Static Analysis & Dataflow Analysis

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Static Analysis

Static analyses consider *all* possible behaviors of a program without running it.
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- Look for a property of interest
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  - Do I dereference NULL pointers?
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  - Do I dereference NULL pointers?
  - Do I leak memory?
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- **Look for a property of interest**
  - Do I dereference NULL pointers?
  - Do I leak memory?
  - Do I violate a protocol specification?
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  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
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  - Is this file open?
  - Does my program terminate?
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  - Do I leak memory?
  - Do I violate a protocol specification?
  - Is this file open?
  - Does my program terminate?

But wait? Isn't that impossible?
Static Analysis

Brief Review of Undecidability

“Does my program terminate?”
Static Analysis

Brief Review of Undecidability

P

HALT?
Static Analysis

Brief Review of Undecidability

P

HALT?

or
Brief Review of Undecidability

\[
\text{if } \text{HALT?(P, P)}: \\
\quad \text{while True: \{ \}} \\
\text{else} \\
\quad \text{return True}
\]

\[
H' = P
\]

P

HALT?

or

P

P

or
Brief Review of Undecidability

if HALT?(P, P):
    while True:
    else
        return True

H' = if HALT?(P, P):
    while True: {}
else
    return True

P

H'?
Static Analysis

Brief Review of Undecidability

It's a classic paradox!
Static Analysis

Static analyses consider *all* possible behaviors of a program without running it.

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  - Do I leak memory?
  - Do I violate a protocol specification?
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- **Only if answers must be perfect.**
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  – Do I dereference NULL pointers?
  – Do I leak memory?
  – Do I violate a protocol specification?
  – Is this file open?
  – Does my program terminate?

But wait? Isn't that impossible?

• Only if answers must be perfect.
Overapproximate or underapproximate the problem, and try to solve this simpler version.
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- **Sound analyses**
  - Overapproximate
  - Guaranteed to find violations of property
  - May raise false alarms
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- **Sound analyses**
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- **Complete analyses**
  - Underapproximate
  - Reported violations are real
  - May miss violations
Static Analysis

Overapproximate or underapproximate the problem, and try to solve this simpler version.

- **Sound analyses**
  - Overapproximate
  - Guaranteed to find violations of property
  - May raise false alarms

- **Complete analyses**
  - Underapproximate
  - Reported violations are real
  - May miss violations

Striking the right balance is key to a useful analysis
Approximation

Modeled program behaviors
Approximation

Modeled program behaviors

- Consider some behaviors possible when they are not.
Approximation

Modeled program behaviors

Overapproximate

Possible Program Behavior

Underapproximate

Ignore some behaviors that *are* possible.
Approximation

- **Dynamic Analysis**
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible
Approximation

- **Dynamic Analysis**
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible

- **Static Analysis**
  - Feasible $\subseteq$ Analyzed
Approximation

- **Dynamic Analysis**
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible

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Approximation

- **Dynamic Analysis**
  - Analyzed $\subseteq$ Feasible
  - As # tests $\uparrow$, Analyzed $\rightarrow$ Feasible

- **Static Analysis**
  - Feasible $\subseteq$ Analyzed
  - As infeasible paths $\downarrow$, Analyzed $\rightarrow$ Feasible
Approximation

- **Dynamic Analysis**
  - Analyzed $\subseteq$ Feasible
  - As # tests ↑, Analyzed $\rightarrow$ Feasible

- **Static Analysis**
  - Feasible $\subseteq$ Analyzed
  - As infeasible paths ↓, Analyzed $\rightarrow$ Feasible

- **The two areas complement each other**
  - Static analysis can help generate useful tests
  - Dynamic analysis can help identify infeasibility
Abstract Interpretation

Q: Does a particular variable ever have a negative value?
   – Might be an offset into invalid memory!

Approximate the program's behavior
Abstract Interpretation

Q: Does a particular variable ever have a negative value?
   – Might be an offset into invalid memory!

