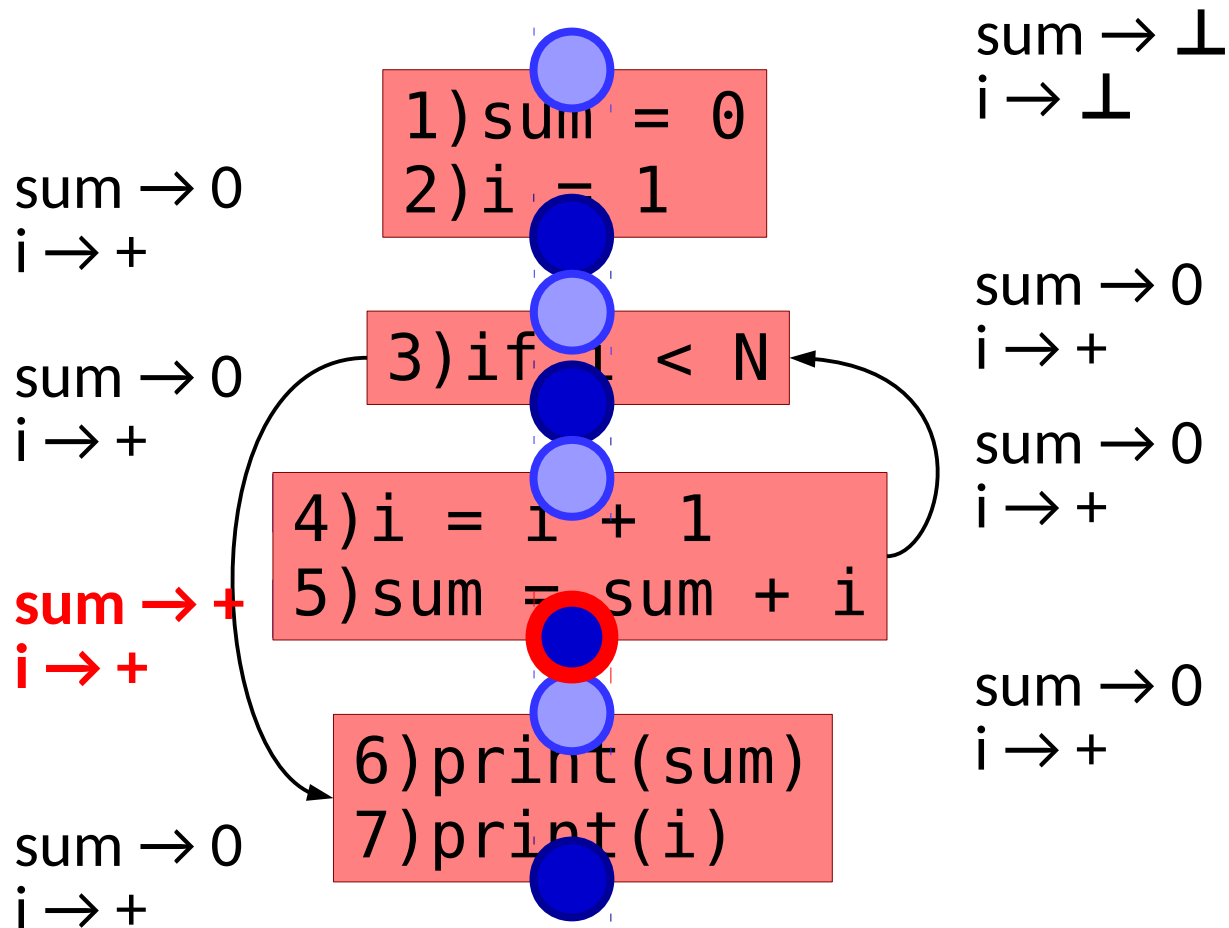
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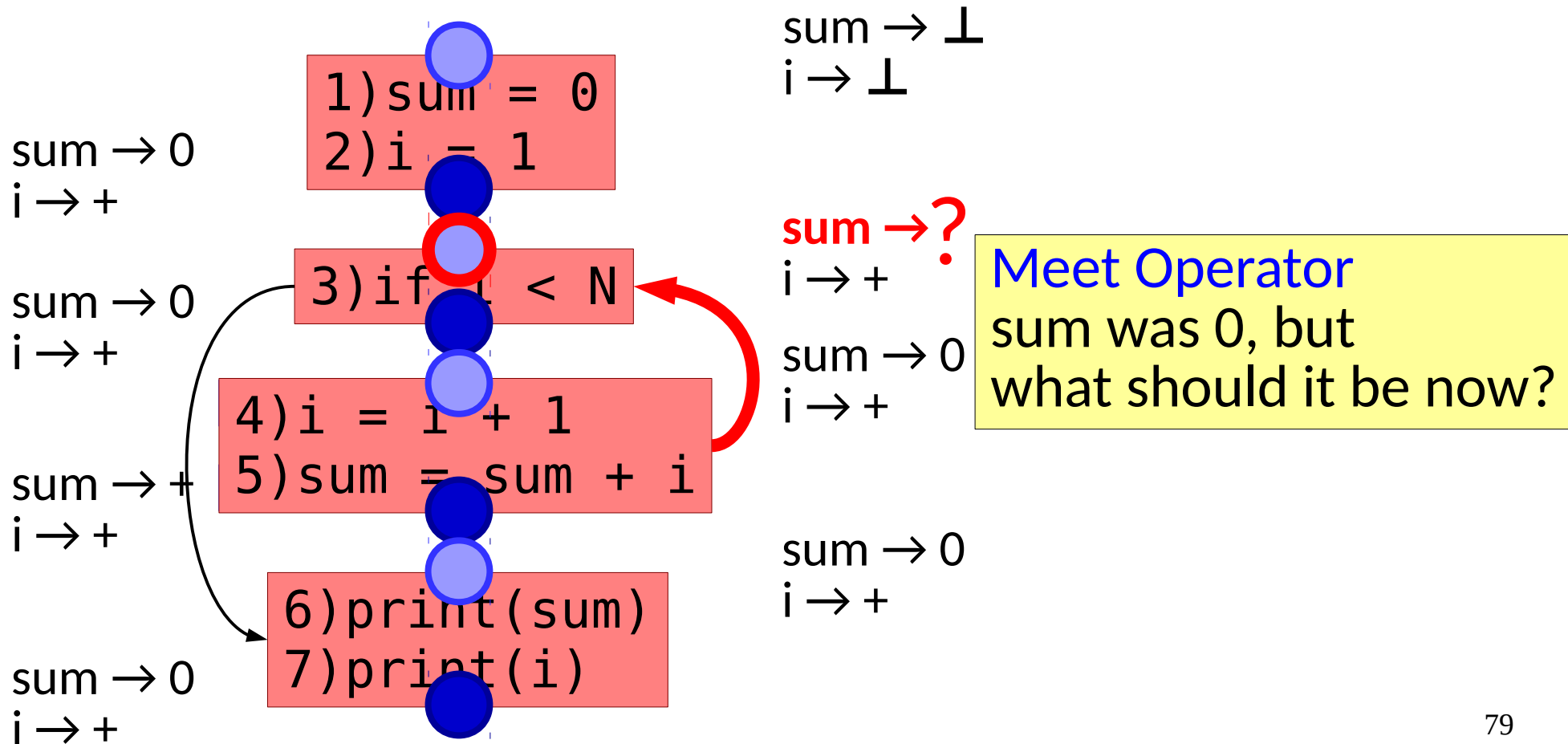
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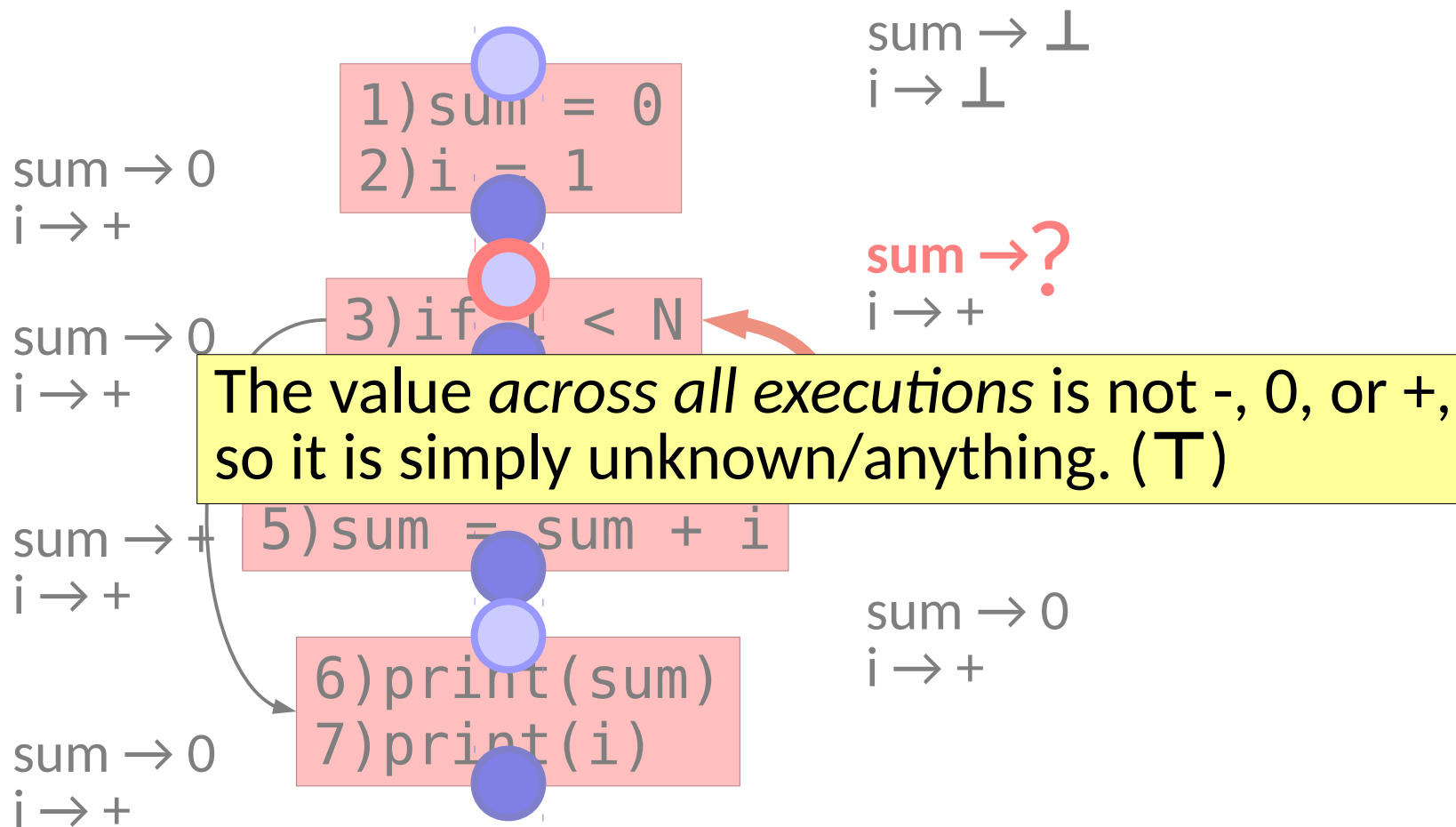
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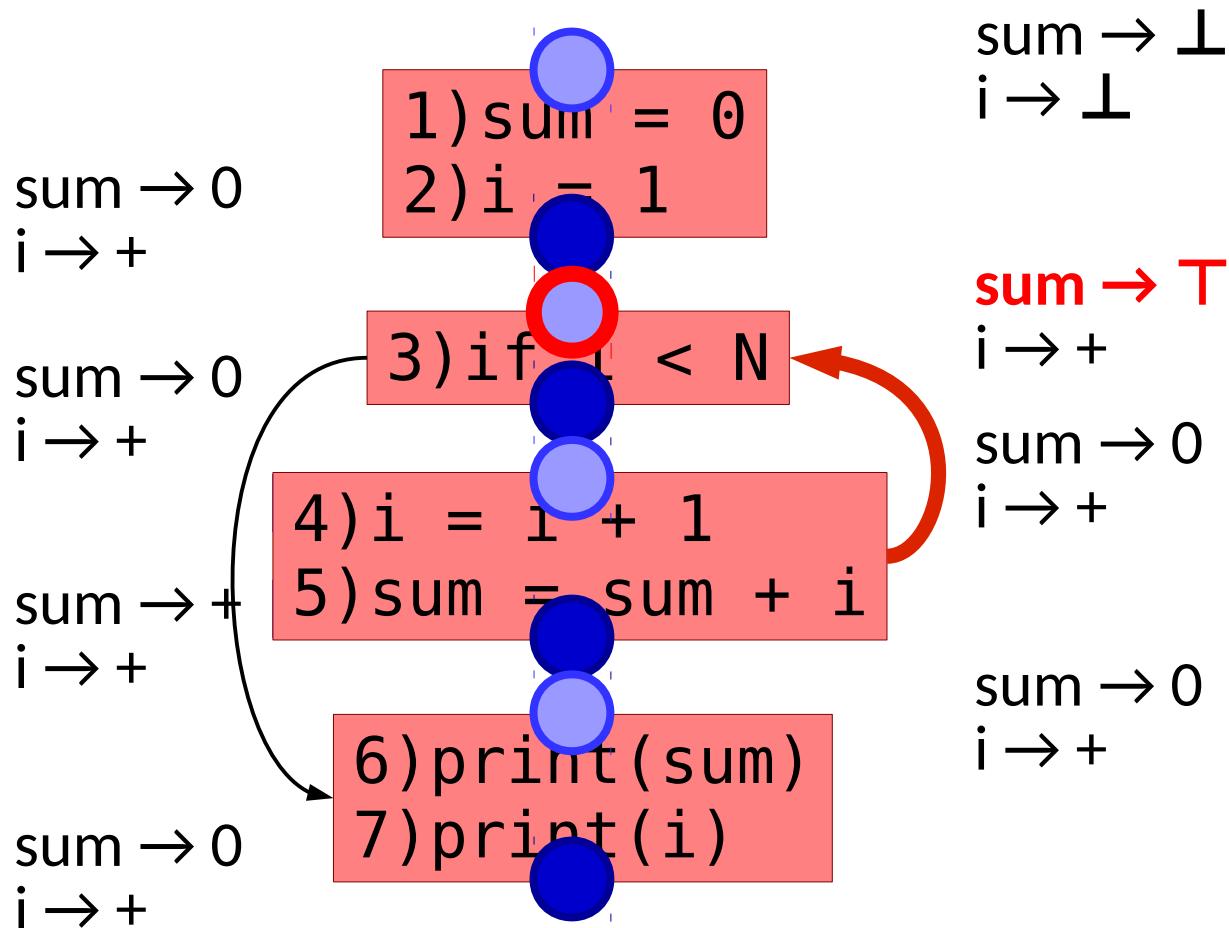
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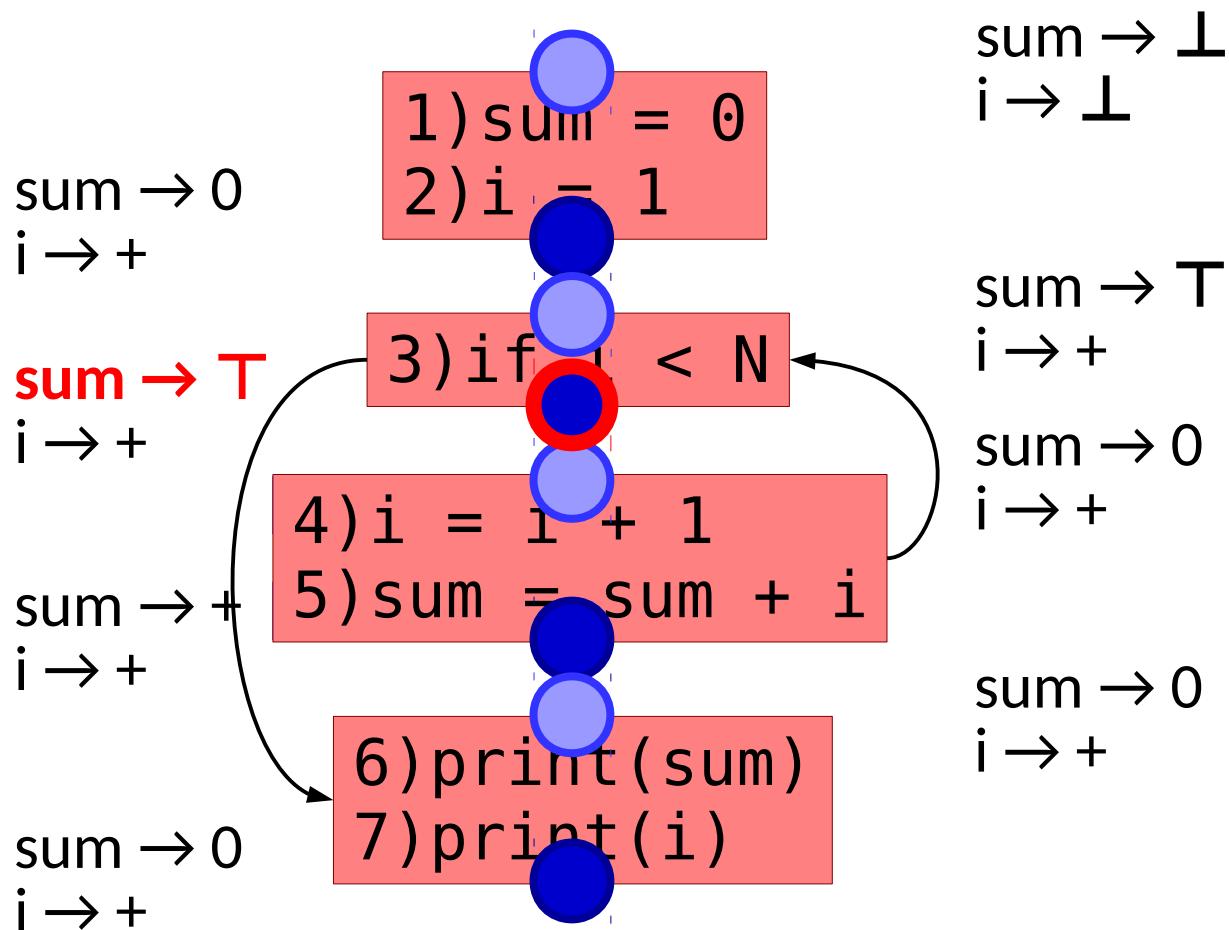
Dataflow Analysis