Approximate the program's behavior

• Concrete domain: integers
• Abstract domain: \{-,0,+\} \cup \{\top,\bot\}
Abstract Interpretation

Q: Does a particular variable ever have a negative value?
   – Might be an offset into invalid memory!

Approximate the program's behavior

- Concrete domain: integers
- Abstract domain: \{-,-,\} \cup \{\top,\bot\}

\[
\begin{align*}
    \text{concrete}(x) &= 5 \quad \Leftrightarrow \quad \text{abstract}(x) = + \\
    \text{concrete}(y) &= -3 \quad \Leftrightarrow \quad \text{abstract}(y) = - \\
    \text{concrete}(z) &= 0 \quad \Leftrightarrow \quad \text{abstract}(z) = 0
\end{align*}
\]

Combines sets of the concrete domain
Abstract Interpretation

- Transfer Functions show how to evaluate this approximated program:
Abstract Interpretation

- **Transfer Functions** show how to evaluate this approximated program:
  - $+++ \rightarrow +$
  - $+-+ \rightarrow -$
  - $0+0 \rightarrow 0$
  - $0+- \rightarrow -$
  - $...$
  - $++- \rightarrow T$ (unknown / might vary)
  - $.../0 \rightarrow \perp$ (undefined)
Abstract Interpretation

- Transfer Functions show how to evaluate this approximated program:
  - \( + + + \rightarrow + \)
  - \( - + - \rightarrow - \)
  - \( 0 + 0 \rightarrow 0 \)
  - \( 0 + - \rightarrow - \)
  - \( \ldots \)
  - \( + + - \rightarrow \top \)(unknown / might vary)
  - \( \ldots / 0 \rightarrow \bot \)(undefined)

This type of approximation is called *abstract interpretation*. 
Abstract Interpretation

1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)
Abstract Interpretation

1) $\text{sum} = 0$
2) $i = 1$
3) if $i < N$
4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$
6) print($\text{sum}$)
7) print($i$)
Abstract Interpretation

1) sum = 0
2) i = 1
3) if i < N
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Abstract Interpretation

1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)

1 \mapsto \perp
i \mapsto \perp
2 \mapsto 0
i \mapsto \perp
3 \mapsto 0
i \mapsto +
6 \mapsto 0
i \mapsto +
7 \mapsto 0
i \mapsto +
Abstract Interpretation

1) $\text{sum} = 0$
2) $i = 1$
3) $\text{if } i < N$
4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$
6) $\text{print}(\text{sum})$
7) $\text{print}(i)$
Abstract Interpretation

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) \( \text{if } i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) \( \text{print(sum)} \)
7) \( \text{print(i)} \)
Abstract Interpretation

1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
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7) print(i)
Abstract Interpretation

1) sum = 0
2) i = 1
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4) i = i + 1
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6) print(sum)
7) print(i)
Abstract Interpretation

1) $\text{sum} = 0$
2) $i = 1$
3) if $i < N$
4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$
6) print($\text{sum}$)
7) print($i$)

Does the process ever end?
Abstract Interpretation

1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)
1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)

Can the final sum ever be negative?
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function
- For basic analyses, use a monotone framework
  Loosely: \(<\text{CFG, Transfer Function, Lattice Abstraction}>\)
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function

- For basic analyses, use a monotone framework
  - \{-,0,\} U \{\top, \perp\}
  - They define a partial order
Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function

- For basic analyses, use a monotone framework
  - \{-,0,+\} ∪ \{⊤,⊥\}
  - They define a partial order

Why does this specific example terminate?
Abstract Interpretation

• Guarantee termination by carefully choosing
  – The abstract domain
  – The transfer function

• For basic analyses, use a monotone framework

• But in theory a lattice need not be finite!
  (ranges/intervals, linear constraints, ...)