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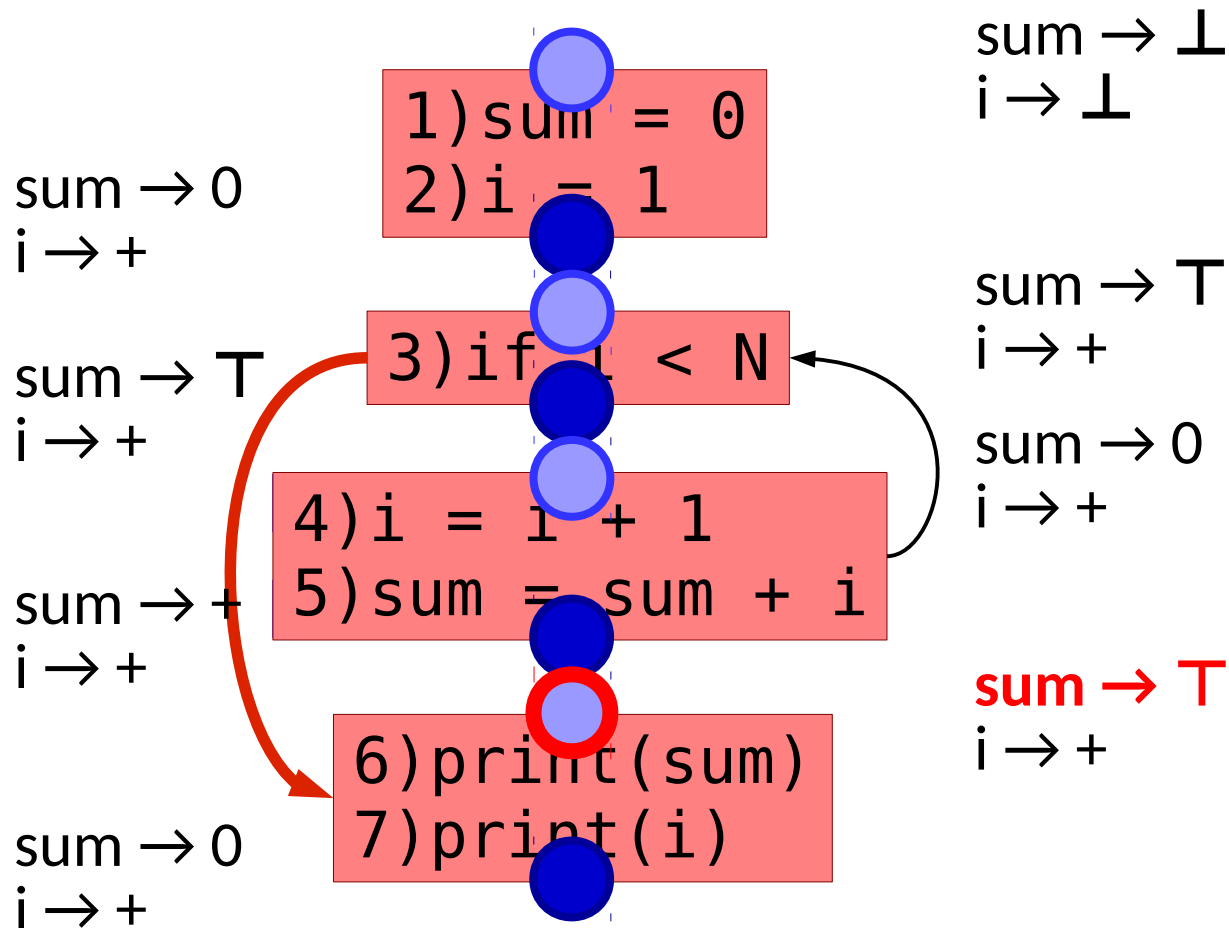
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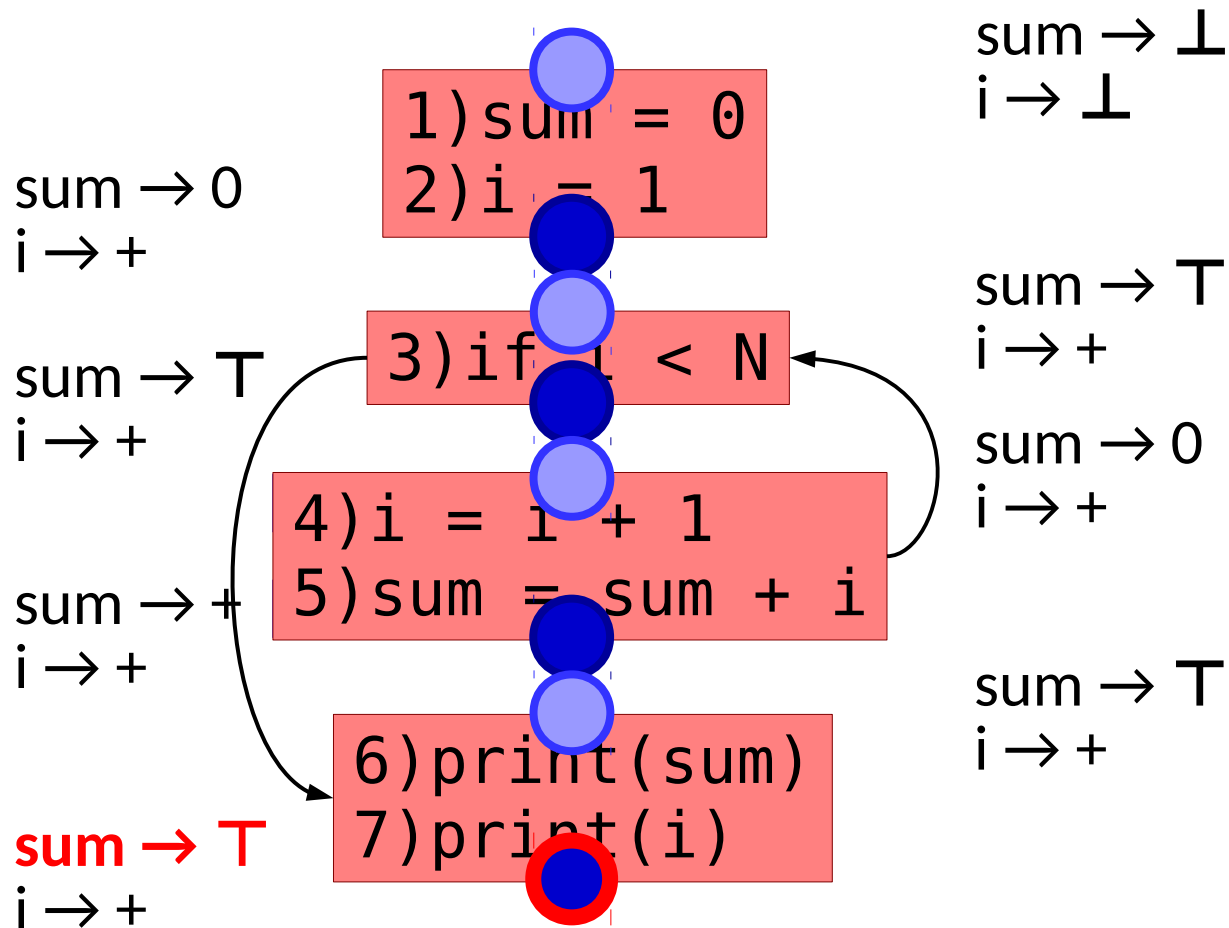
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(Topological Order, Strongly Connected Components)

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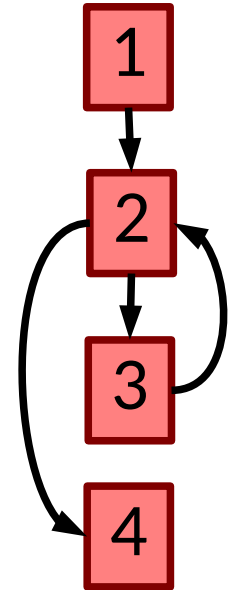
Will it always terminate?

Dataflow Analysis

- Note: need to model program state before and after each statement
- Proper ordering & a work list algorithm improves the efficiency

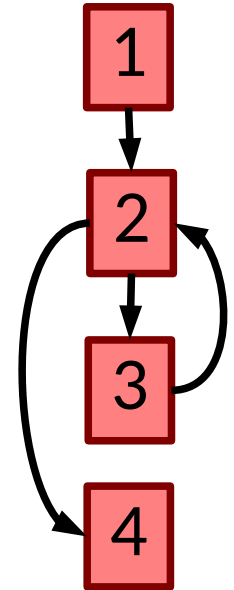
Worklist Algorithms

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work: 1 2 3 4

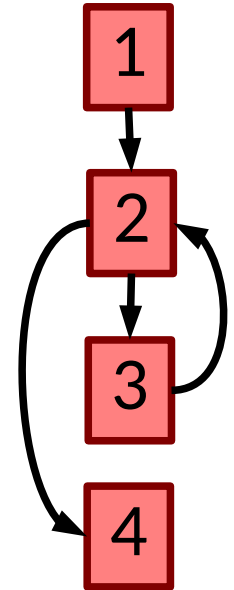
state: $\left\{ \begin{array}{l} \left(\begin{array}{c} \text{1} \\ \text{2} \end{array} \mapsto \perp \right) \\ \left(\begin{array}{c} \text{3} \\ \text{4} \end{array} \mapsto \perp \right) \end{array} \right\}$

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```

unit = 1



work: 2 3 4

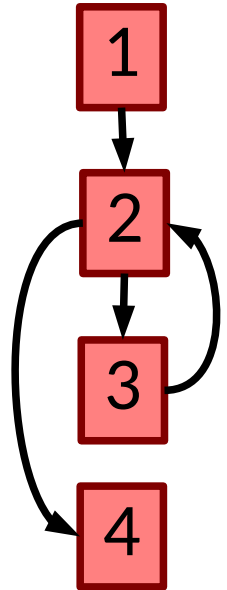
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unit = 1
old = \perp



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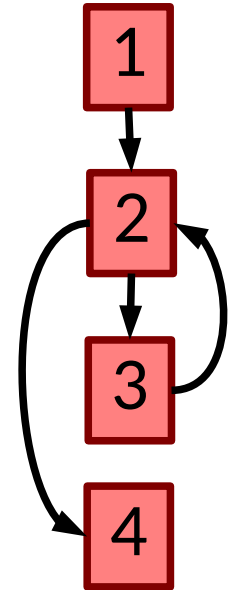
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```

```

unit = 1
old =  $\perp$ 
new = sum  $\rightarrow$  0  
i  $\rightarrow$  +
=
    
```



work: 2 3 4

state: $\left\{ \begin{array}{l} \left(\begin{array}{l} \text{1} \\ \text{1} \end{array} \mapsto \perp \right) \quad \left(\begin{array}{l} \text{3} \\ \text{3} \end{array} \mapsto \perp \right) \\ \left(\begin{array}{l} \text{2} \\ \text{2} \end{array} \mapsto \perp \right) \quad \left(\begin{array}{l} \text{4} \\ \text{4} \end{array} \mapsto \perp \right) \end{array} \right\}$

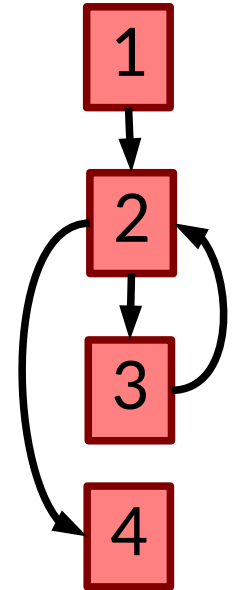
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state: { ( 1  $\mapsto$  sum  $\rightarrow$  0  
i  $\rightarrow$  + ) ( 3  $\mapsto$   $\perp$  )
        { ( 2  $\mapsto$   $\perp$  ) ( 4  $\mapsto$   $\perp$  ) }
    }
    
```

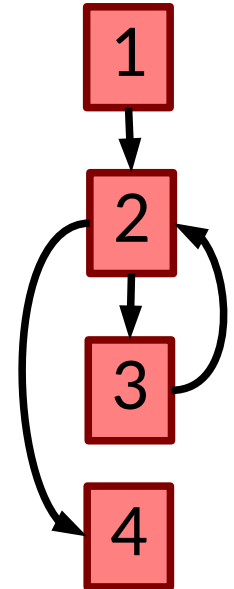
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```

```

unit = 2
old =  $\perp$ 
new = { sum  $\rightarrow$  0
       i  $\rightarrow$  + }
=
    
```



```

work: [ ] 3 4
state: { ( 1  $\mapsto$  { sum  $\rightarrow$  0
                  i  $\rightarrow$  + } ) ( 3  $\mapsto$   $\perp$  )
        ( 2  $\mapsto$  { sum  $\rightarrow$  0
                  i  $\rightarrow$  + } ) ( 4  $\mapsto$   $\perp$  ) }
    
```

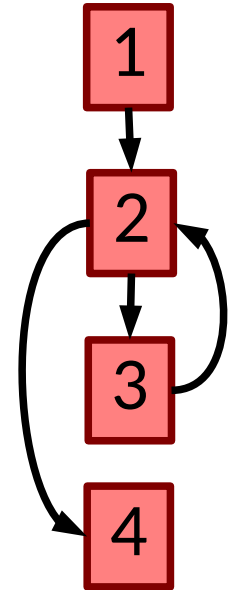
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```

```

unit = 3
old =  $\perp$ 
new = { sum  $\rightarrow$  +
       i  $\rightarrow$  + }
=
    
```



work: 4 2 ← 2 was added back to the list

state: $\left\{ \left(\begin{array}{l} \boxed{1} \mapsto \begin{array}{l} \text{sum} \rightarrow 0 \\ i \rightarrow + \end{array} \\ \boxed{2} \mapsto \begin{array}{l} \text{sum} \rightarrow 0 \\ i \rightarrow + \end{array} \end{array} \right) \left(\begin{array}{l} \boxed{3} \mapsto \begin{array}{l} \text{sum} \rightarrow + \\ i \rightarrow + \end{array} \\ \boxed{4} \mapsto \perp \end{array} \right) \right\}$

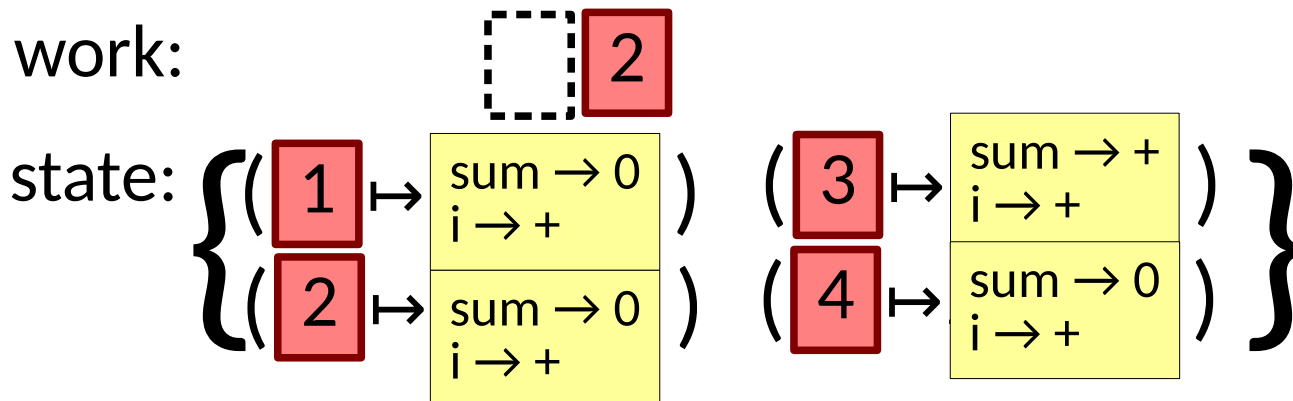
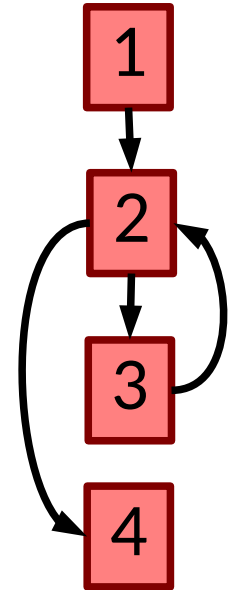
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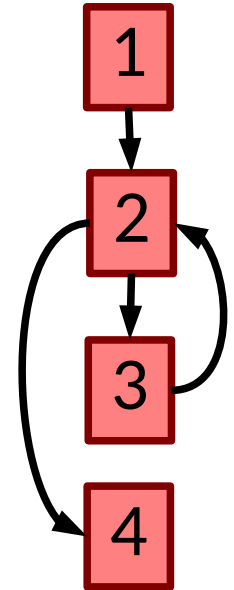
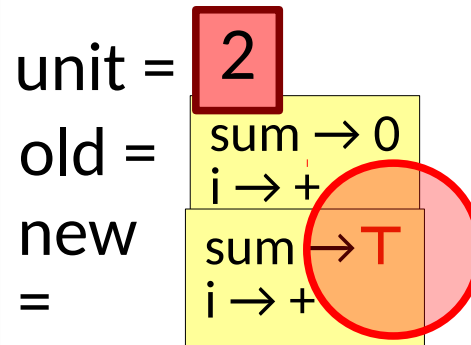
unit = 4
old =  $\perp$ 
new = { sum  $\rightarrow$  0
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=
    
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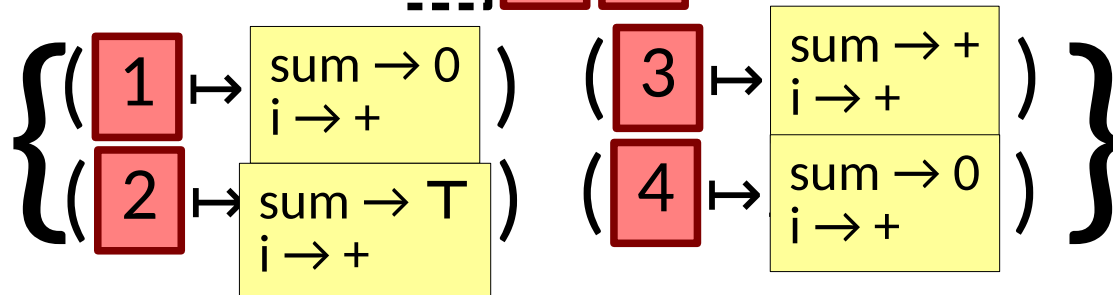


4,3 were added back to the list

work:



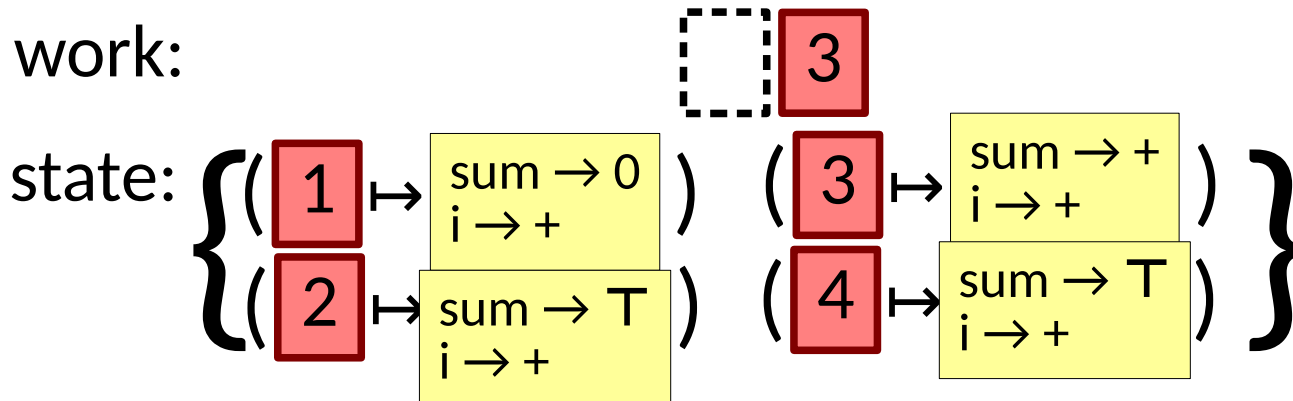
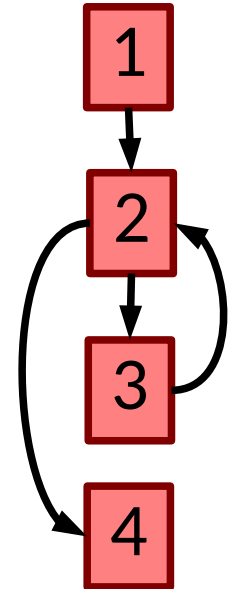
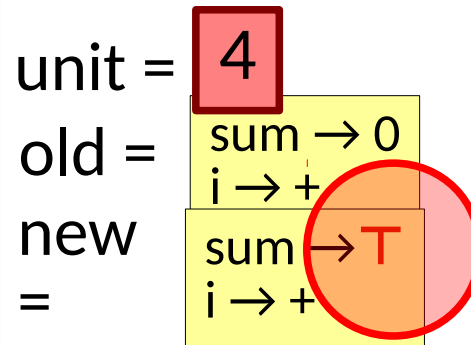
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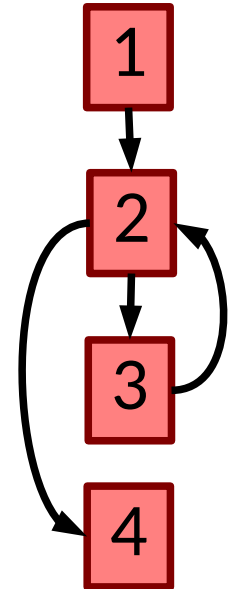
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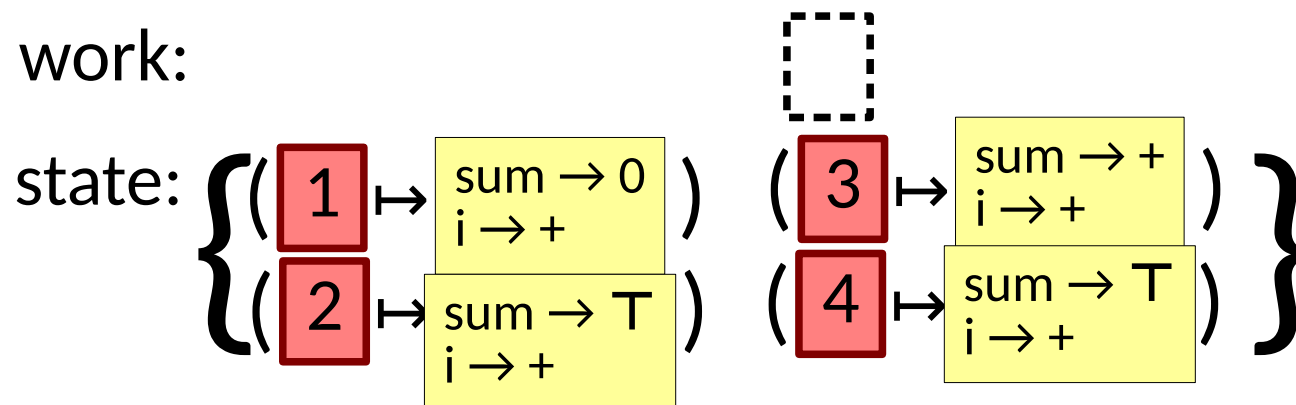
```