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Abstract Interpretation

- Guarantee termination by carefully choosing
  - The abstract domain
  - The transfer function

- For basic analyses, use a monotone framework

- But in theory a lattice need not be finite!
  - Widening operators can still make it feasible
    (e.g., heuristically raise to $\top$)
Abstract Interpretation

- What properties should a good abstraction have?
Abstract Interpretation

- What properties should a good abstraction have?

Concrete

\{1, 4\}  \{1, 5\}
What properties should a good abstraction have?
Abstract Interpretation

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Abstract Interpretation

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Concrete

\[ \mathbb{Z} \]

\{1, 2, 3, \ldots\}

\{1, 4, 5\}

\{1, 4\} \rightarrow \{1, 5\}

Abstract

\[ \top \]

\[ \bot \]

\[ 0 \]

\[ + \]

\[ - \]
Abstract Interpretation

- What properties should a good abstraction have?

Concrete $\mathbb{Z}$ $\{1, 2, 3, \ldots\}$ $\{1, 4, 5\}$ $\{1, 4\}$ $\{1, 5\}$

Abstract $\alpha$ $\top$ $\bot$ $\neg$ $0$ $\pm$
Abstract Interpretation

- What properties should a good abstraction have?
Abstract Interpretation

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Abstract Interpretation

- What properties should a good abstraction have?

\[ \mathbb{Z} \]

\{1, 2, 3, ...\}

\{1, 4, 5\}

\{1, 4\}

\( s \subseteq \gamma(\alpha(s)) \)

No concrete values were discarded by abstraction
Dataflow Analysis

- Dataflow analysis performs *model checking* of abstract interpretations
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator ($\sqcap$) combines results across program paths
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator (\(\sqcap\)) combines results across program paths

\[
x_A \mapsto + \quad x_B \mapsto - \quad x_C \mapsto ?
\]

\[
x_A \sqcap x_B = ?
\]
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator ($\sqcap$) combines results across program paths

\[
x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = ?
\]
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator ($\sqcap$) combines results across program paths

$$x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = ?$$
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator ($\sqcap$) combines results across program paths

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x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = ?
\]
Dataflow Analysis

- Dataflow analysis performs model checking of abstract interpretations
- Meet Operator ($\sqcap$) combines results across program paths

\[
x_A \sqcap x_B = \alpha(x_A) \sqcup \alpha(x_B) = T
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

\[
\begin{align*}
1) &\quad \text{sum} = 0 \\
2) &\quad i = 1 \\
3) &\quad \text{if } i < N \\
4) &\quad i = i + 1 \\
5) &\quad \text{sum} = \text{sum} + i \\
6) &\quad \text{print(sum)} \\
7) &\quad \text{print(i)}
\end{align*}
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

State transitions:
- sum → 0
- i → +
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   sum → ⊥
   i → ⊥
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)
```

```plaintext
sum → 0
i → +
sum → 0
i → +
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

<table>
<thead>
<tr>
<th>sum</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>⊥</td>
</tr>
<tr>
<td>+</td>
<td>⊥</td>
</tr>
<tr>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>0</td>
<td>+</td>
</tr>
</tbody>
</table>

```
sum → ⊥
i → ⊥
sum → 0
i → +
sum → 0
i → +
```
Dataflow Analysis

• Now model the abstract program state and propagate through the CFG.

1) $\text{sum} = 0$
2) $i = 1$
3) if $i < N$
4) $i = i + 1$
5) $\text{sum} = \text{sum} + i$
6) \text{print}(\text{sum})
7) \text{print}(i)$
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) \( \text{sum} = 0 \)
2) \( i = 1 \)
3) if \( i < N \)
4) \( i = i + 1 \)
5) \( \text{sum} = \text{sum} + i \)
6) print(\( \text{sum} \))
7) print(\( i \))
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

```
sum → ⊥
i → ⊥
sum → 0
i → +
sum → 0
i → +
sum → +
i → +
sum → 0
i → +
sum → 0
i → +
```
Now model the abstract program state and propagate through the CFG. Meet Operator

sum → ⊥
i → ⊥

sum → ?
i → +

sum → 0
i → +

sum → 0
i → +

sum → 0
i → +

sum → 0
i → +
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

The value *across all executions* is not -, 0, or +, so it is simply unknown/anything. $\top$
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

\[
\begin{align*}
1) \text{sum} &= 0 \\
2) i &= 1 \\
3) \text{if } i < N \quad \text{sum} &\rightarrow \bot \\
\text{i} &\rightarrow \bot \\
4) i &= i + 1 \\
5) \text{sum} &= \text{sum} + i \\
\text{sum} &\rightarrow \top \\
\text{i} &\rightarrow + \\
6) \text{print(sum)} \\
7) \text{print(i)} \\
\text{sum} &\rightarrow 0 \\
\text{i} &\rightarrow + \\
\end{align*}
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

\[
\begin{align*}
1) \quad & \text{sum} = 0 \\
2) \quad & \text{i} = 1 \\
3) \quad & \text{if i < N} \\
4) \quad & \text{i} = \text{i} + 1 \\
5) \quad & \text{sum} = \text{sum} + \text{i} \\
6) \quad & \text{print(sum)} \\
7) \quad & \text{print(i)} \\
\end{align*}
\]
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
   6) print(sum)
   7) print(i)
```

<table>
<thead>
<tr>
<th>Instruction</th>
<th>Sum</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum = 0</td>
<td>⊤</td>
<td>⊥</td>
</tr>
<tr>
<td>i = 1</td>
<td>⊥</td>
<td>⊥</td>
</tr>
<tr>
<td>if i &lt; N</td>
<td>⊤</td>
<td>⊤</td>
</tr>
<tr>
<td>i = i + 1</td>
<td>⊤</td>
<td>⊤</td>
</tr>
<tr>
<td>sum = sum + i</td>
<td>⊤</td>
<td>⊤</td>
</tr>
<tr>
<td>print(sum)</td>
<td>⊥</td>
<td>⊥</td>
</tr>
<tr>
<td>print(i)</td>
<td>⊥</td>
<td>⊥</td>
</tr>
</tbody>
</table>

```
sum → ⊥
i → ⊥
sum → ⊤
i → +
sum → 0
i → +
sum → ⊤
i → +
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```plaintext
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

<table>
<thead>
<tr>
<th>sum</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>⊥</td>
<td>⊥</td>
</tr>
<tr>
<td>⊤</td>
<td>+</td>
</tr>
<tr>
<td>⊤</td>
<td>⊤</td>
</tr>
<tr>
<td>⊤</td>
<td>+</td>
</tr>
<tr>
<td>⊤</td>
<td>+</td>
</tr>
<tr>
<td>⊤</td>
<td>+</td>
</tr>
</tbody>
</table>

```
sum → ⊥
i → ⊥
sum → T
i → +
sum → 0
i → +
sum → T
i → +
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
4) i = i + 1
5) sum = sum + i
6) print(sum)
7) print(i)

But we’re not done yet!
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   sum → ⊥
   i → ⊥

4) i = i + 1
5) sum = sum + i
   sum → ⊤
   i → +

6) print(sum)
   sum → ⊤
   i → +

7) print(i)
   sum → ⊤
   i → +
```
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

1) $\text{sum} = 0$
2) $\text{i} = 1$
3) if $\text{i} < \text{N}$
4) $\text{i} = \text{i} + 1$
5) $\text{sum} = \text{sum} + \text{i}$
6) $\text{print}(\text{sum})$
7) $\text{print}(\text{i})$