unit = 3
old = sum  $\rightarrow$  +
      i  $\rightarrow$  +
new = sum  $\rightarrow$  +
      i  $\rightarrow$  +
=
    
```

No change



work:



Worklist Algorithms

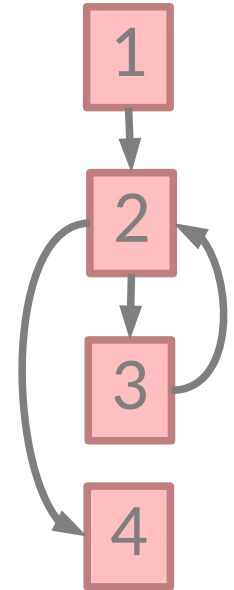
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    new = transfer(before, old)
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        state(unit) = new
    
```

```

unit = 4
old = sum  $\rightarrow$  0
      i  $\rightarrow$  +
new = sum  $\rightarrow$  T
    
```

Done!



work:

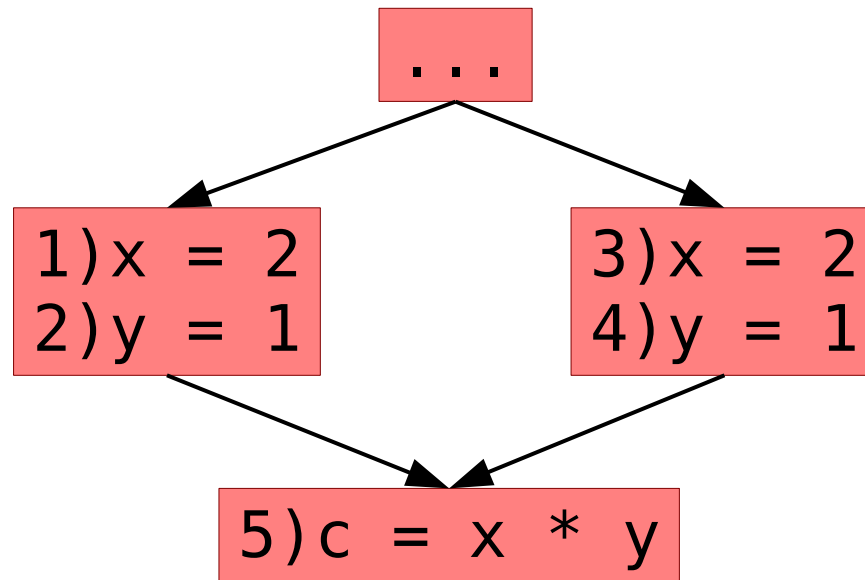
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Effect of Approximation

- There are several possible sources of imprecision

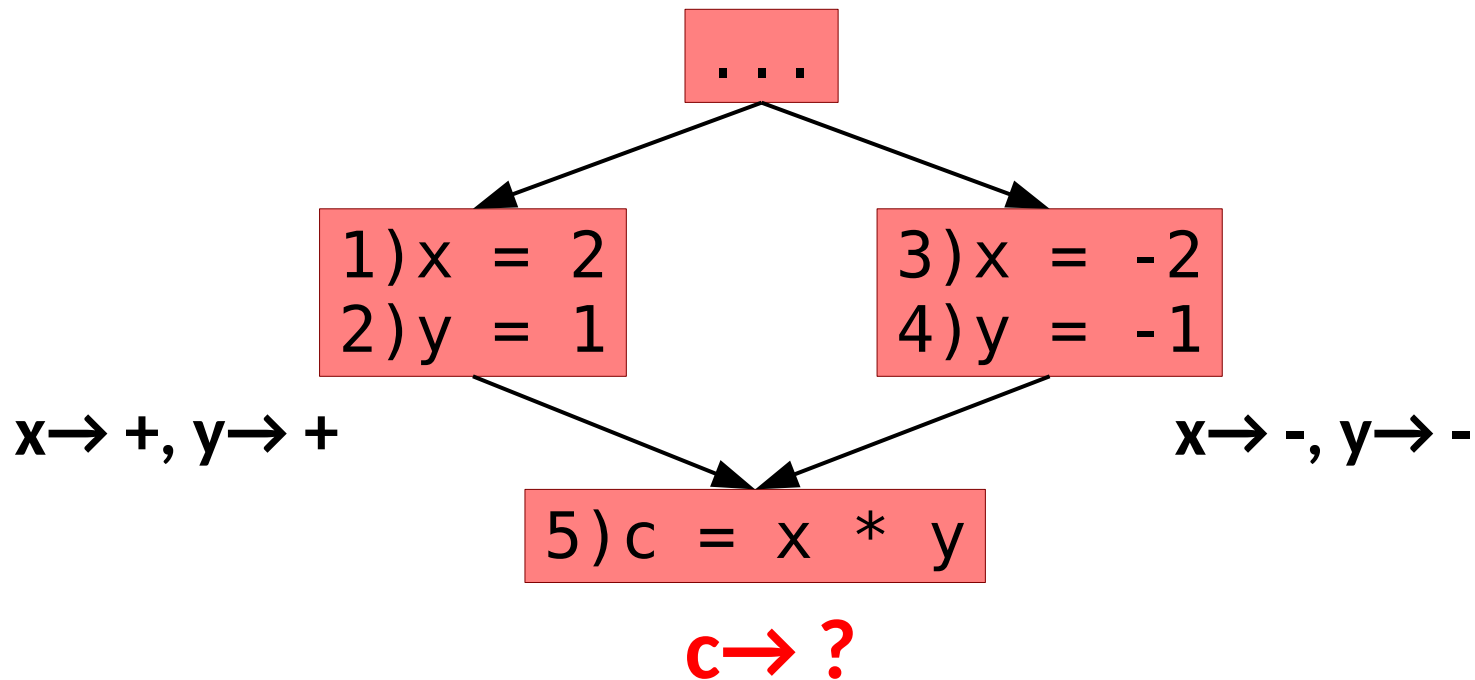
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- 2 Key sources are
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 - Abstraction
 - Deliberately throwing away information
 - Granularity of program state affects correlations across variables

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For all paths p : $\bigwedge_p f_p(\perp)$

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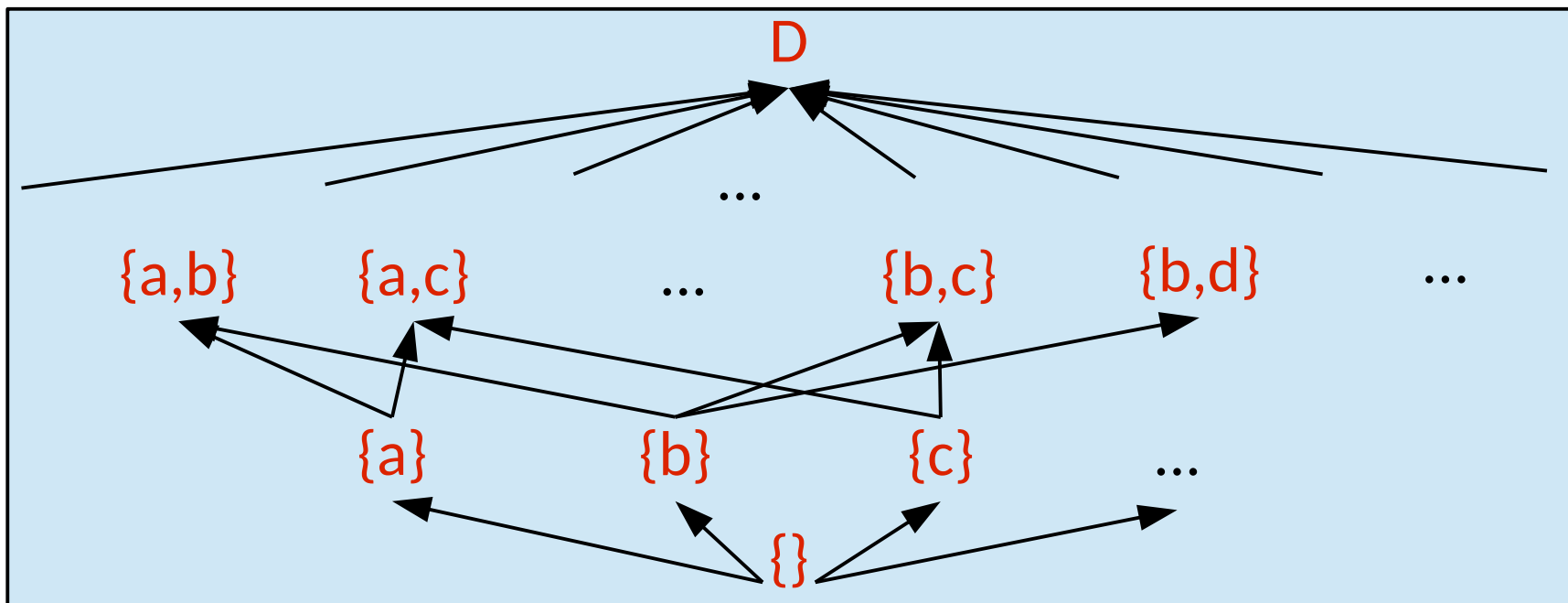
- We compute results with maximal fixed points (MFP) in the lattice
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 $f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z)$

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 - When $f()$ is distributive, $MFP=MOP$
 $f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z)$
 - This applies to an important class of problems called *bitvector frameworks*.

Bitvector Frameworks

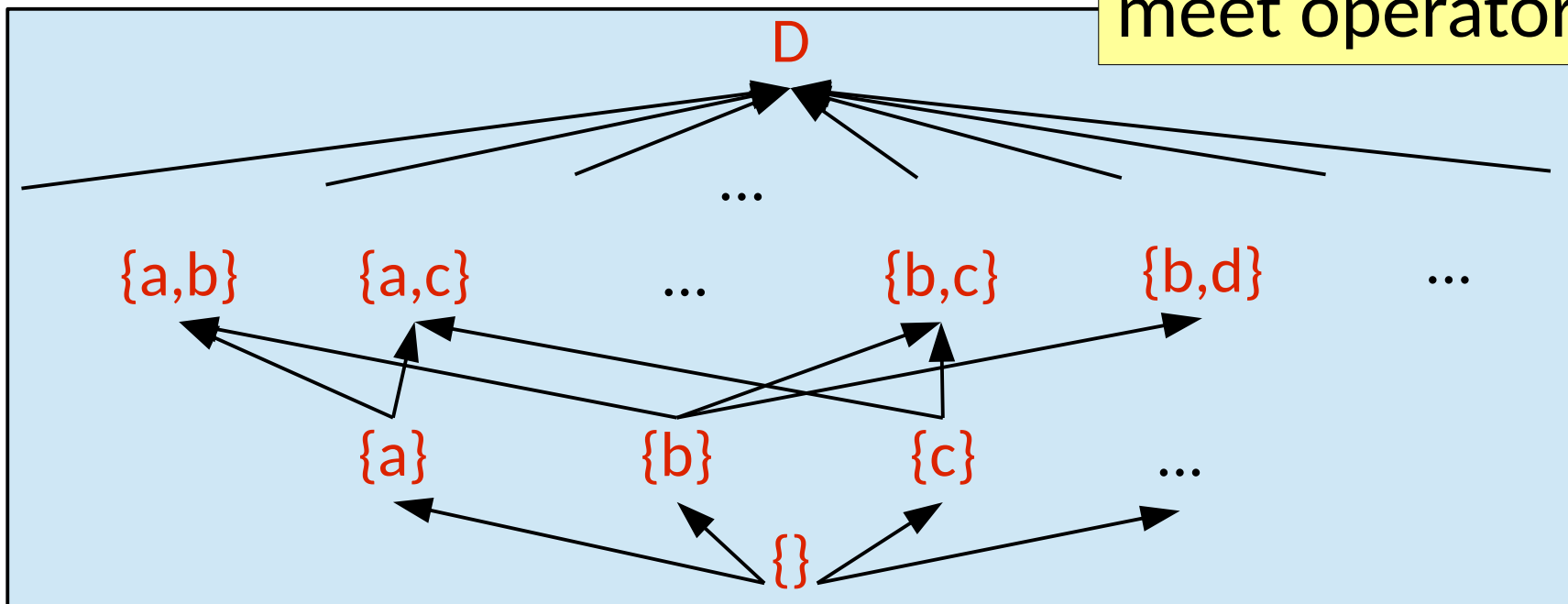
- When the property concerns subsets of a finite set, the abstract domain & lattice are easy:
 - Concrete: $D = \{a, b, c, d, \dots\}$
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What would the meet operator be?



Bitvector Frameworks

- Why is this convenient?
 - Hint: *bitvector* frameworks

Bitvector Frameworks

- Why is this convenient?
 - Hint: **bitvector** frameworks
 - $X=\{a,b\}, Y=\{c,d\} \rightarrow X \sqcup Y = \{a,b\} \cup \{c,d\} = \{a,b,c,d\}$
 - We can implement the abstract state using efficient bitvectors!

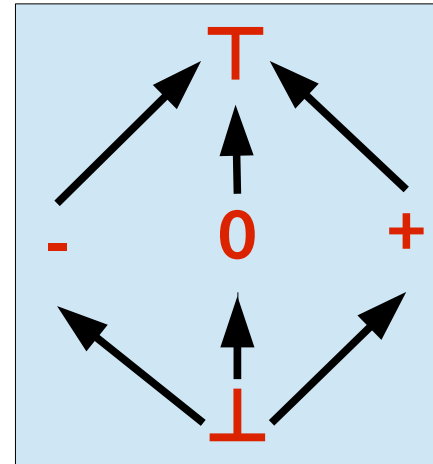
Effect of Approximation

- If approximation yields imprecise results, why do we do it?

Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- **Abstract domain:** e.g. $\{-, 0, +\} \cup \{\top, \perp\}$
- **Transfer functions:** $- + + \rightarrow \top$
- Bounded domain lattice height:
- Concern for false + & -



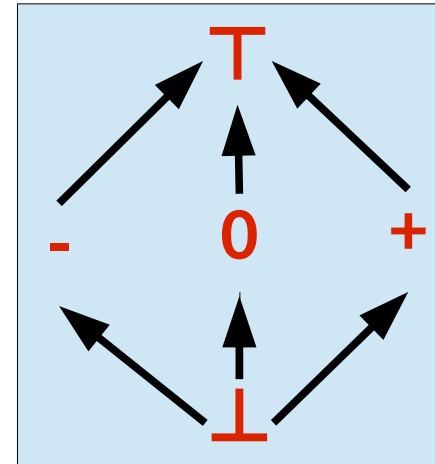
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Implementation:

- Computing using work lists
- Speeding up by sorting CFG nodes



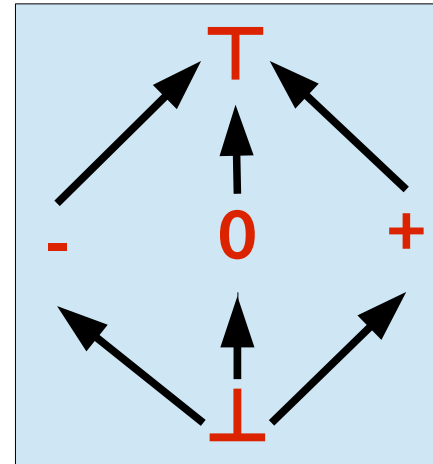
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Let's see an example

File Policy Analysis

Goal: Identify potential misuses of open/closed files

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[DEMO]

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- Saw *flow sensitive* analysis
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Flow Insensitive Analysis

- Saw *flow sensitive* analysis
 - Modeling state at each statement is expensive
 - Scales to functions and small components
 - Usually not beyond 1000s of lines without care
- *Flow insensitive* analyses aggregate into a global state
 - Better scalability
 - Less precision
 - “Does this function modify global variable X?”