$\text{sum} \rightarrow \bot$
$\text{i} \rightarrow \bot$

$\text{sum} \rightarrow \top$
$\text{i} \rightarrow +$

$\text{sum} \rightarrow \top$
$\text{i} \rightarrow +$

$\text{sum} \rightarrow \top$
$\text{i} \rightarrow +$

$\text{sum} \rightarrow \top$
$\text{i} \rightarrow +$

$\text{sum} \rightarrow \top$
$\text{i} \rightarrow +$
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.

```
1) sum = 0
2) i = 1
3) if i < N
   4) i = i + 1
   5) sum = sum + i
6) print(sum)
7) print(i)
```

\[
\begin{align*}
\text{sum} & \rightarrow 0 \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow T \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow T \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow T \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow \bot \\
\text{i} & \rightarrow \bot \\
\text{sum} & \rightarrow T \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow T \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow \bot \\
\text{i} & \rightarrow \bot \\
\text{sum} & \rightarrow T \\
\text{i} & \rightarrow + \\
\text{sum} & \rightarrow \bot \\
\text{i} & \rightarrow \bot
\end{align*}
\]

Could we have done better?
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point
    (No more changes)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point
    (No more changes)
  - Proper ordering can improve the efficiency.
    (Topological Order, Strongly Connected Components)
Dataflow Analysis

- Now model the abstract program state and propagate through the CFG.
  - Continue until we reach a fixed point (No more changes)
  - Proper ordering can improve the efficiency.
    - (Topological Order, Strongly Connected Components)

Will it always terminate?
Dataflow Analysis

- Note: need to model program state before and after each statement
- Proper ordering & a work list algorithm improves the efficiency
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ⨍ state(p)
        ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
             ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work: 1 2 3 4
state: 

{(1 ↦ ⊥), (2 ↦ ⊥), (3 ↦ ⊥), (4 ↦ ⊥)}
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: 2 3 4
state: { (1 ↦ ⊥), (2 ↦ ⊥), (3 ↦ ⊥), (4 ↦ ⊥) }
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)  
            ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: \{2\{\{1\mapsto⊥\}\{2\mapsto⊥\}\}\{3\\\{3\mapsto⊥\}\}\{4\\{4\mapsto⊥\}\}\}\}
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = \prod_{p ∈ \text{preds(unit)}} state(p)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: 2 3 4
state: \{(1 \mapsto \bot), (2 \mapsto \bot), (3 \mapsto \bot), (4 \mapsto \bot)\}

unit = 1
old = ⊥
new = ⊥
sum → 0
i → +

1 → 2
2 → 3
3 → 4
4 → 1
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ⋂ state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
    state(unit) = new

work: \{ (1 ↦ ⊥), (2 ↦ +), (3 ↦ +), (4 ↦ ⊥) \}
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = \prod state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work: 3 4
state: 

{(1) 2
  ▸ sum → 0
  i → +

(3) 4
  ▸ sum → 0
  i → +
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work: {1, 2, 4, 2}
state: {1 → ⊥, 2 → ⊥, 3 → ⊥, 4 → ⊥}
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p) ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work:
{1→2}{2→3}{3→4}{4→

state:
{1→0 2→+ sum→0 i→+}{3→+ sum→+ i→+}{4→0 sum→+ i→+}
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

4,3 were added back to the list
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
  unit = take(work)
  old = state(unit)
  before = ⋂ state(p)
  ∀ p ∈ preds(unit)
  new = transfer(before, unit)
  if old ≠ after:
    work = work ∪ succs(unit)
    state(unit) = new

work: {1
  2
  3
  4
  state: (1
  sum → 0
  i → +
  sum → T
  i → +
  (2
  (3
  3
  4
  sum → +
  i → +
  sum → T
  i → +
  (4
  sum → 0
  i → +
  sum → T
  i → +

unit = 4
old = ⊤
new = T
sum → +
i → +
Worklist Algorithms

work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
    unit = take(work)
    old = state(unit)
    before = ∏ state(p)
    ∀ p ∈ preds(unit)
    new = transfer(before, unit)
    if old ≠ after:
        work = work ∪ succs(unit)
        state(unit) = new

work:
state:

No change
work = nodes()
state(n) = ⊥ ∀ n ∈ nodes()
while work ≠ ∅:
  unit = take(work)
  old = state(unit)
  before = ⋂ state(p)
    ∀ p ∈ preds(unit)
  new = transfer(before, unit)
  if old ≠ after:
    work = work ∪ successors(unit)
  state(unit) = new

Done!
Effect of Approximation

- There are several possible sources of imprecision
Effect of Approximation

- There are several possible sources of imprecision

1) $x = 2$
2) $y = 1$
3) $x = 2$
4) $y = 1$
5) $c = x \ast y$
Effect of Approximation

- There are several possible sources of imprecision

\[
\begin{align*}
1) & \quad x = 2 \\
2) & \quad y = 1 \\
3) & \quad x = -2 \\
4) & \quad y = -1 \\
5) & \quad c = x \times y
\end{align*}
\]

\[
\begin{align*}
x & \rightarrow +, \ y \rightarrow + \\
x & \rightarrow -, \ y \rightarrow -
\end{align*}
\]
Effect of Approximation

- There are several possible sources of imprecision
- 2 Key sources are
  - Control flow
    - Many different paths are summarized together
Effect of Approximation

• There are several possible sources of imprecision

• 2 Key sources are
  – Control flow
    • Many different paths are summarized together
  – Abstraction
    • Deliberately throwing away information
    • Granularity of program state affects correlations across variables
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)

\[ \text{For one path } p: \quad fp(\bot) = f_n(f_{n-1}(\ldots f_1(f_0(\bot)))) \]
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)

For one path $p$: $fp(\perp) = fn(fn-1(...f1(f0(\perp))))$
For all paths $p$: $\bigwedge fp(\perp)$
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
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- Are they different?
  - Sometimes. But sometime solutions are perfect.
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
  - Sometimes. But sometime solutions are perfect.
  - When \( f() \) is *distributive*, MFP=MOP
    \[
    f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z)
    \]
Effect of Approximation

- We compute results with maximal fixed points (MFP) in the lattice
- Ideal solution is a Meet Over all Paths (MOP)
- Are they different?
  - Sometimes. But sometime solutions are perfect.
  - When $f()$ is distributive, MFP=MOP
    \[ f(x \sqcap y \sqcap z) = f(x) \sqcap f(y) \sqcap f(z) \]
  - This applies to an important class of problems called *bitvector frameworks*. 
Bitvector Frameworks

- When the property concerns subsets of a finite set, the abstract domain & lattice are easy:
  - Concrete: \( D = \{a, b, c, d, \ldots \} \)
  - Abstract: \( \mathcal{P}(D) = \{\{\}, \{a\}, \{b\}, \ldots, \{a, b\}, \{a, c\}, \ldots\} \)
  - Lattice: Defined by subset relation:

![Diagram showing the lattice structure of subsets of a finite set.](image-url)
When the property concerns subsets of a finite set, the abstract domain & lattice are easy:

- Concrete: $D = \{a, b, c, d, \ldots \}$
- Abstract: $\wp(D) = \{\emptyset, \{a\}, \{b\}, \ldots, \{a, b\}, \{a, c\}, \ldots\}$
- Lattice: Defined by subset relation:

What would the meet operator be?
Bitvector Frameworks

- Why is this convenient?
  - Hint: *bitvector* frameworks
Bitvector Frameworks

• Why is this convenient?
  – Hint: *bitvector* frameworks
  – \( X=\{a,b\}, \ Y=\{c,d\} \rightarrow X \uplus Y = \{a,b\} \cup \{c,d\} = \{a,b,c,d\} \)
  – We can implement the abstract state using efficient bitvectors!
Effect of Approximation

- If approximation yields imprecise results, why do we do it?
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- **Abstract domain**: e.g. \{-,0,+\} \cup \{\top, \bot\}
- **Transfer functions**: - + + \rightarrow \top
- **Bounded domain lattice height**:
- **Concern for false + & -**
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- Abstract domain: e.g. \{-,0,+\} \bigcup \{\top,\bot\}
- Transfer functions: - + + \rightarrow \top
- Bounded domain lattice height:
- Concern for false + & -