Context Sensitive Analyses

- Program behavior may be dependent on the call stack / **calling context**.
 - “If bar() is called by foo(), then it is exception free.”
 - Can enable more precise *interprocedural* analyses

Context Sensitive Analyses

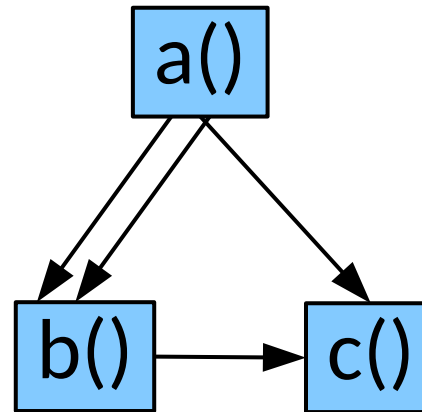
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Can you imagine how to solve this?
What problems might arise?

Context Sensitivity

- Recall that we can extract a call graph
 - Just as you are doing in your first project!

```
def a():  
    b()  
    ...  
    b()  
  
def b():  
    ...  
    c()  
  
def c():  
    ...
```



The behavior of `c()` could be affected by each “...”

Modeling them can make analysis more precise.

Context Sensitivity

- Simplest Approach
 - Add edges between call sites & targets
 - Perform data flow on this larger graph

```
def main():  
    x = 7  
    r = p(x)  
    x = r  
    z = p(x+10)
```

```
def p(a):  
    if a < 9:  
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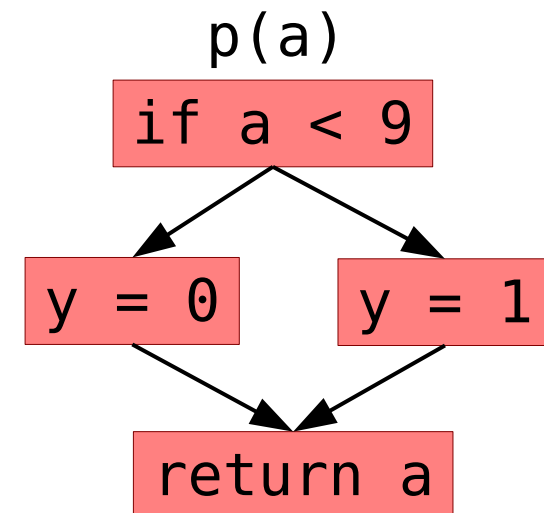
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main()

```
x = 7  
call p(x)
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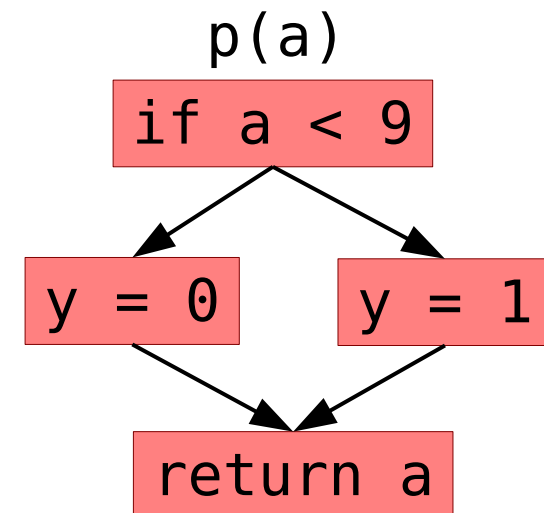
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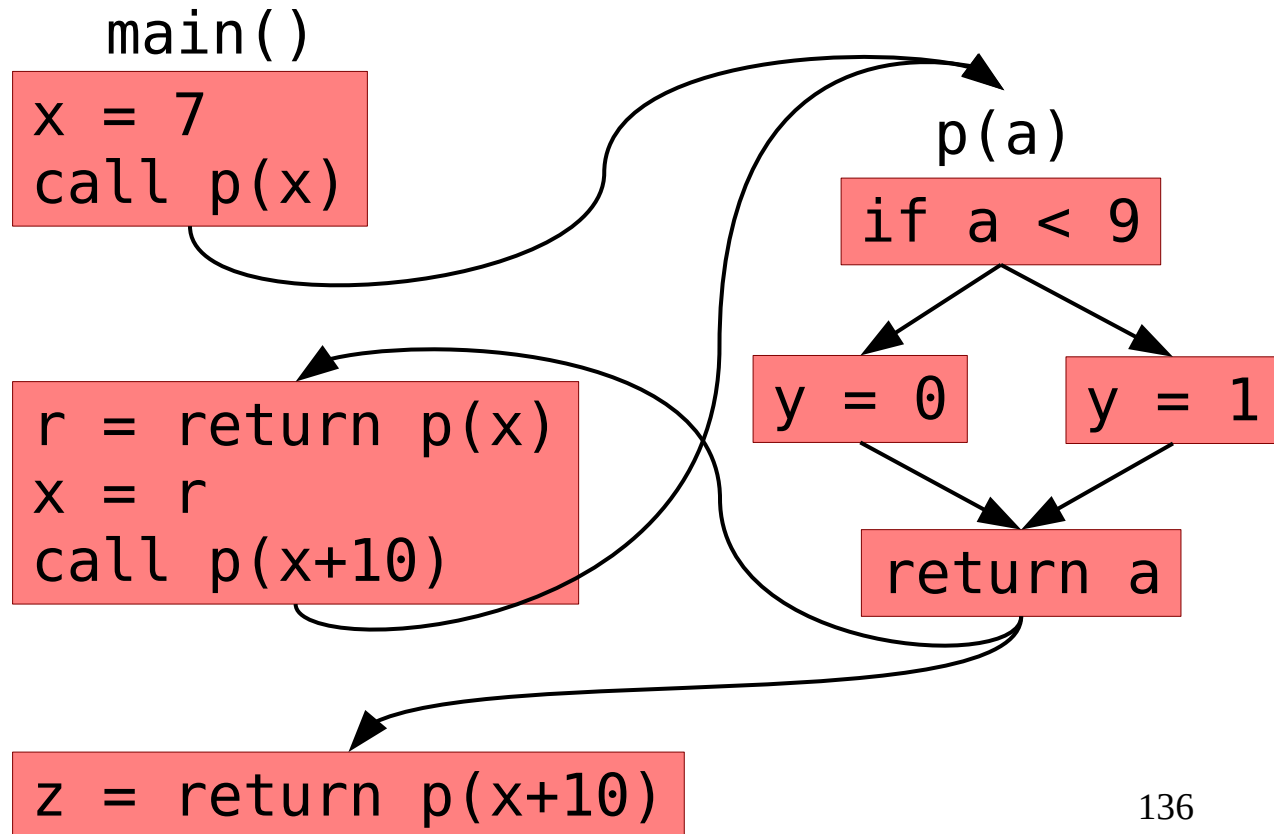


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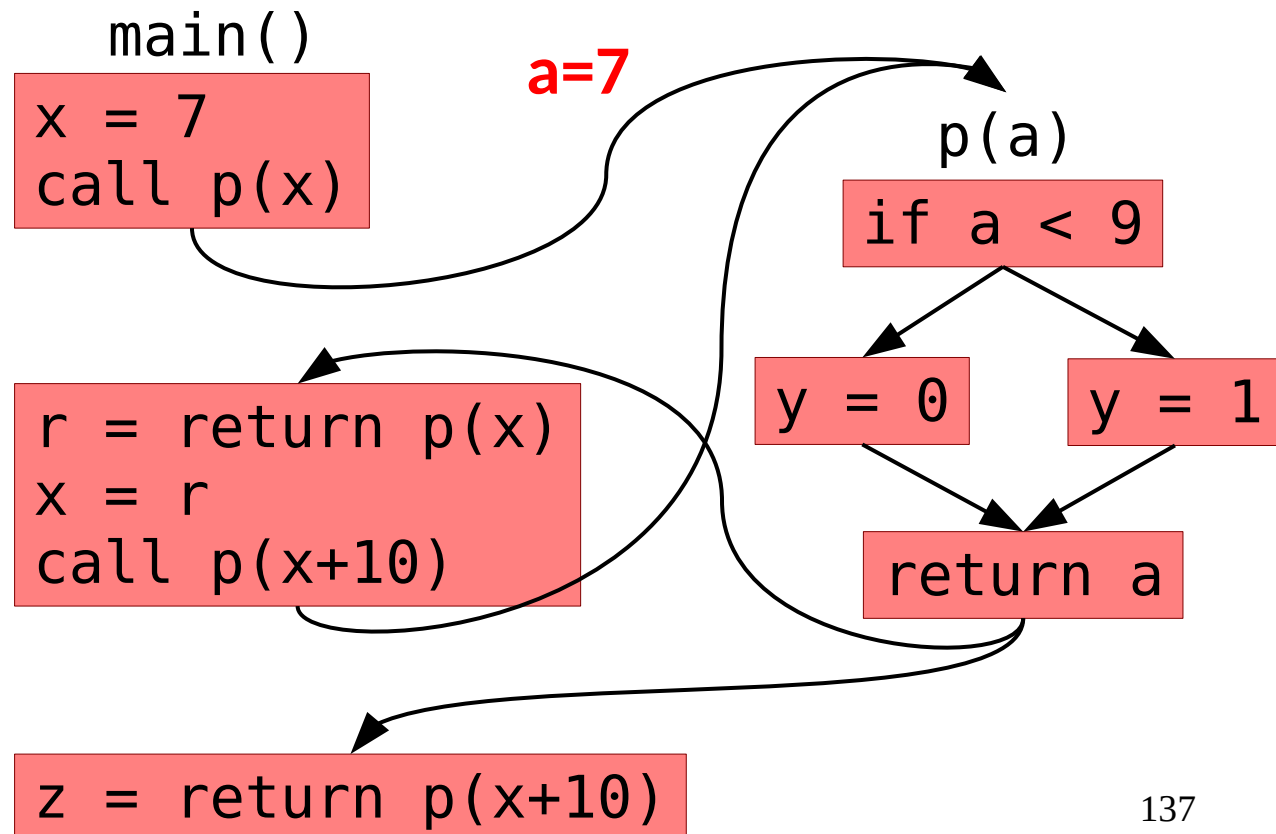


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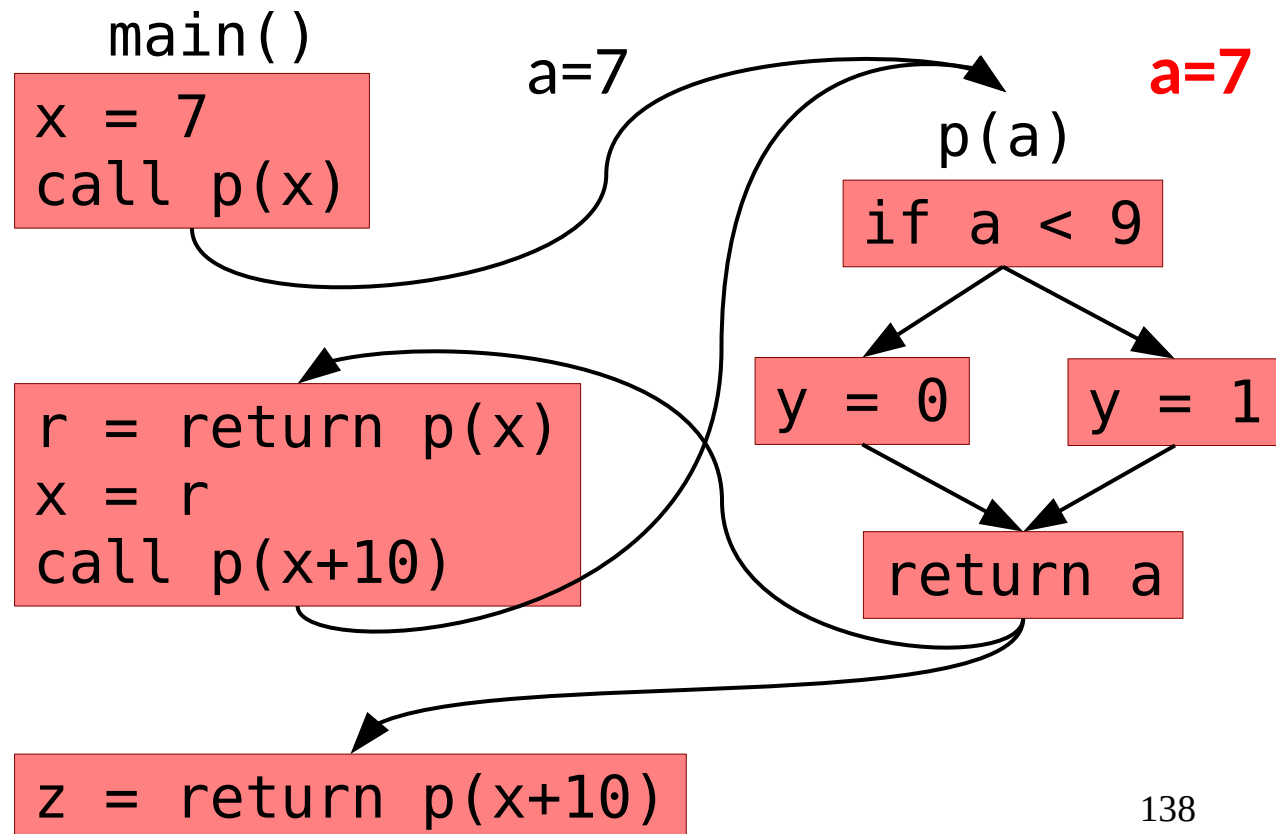


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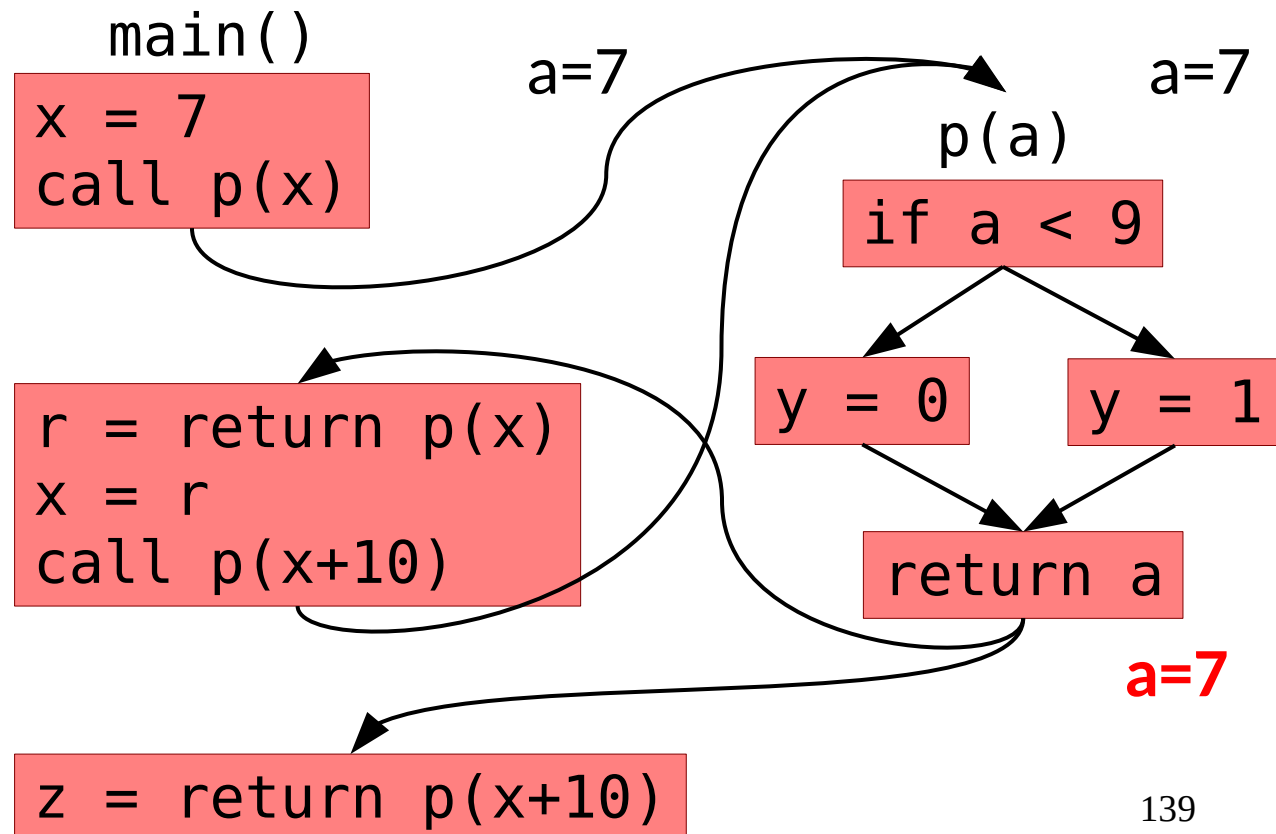


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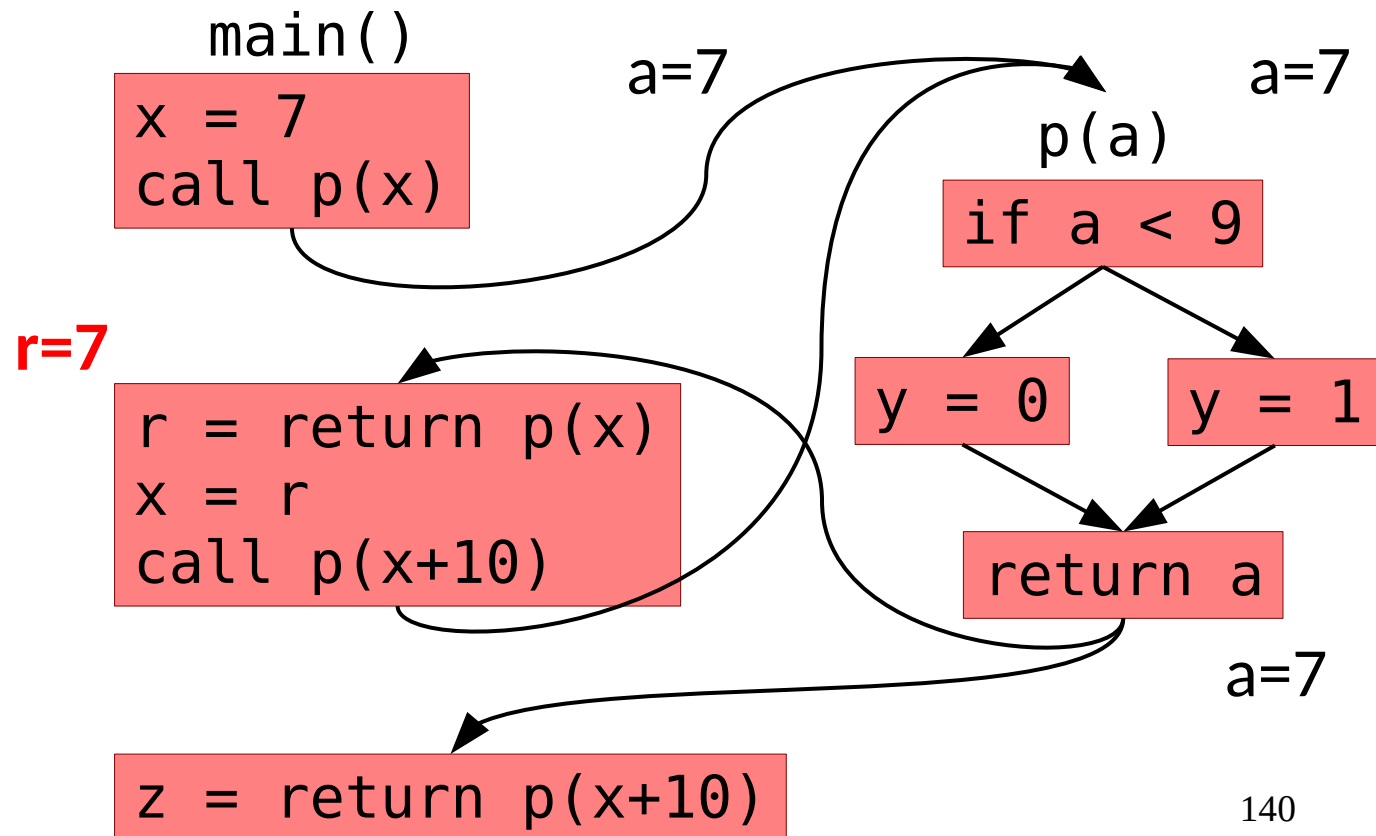