Implementation:
- Computing using work lists
- Speeding up by sorting CFG nodes
Recap: Dataflow Analysis

Analyze complex behavior with approximation:

- Abstract domain: e.g. \{-,0,+\} \cup \{T,\bot\}
- Transfer functions: \(- + + \rightarrow T\)
- Bounded domain lattice height:
- Concern for false + & -

Implementation:
- Computing using work lists
- Speeding up by sorting CFG nodes

Let's see an example
File Policy Analysis

**Goal:** Identify potential misuses of open/closed files
File Policy Analysis

Goal: Identify potential misuses of open/closed files
• Files may be open or closed
File Policy Analysis

Goal: Identify potential misuses of open/closed files
- Files may be open or closed
- Many operations may only occur on open files
e.g. read, write, print, flush, close, ...
File Policy Analysis

**Goal:** Identify potential misuses of open/closed files

- Files may be open or closed
- Many operations may only occur on open files
  e.g. read, write, print, flush, close, ...

What should our design actually be?

- Abstract domain?
- Transfer functions?
- Lattice?
File Policy Analysis

Goal: Identify potential misuses of open/closed files
- Files may be open or closed
- Many operations may only occur on open files
  e.g. read, write, print, flush, close, ...

What should our design actually be?
- Abstract domain?
- Transfer functions?
- Lattice?

[DEMO]
Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care
Flow Insensitive Analysis

- Saw *flow sensitive* analysis
  - Modeling state at each statement is expensive
  - Scales to functions and small components
  - Usually not beyond 1000s of lines without care

- *Flow insensitive* analyses aggregate into a global state
  - Better scalability
  - Less precision
  - “Does this function modify global variable X?”
Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
  - “If bar() is called by foo(), then it is exception free.”
  - Can enable more precise *interprocedural* analyses
Context Sensitive Analyses

- Program behavior may be dependent on the call stack / calling context.
  - “If bar() is called by foo(), then it is exception free.”
  - Can enable more precise *interprocedural* analyses

Can you imagine how to solve this?
What problems might arise?
Context Sensitivity

- Recall that we can extract a call graph
  - Just as you are doing in your first project!

```python
def a():
    b()
    ...
    b()
def b():
    ...
    c()
def c():
    ...
```

The behavior of `c()` could be affected by each “...”

Modeling them can make analysis more precise.
Context Sensitivity

- Simplest Approach
  - Add edges between call sites & targets
  - Perform data flow on this larger graph

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
```

Example from Stephen Chong
Context Sensitivity

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def main():
    x = 7
    r = p(x)
    x = r
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def p(a):
    if a < 9:
        y = 0
    else:
        y = 1

r = return p(x)
call p(x)
r = return p(x)
call p(x+10)
main()
p(a)
if a < 9
    y = 0
else:
    y = 1
return a
z = return p(x+10)
```

Example from Stephen Chong
Context Sensitivity

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    return a

main()
```

Example from Stephen Chong
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Example from Stephen Chong
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Example from Stephen Chong
Context Sensitivity

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  - Add edges between call sites & targets
  - Perform data flow on this larger graph

Example from Stephen Chong

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def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1

r = return p(x)
main()
if a < 9
r = return p(x)
call p(x+10)
y = 0
else:
y = 1
return a
z = return p(x+10)
a = T
```
Context Sensitivity

- Simplest Approach
  - Add edges between call sites & targets
  - Perform data flow on this larger graph