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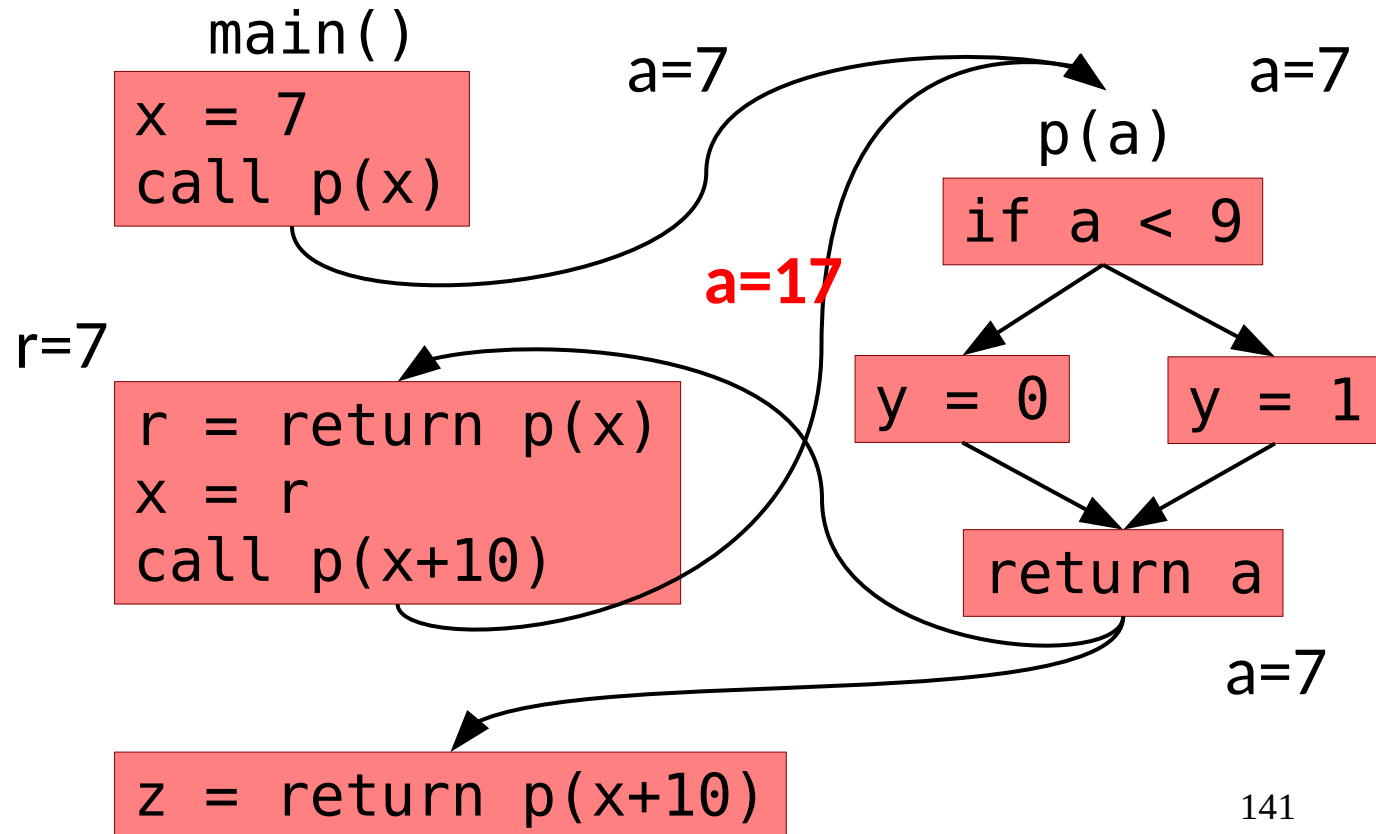


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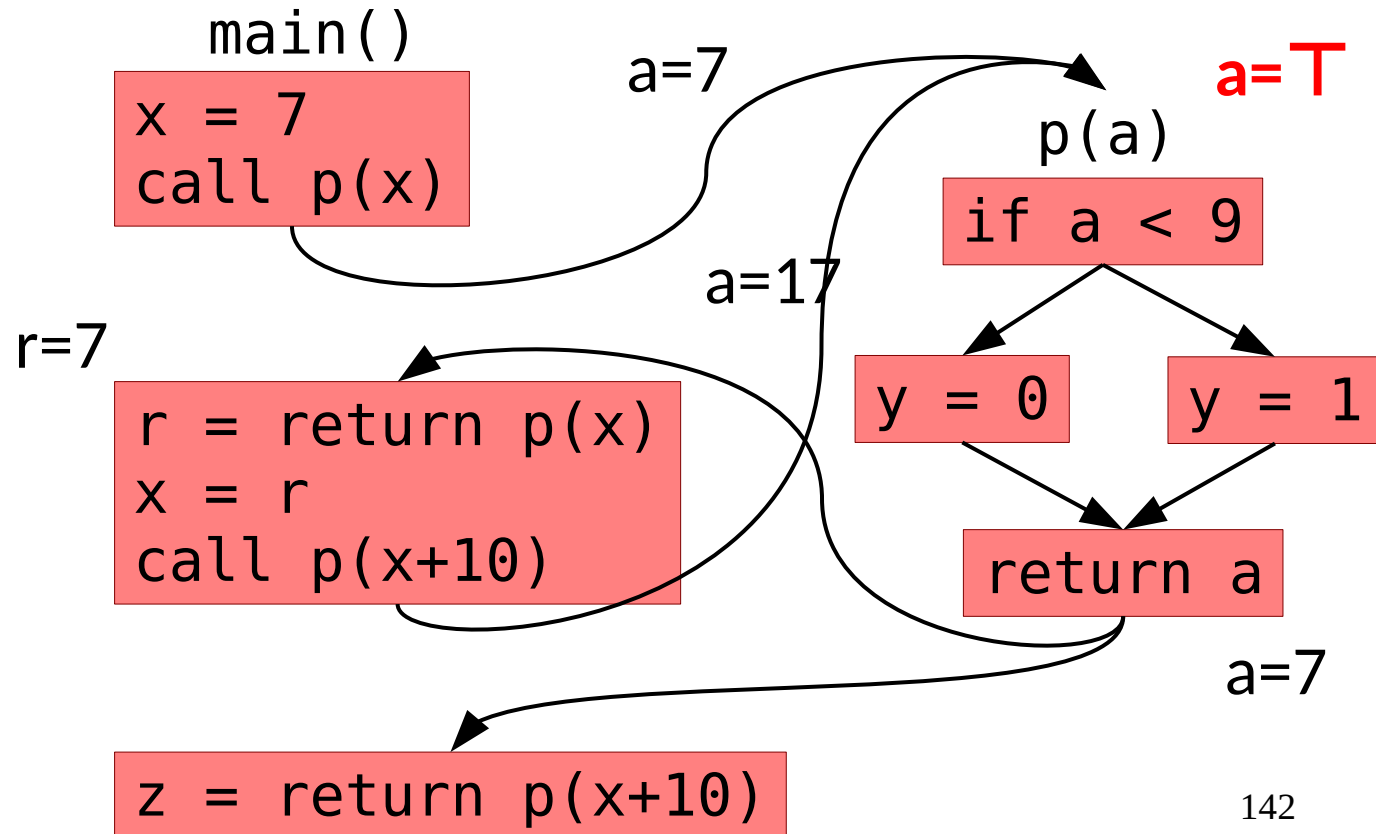


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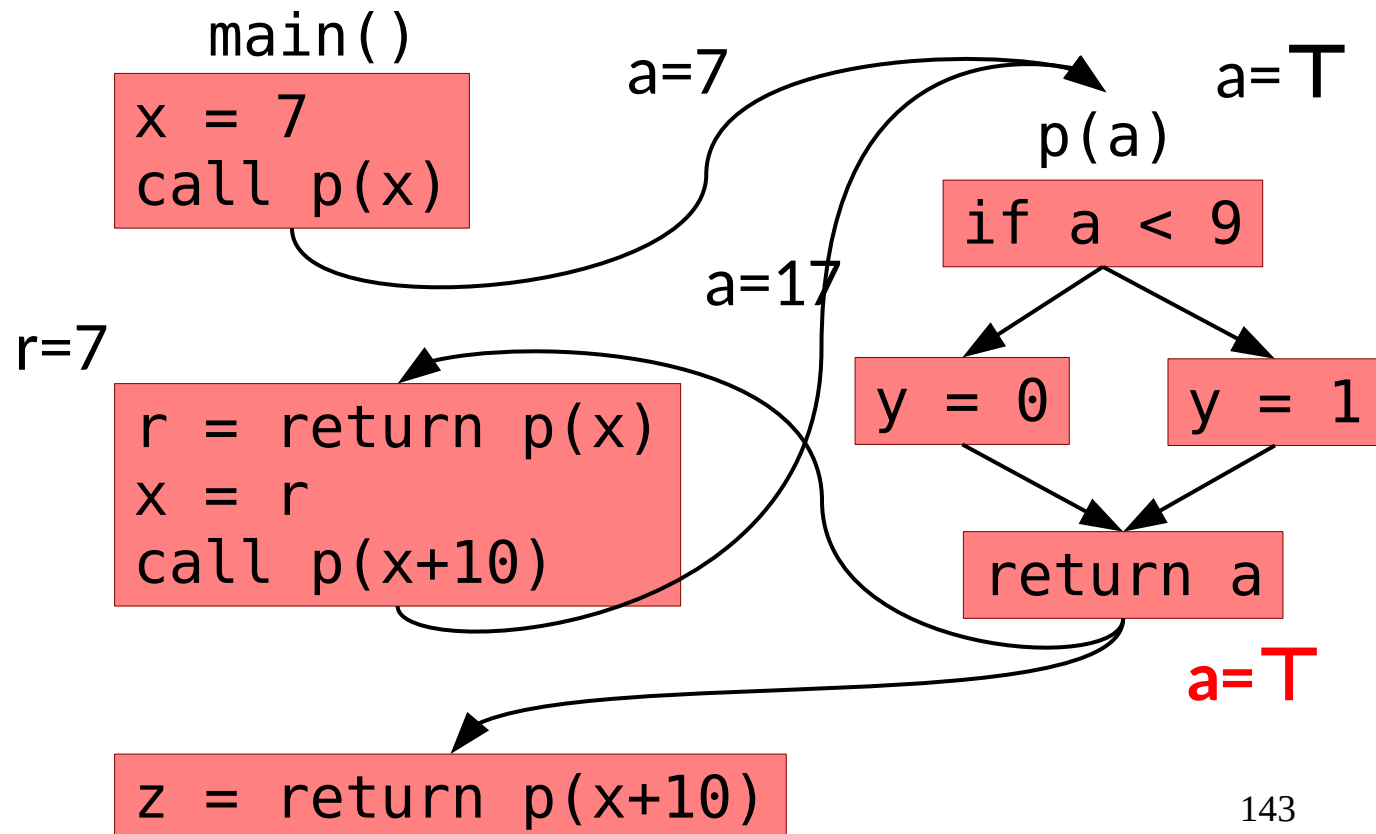


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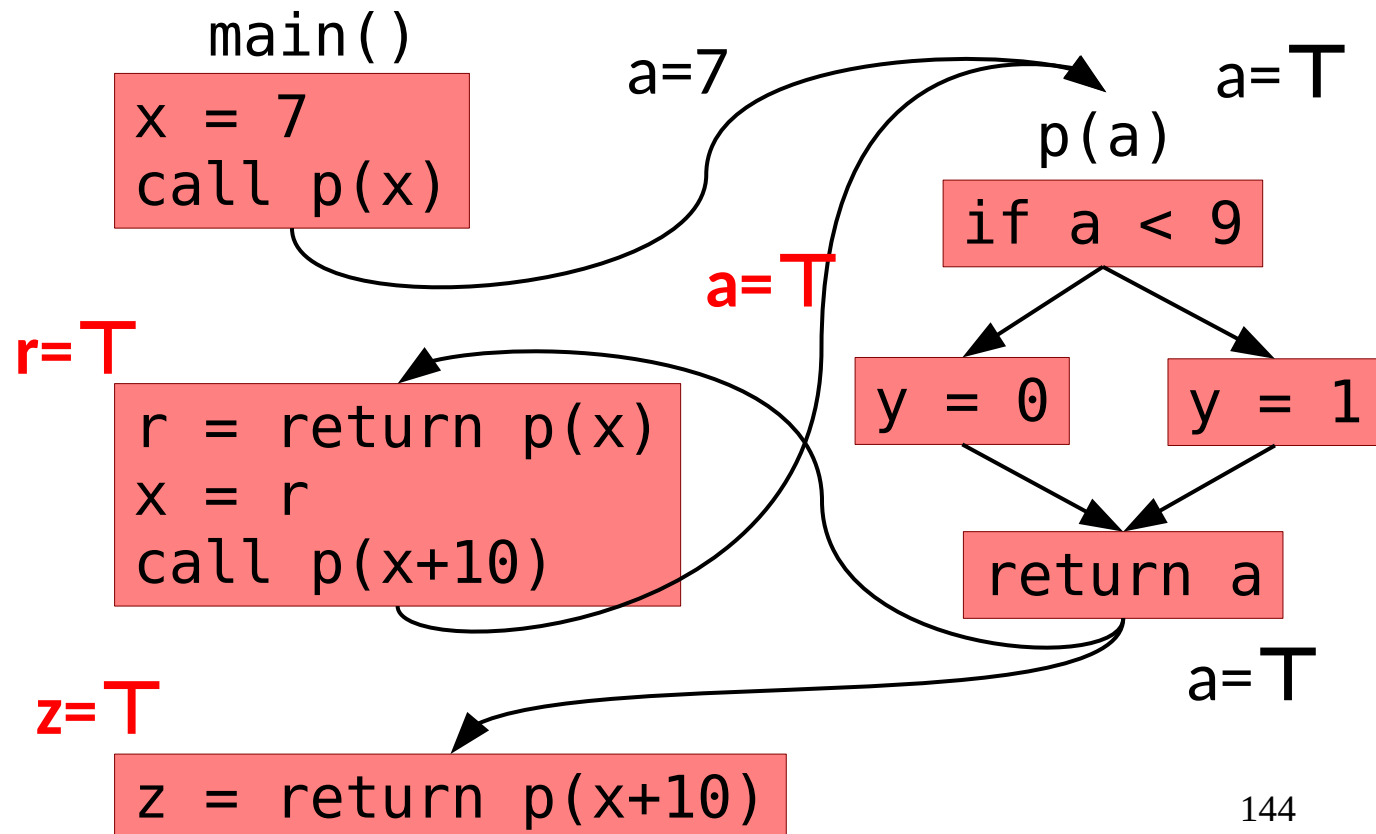


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- How could we address it?

Context Sensitivity

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- What problems arise?
- What other strategies can we use?

Context Sensitivity

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```

```
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```

```
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7)     pass
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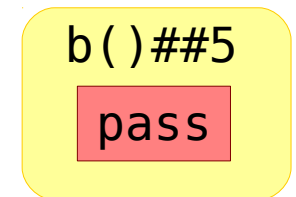
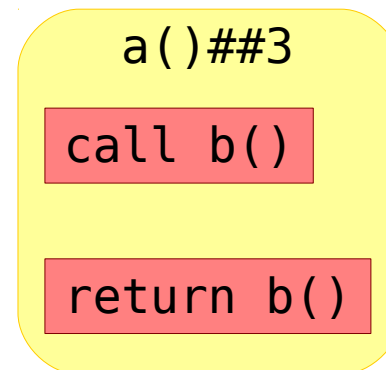
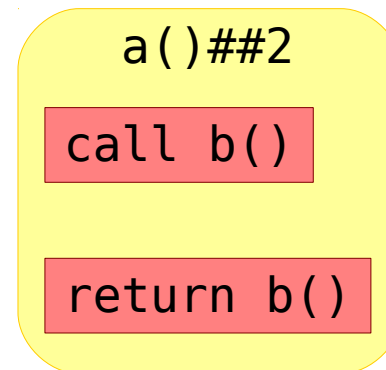
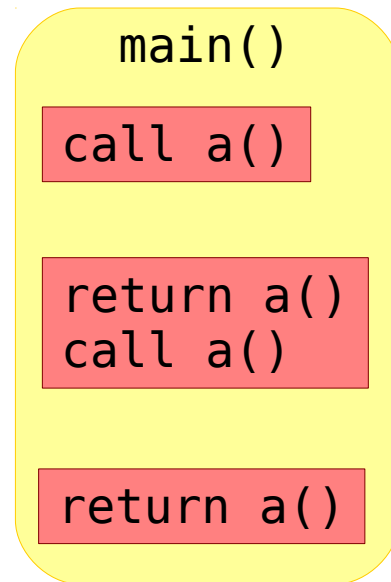
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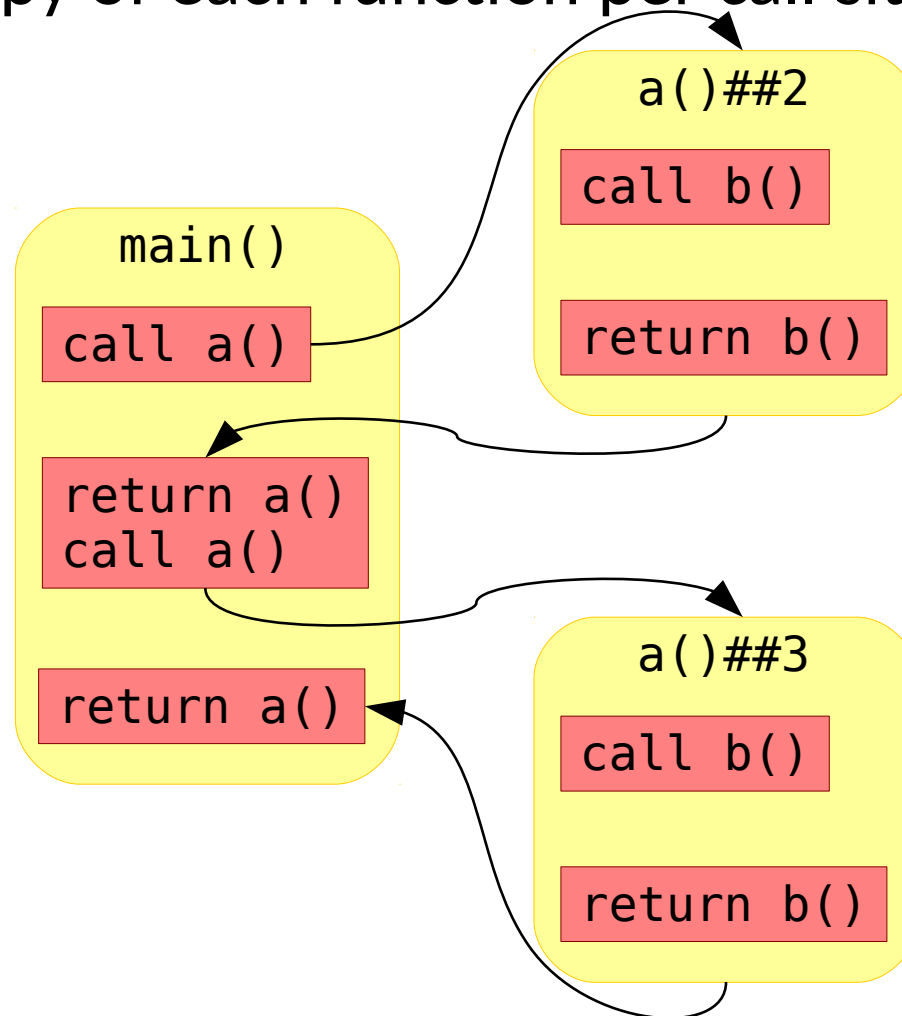
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So far,
so good

b()##5
pass

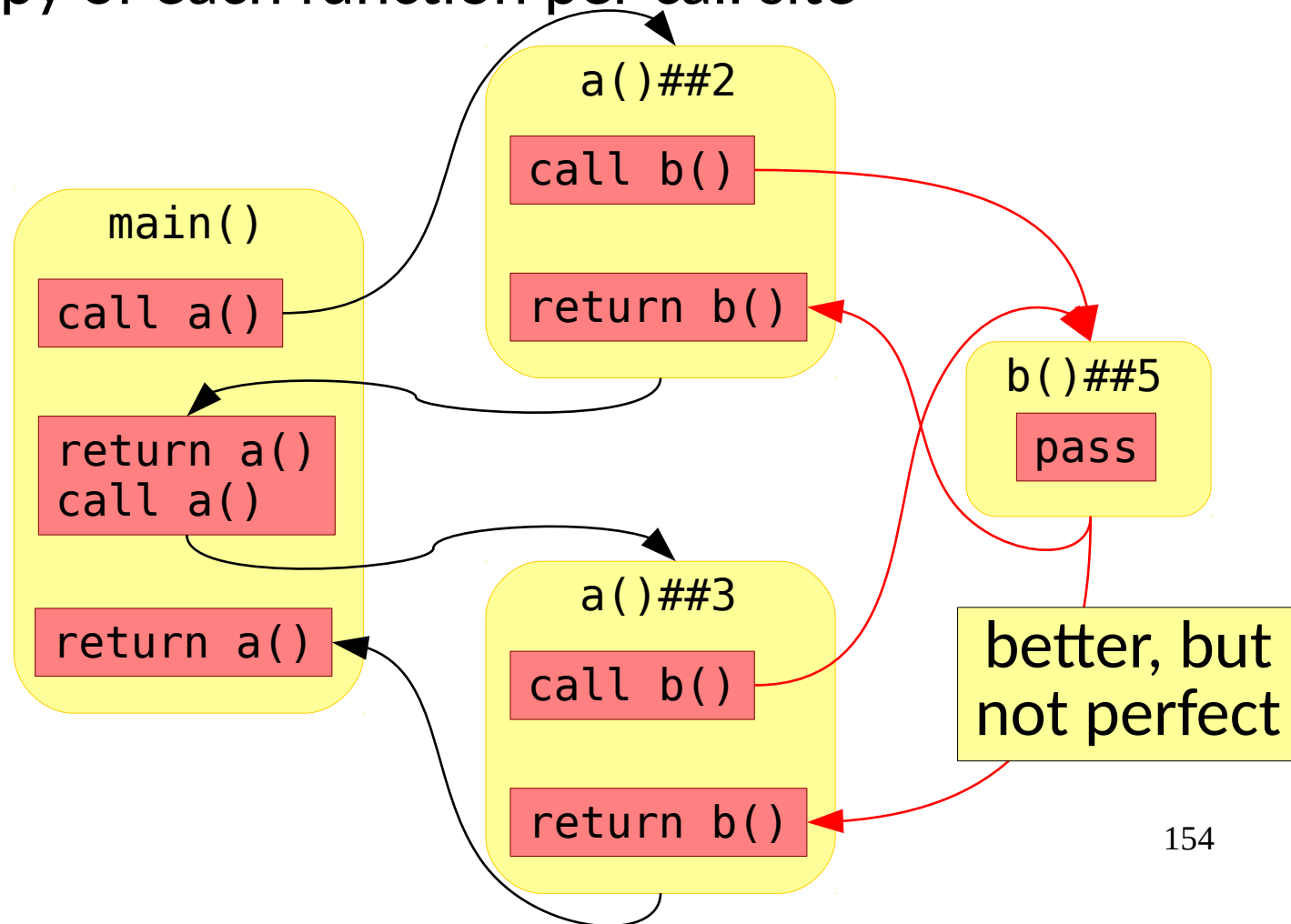
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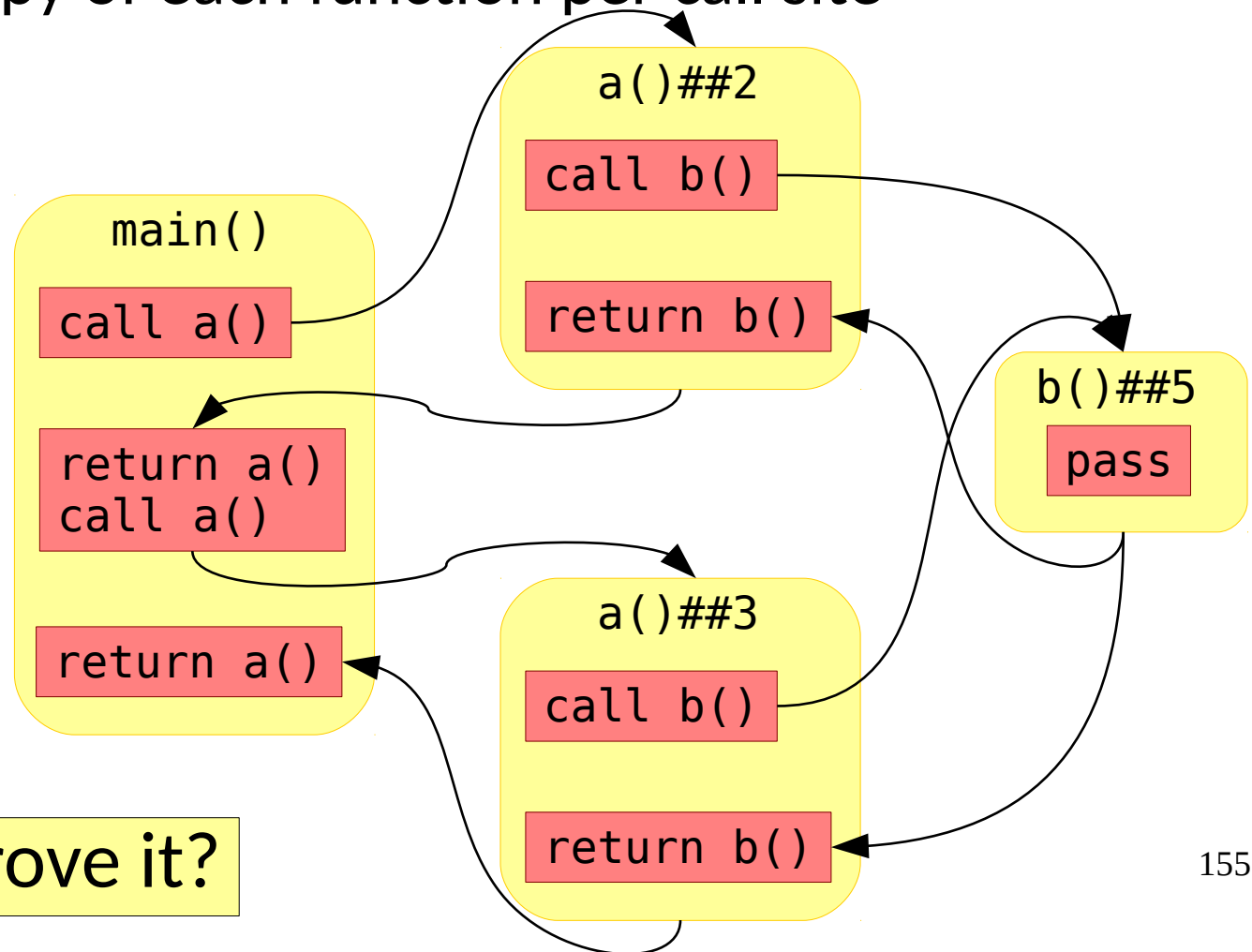
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How can we improve it?

Context Sensitivity

Generalized:

- Make a bounded number of copies

Context Sensitivity

Generalized:

- Make a bounded number of copies
- Choose a key/feature that determines which copy to use
 - Bounded calling context/call stack (*call site sensitivity*)
 - Allocation sites of objects (*object sensitivity*)

Context Sensitivity

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On `foo(in)` with context C:

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If `(foo,C)` doesn't have a summary, process `foo(in)` in C and save the result to S.

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Otherwise, process `foo(in)` in `C` and update `S` with `(in \sqcap S.in)`.

If the result changes, reprocess all callers of `(foo,C)`

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(**I**nterprocedural **F**inite **D**istributive **S**ubsets)

Context Sensitivity - IFDS

- Consider an undefined variable analysis...

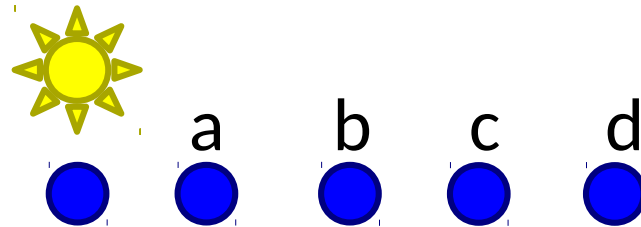
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b = a

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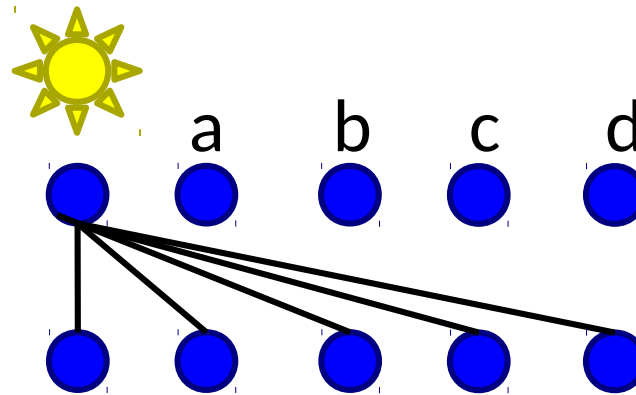


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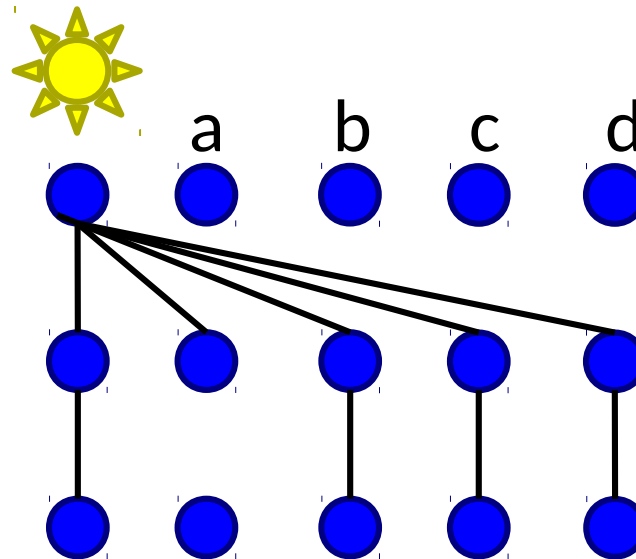
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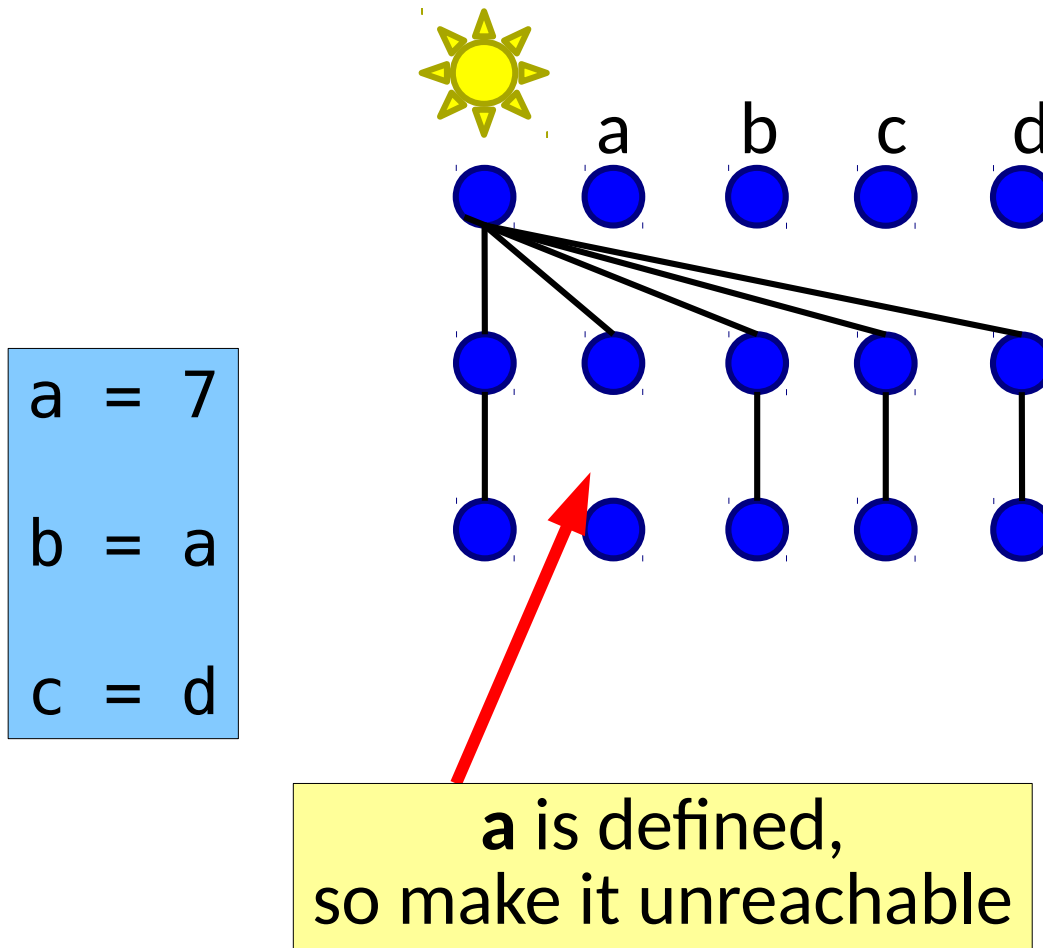
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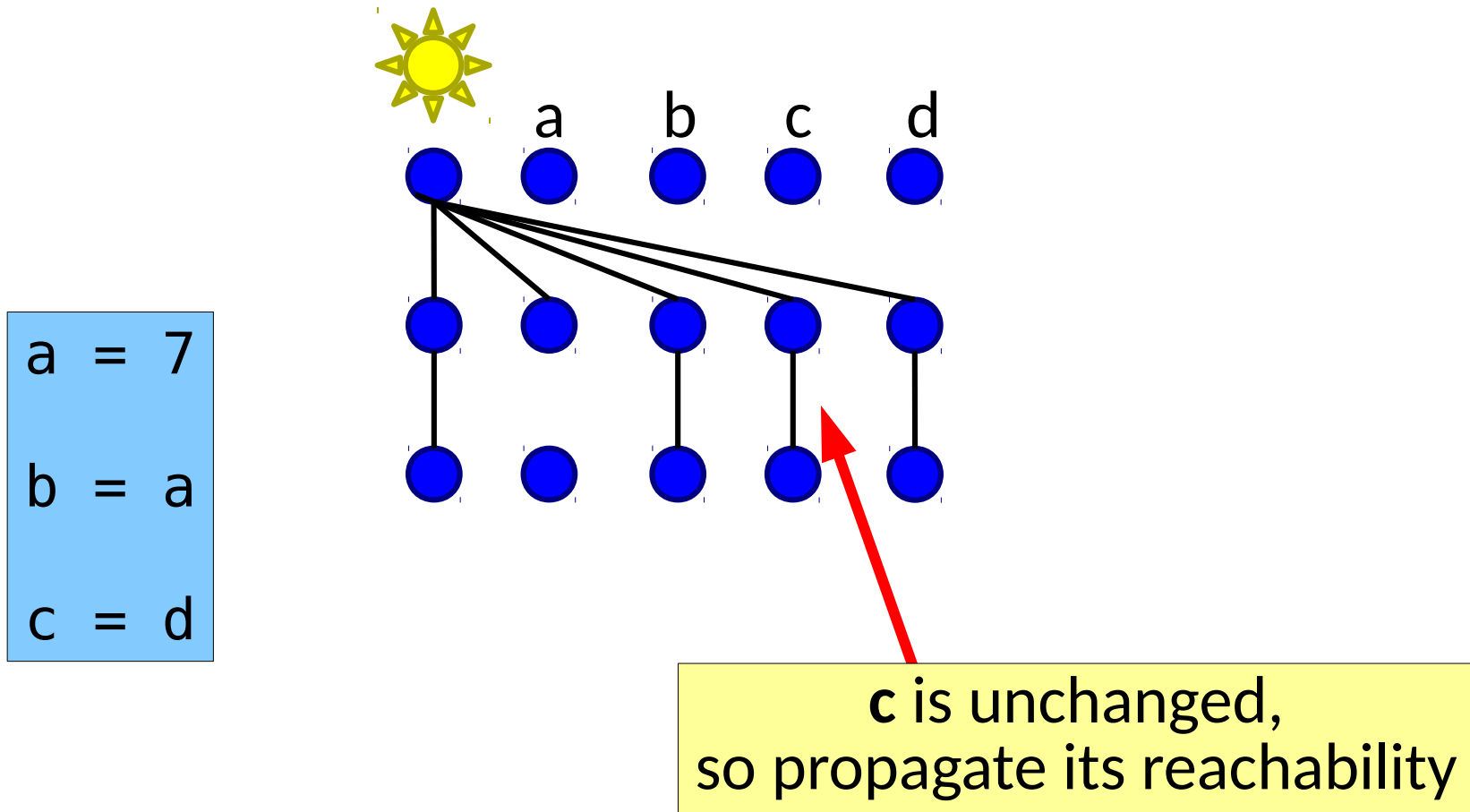
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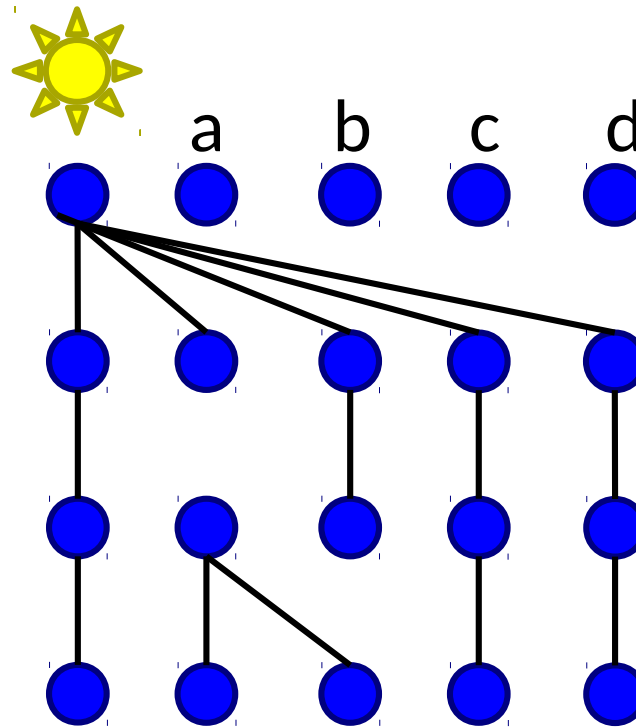
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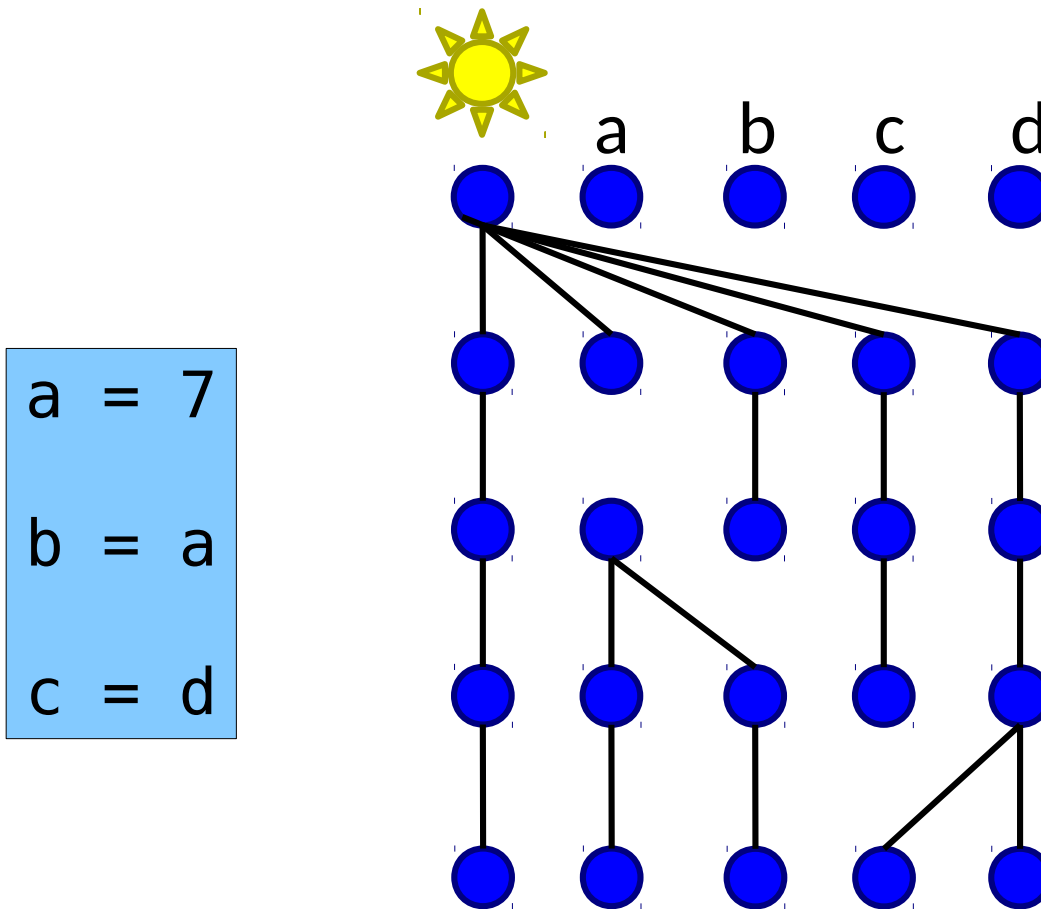
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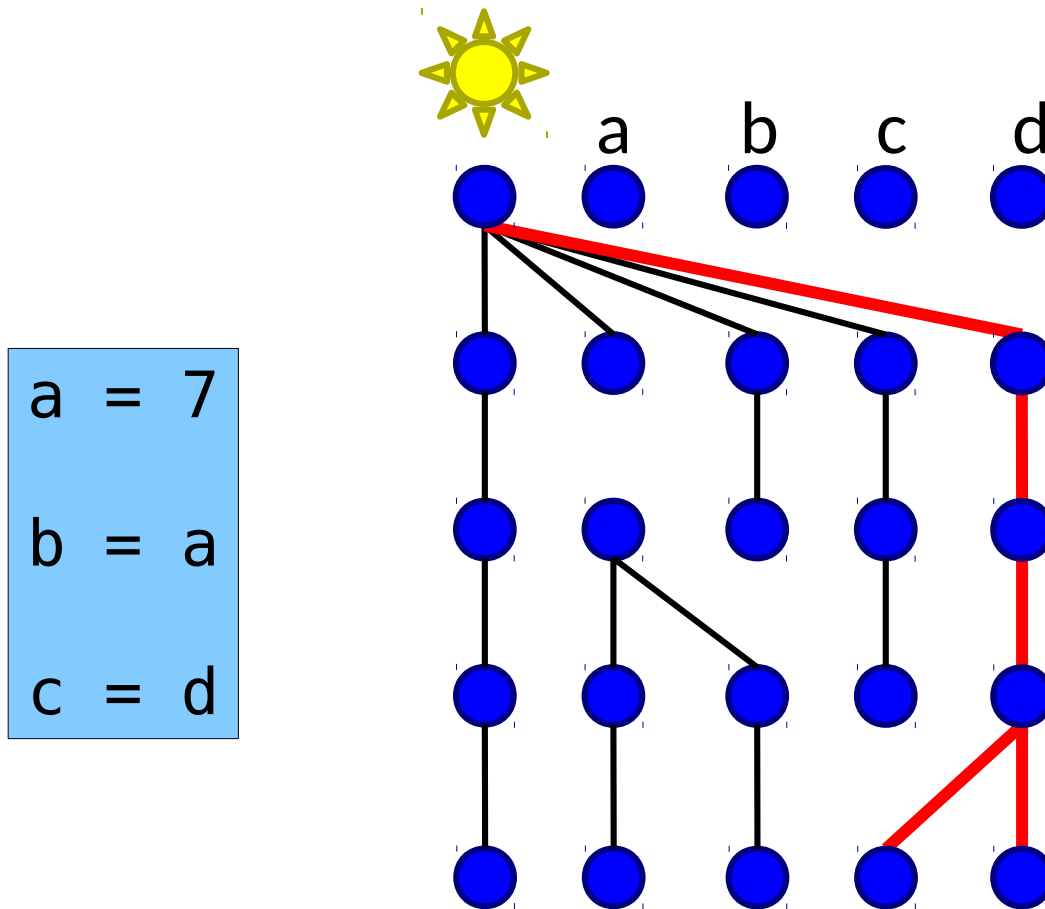
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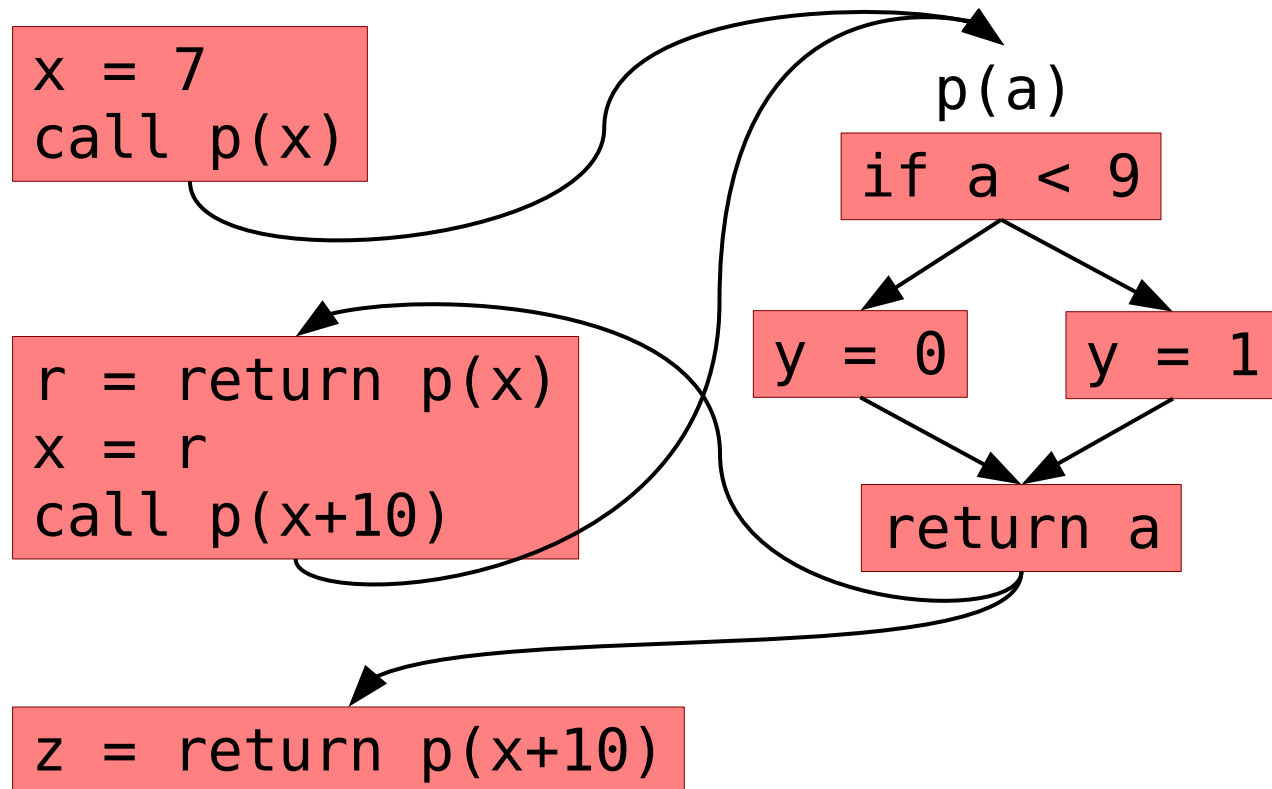
c and d are reachable here.
They are undefined at this point.

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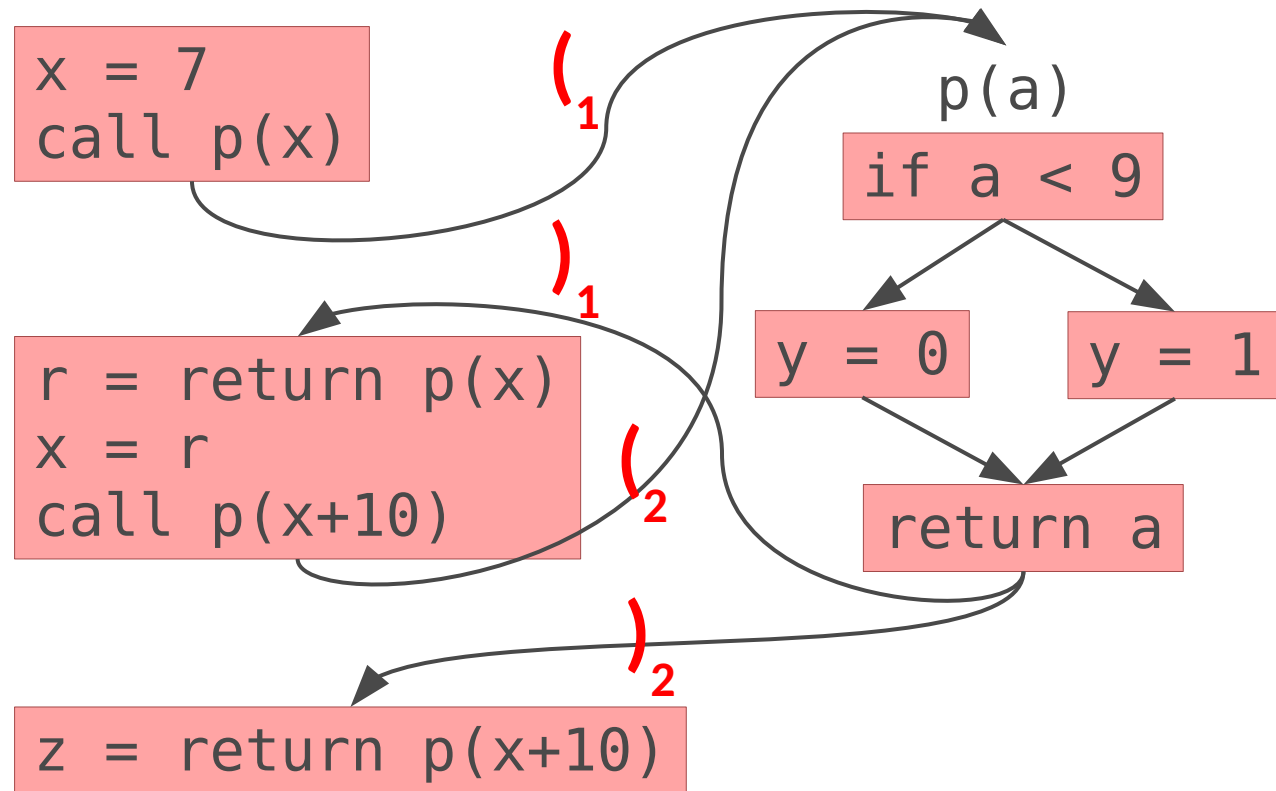


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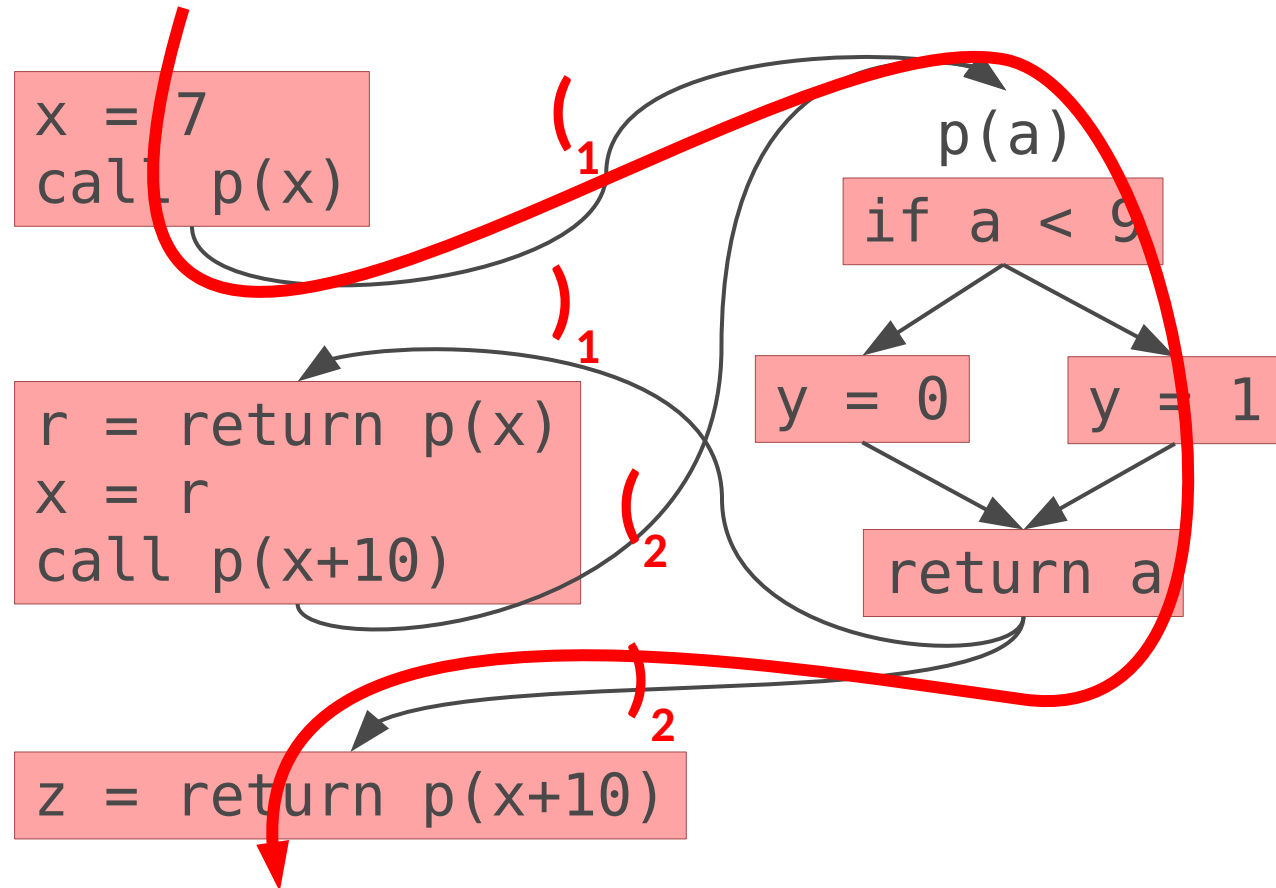


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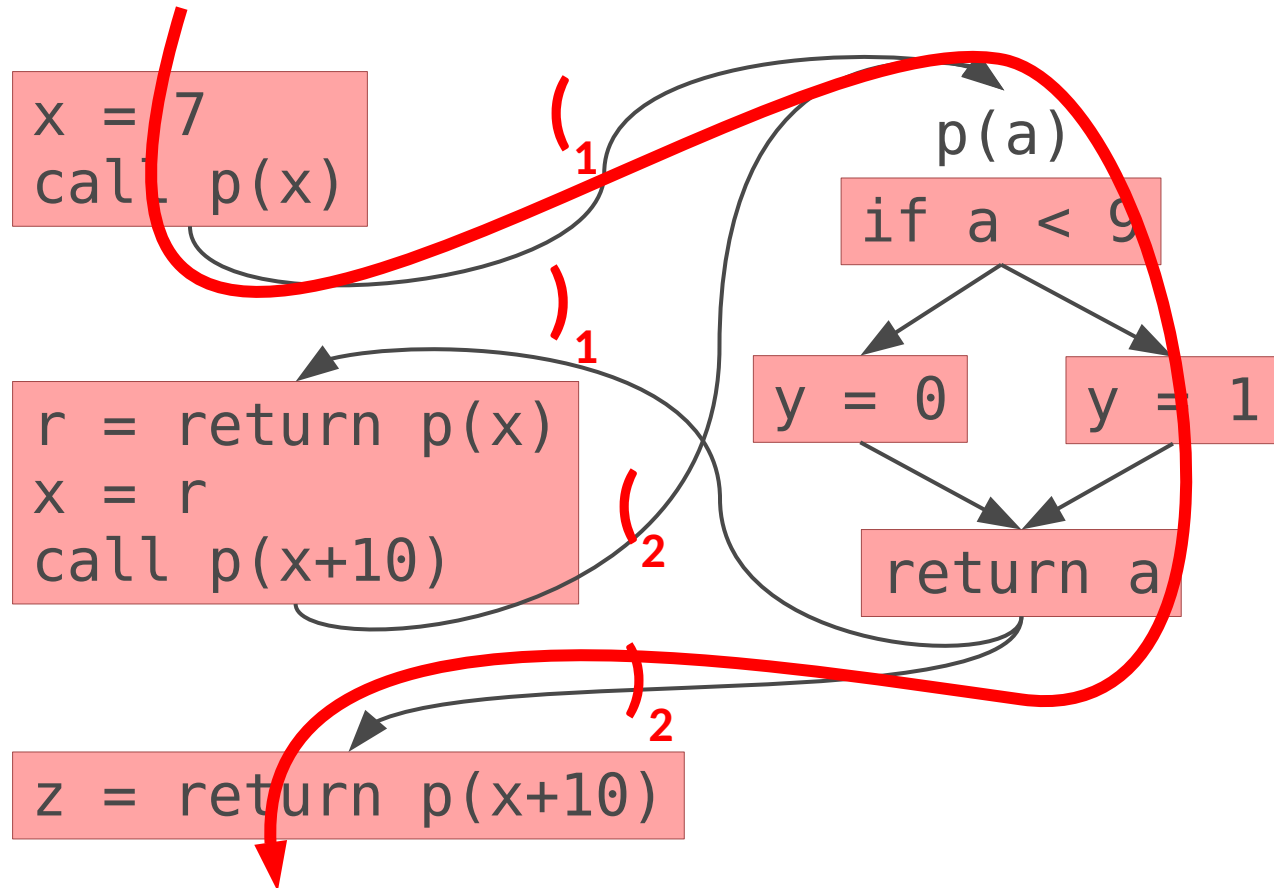
string: $(\)_2$

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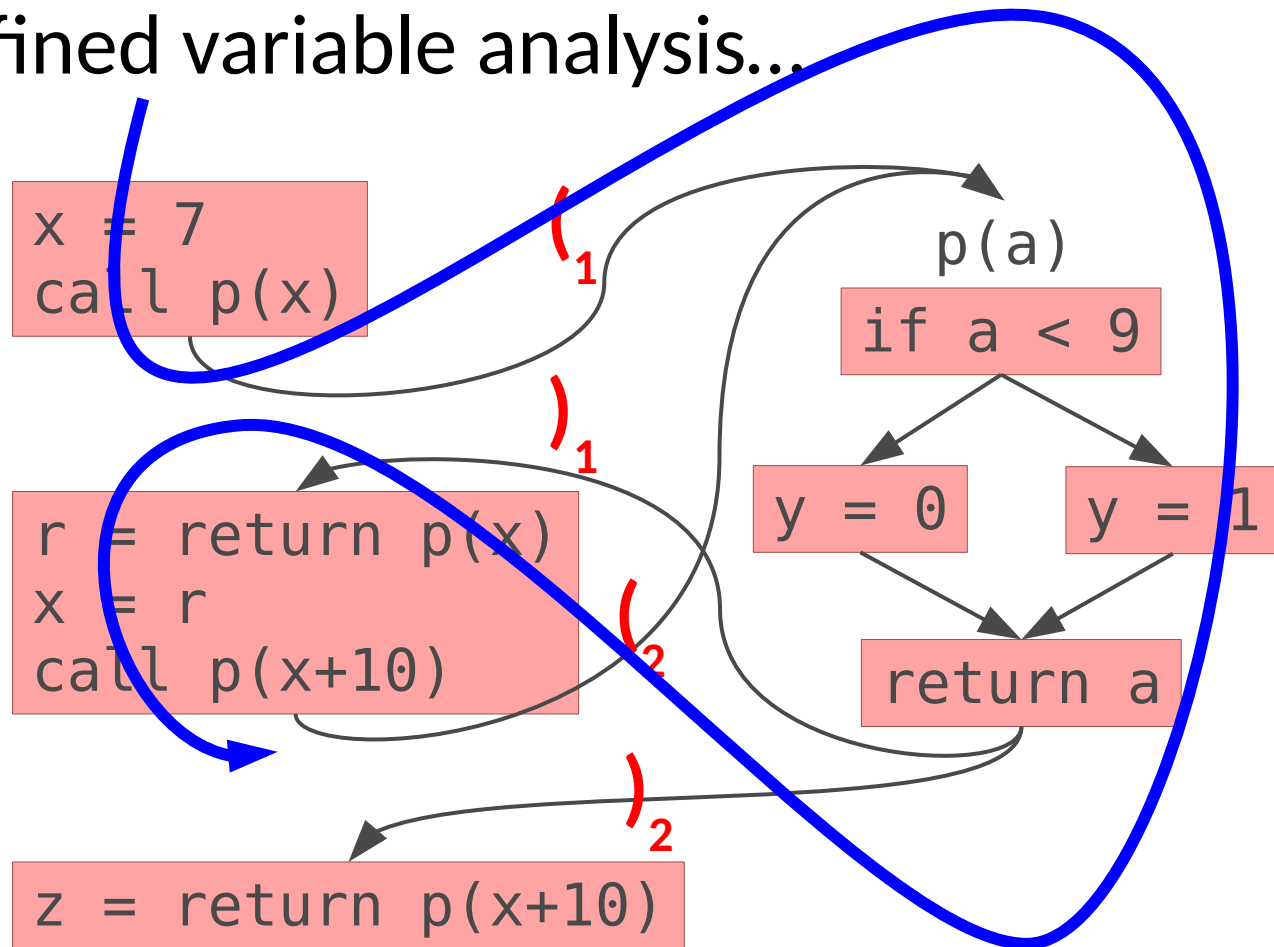
string: $(\)_1)_2$ unreachable

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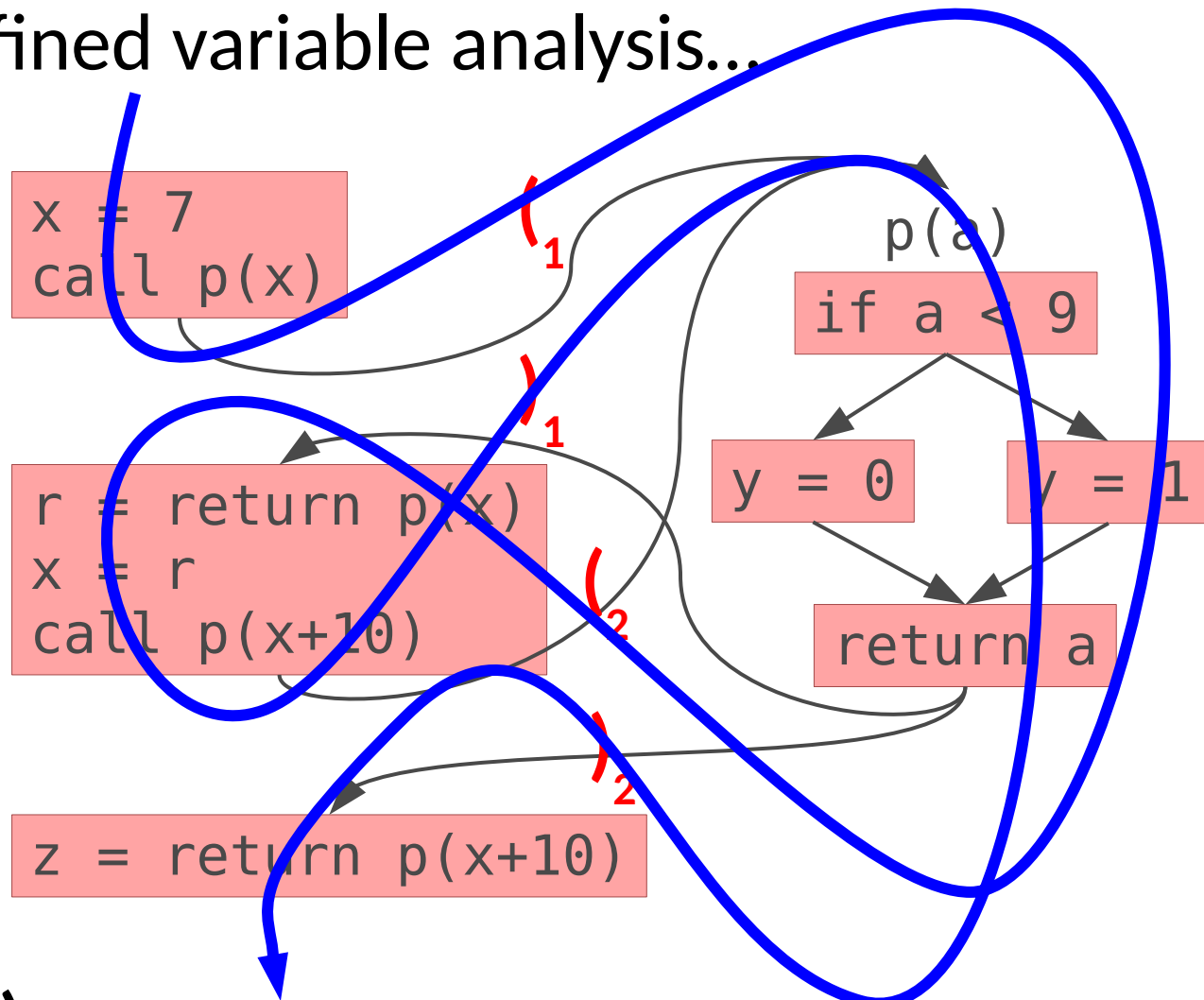
string: $(\)_1$

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string: $(\)_1 (\)_2$

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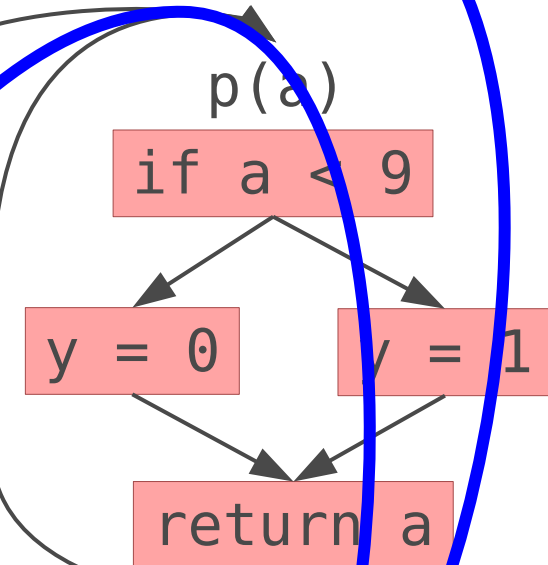
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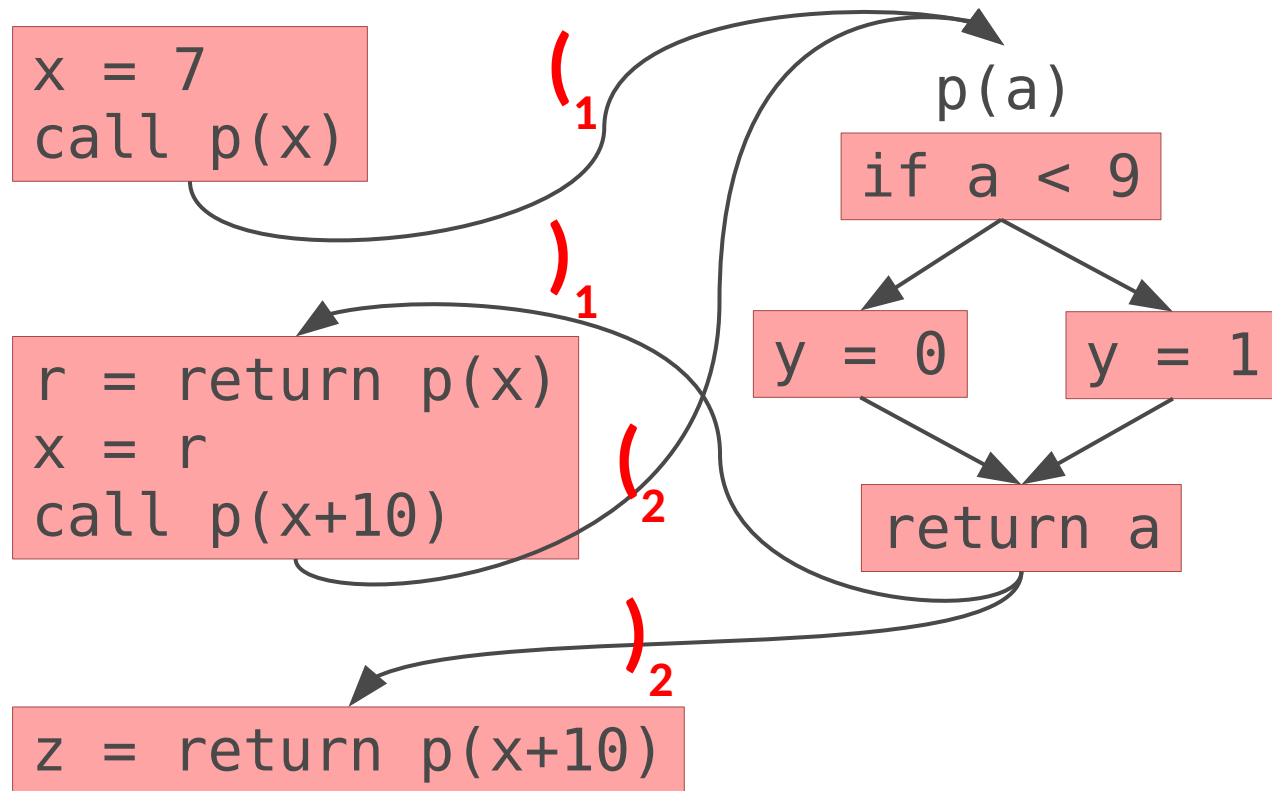
string: $(\)_1 (\)_2$ reachable

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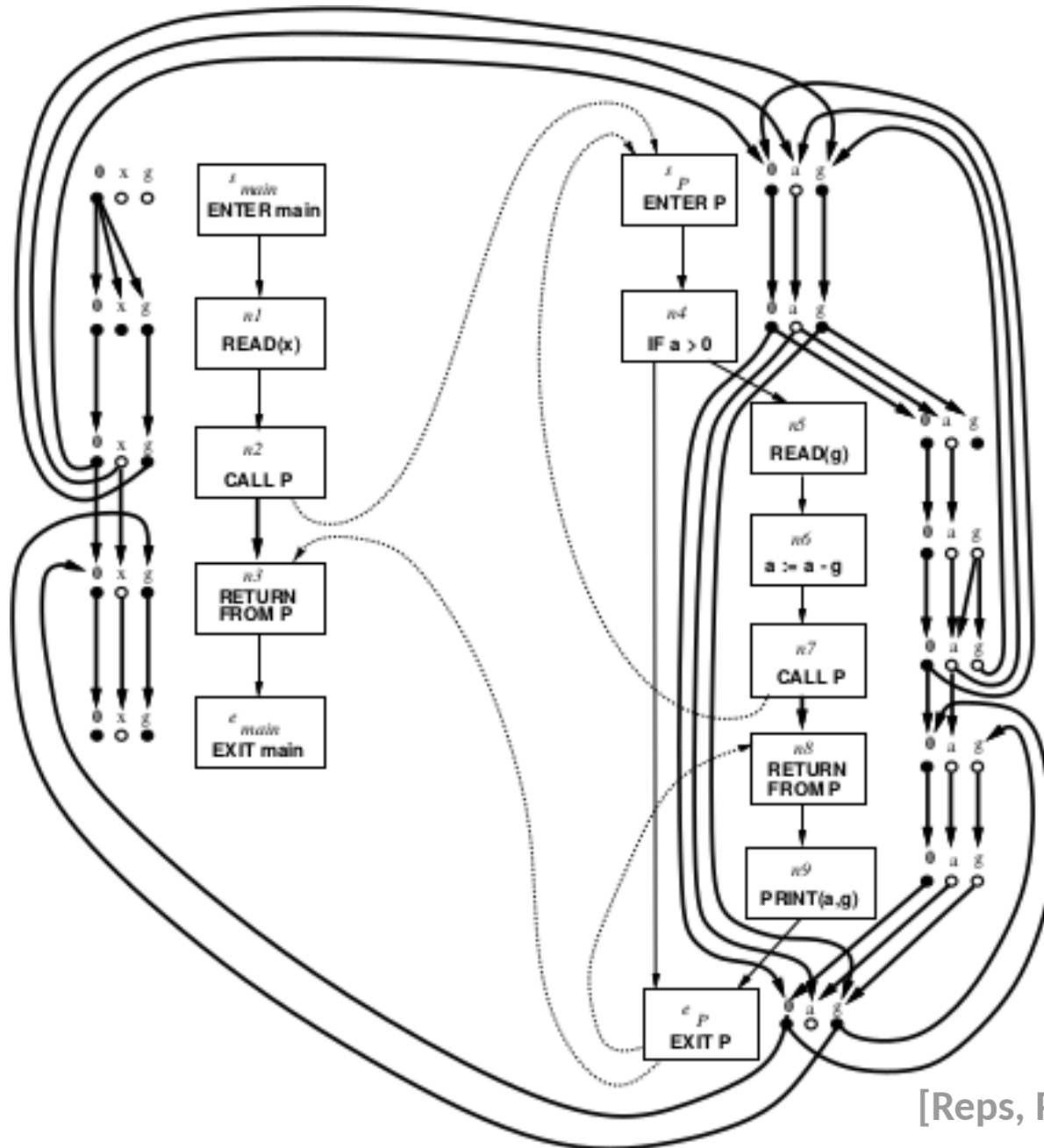
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- A fact f holds before a node if f is *CFL-Reachable* in a language of matched parentheses

Context Sensitivity - IFDS



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- Does constant propagation fit our definition of IFDS?

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- Does constant propagation fit our definition of IFDS?
- Can you think of ways that it could be made to fit into IFDS?

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The configuration is ultimately driven by the property/problem of interest

Static Analysis

- We've already seen a few static analyses:
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 - Call graph construction
 - Points-to graph construction (What are MAY/MUST?)
 - Static slicing
- The choices for approximation are why these analyses are imprecise.

Other (Traditionally) Static Approaches

- Type based analyses
- Bounded state exploration
- Symbolic execution
- Model checking

Many of these have been integrated into *dynamic* analyses, as we shall see over the semester.

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- Dataflow analysis is one common form of static analysis