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def main():
    x = 7
    r = p(x)
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        y = 1
    return a
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Example from Stephen Chong
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  - Add edges between call sites & targets
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    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
    return a
```

Example from Stephen Chong
Context Sensitivity

- Information from one call site can flow to a mismatched return site!
Context Sensitivity

• Information from one call site can flow to a mismatched return site!
• How could we address it?
Context Sensitivity

- Solution 2: Inlining
  - Make a copy of the function at each call site
Context Sensitivity

• Solution 2: Inlining
  – Make a copy of the function at each call site

• What problems arise?
Context Sensitivity

- Solution 2: Inlining
  - Make a copy of the function at each call site
- What problems arise?
- What other strategies can we use?
Context Sensitivity

- Solution 3: Make a Copy
  - Make one copy of each function per call site
Context Sensitivity

• Solution 3: Make a Copy
  – Make one copy of each function per call site

1) def main():
2)   a()
3)   a()
4) def a():
5)   b()
6) def b():
7)   pass
Context Sensitivity

- **Solution 3: Make a Copy**
  - Make one copy of each function per call site

1) def main():
2)   a()
3)   a()
4) def a():
5)   b()
6) def b():
7)   pass

```
1) def main():
2)   a()
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3)   a()
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3)   a()
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```
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3)   a()
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```
1) def main():
2)   a()
3)   a()
4) def a():
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6) def b():
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```
• Solution 3: Make a Copy
  - Make one copy of each function per call site

1) def main():
2)   a()
3)   a()

4) def a():
5)   b()

6) def b():
7)   pass

So far, so good

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Context Sensitivity

- Solution 3: Make a Copy
  - Make one copy of each function per call site

1) def main():
2)   a()
3)   a()

4) def a():
5)   b()

6) def b():
7)   pass

Better, but not perfect
Context Sensitivity

- **Solution 3: Make a Copy**
  - Make one copy of each function per call site

```
1) def main():
2)   a()
3)   a()

4) def a():
5)   b()

6) def b():
7)   pass
```

How can we improve it?
Context Sensitivity

Generalized:

- Make a bounded number of copies
Context Sensitivity

Generalized:

- Make a bounded number of copies
- Choose a key/feature that determines which copy to use
  - Bounded calling context/call stack (*call site sensitivity*)
  - Allocation sites of objects (*object sensitivity*)
Context Sensitivity

- Solution 4: Make a *logical* copy
Context Sensitivity

• Solution 4: Make a *logical* copy
  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
Context Sensitivity

• Solution 4: Make a *logical* copy
  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
  – Compute results (called *procedure summaries*) for each logical copy of a function.
Context Sensitivity

- **Solution 4: Make a *logical* copy**
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:

```
On foo(in) with context C:
```
Context Sensitivity

- Solution 4: Make a *logical* copy
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:

    **On foo(in) with context C:***
    
    If (foo,C) doesn't have a summary, process foo(in) in C and save the result to S.
Context Sensitivity

- Solution 4: Make a *logical* copy
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:

    **On foo(in) with context C:**
    - If (foo,C) doesn't have a summary, process foo(in) in C and save the result to S.
    - If the summary S already approximates foo(in), use S
Context Sensitivity

- **Solution 4: Make a *logical* copy**
  - Instead of actually making a copy, just keep track of the context information (the key) during analysis
  - Compute results (called *procedure summaries*) for each logical copy of a function.
  - Modify the treatment of calls slightly:

  **On** \texttt{foo(in)} \texttt{with context C}:
  
  If \((\texttt{foo,C})\) doesn't have a summary, process \texttt{foo(in)} in \(C\) and save the result to \(S\).
  
  If the summary \(S\) already approximates \texttt{foo(in)}, use \(S\)
  
  Otherwise, process \texttt{foo(in)} in \(C\) and update \(S\) with \((\texttt{in}) \bigcap S.\text{in})\).
Context Sensitivity

• Solution 4: Make a *logical* copy
  – Instead of actually making a copy, just keep track of the context information (the key) during analysis
  – Compute results (called *procedure summaries*) for each logical copy of a function.
  – Modify the treatment of calls slightly:

```
On foo(in) with context C:
  If (foo,C) doesn't have a summary, process foo(in) in C and save the result to S.
  If the summary S already approximates foo(in), use S
  Otherwise, process foo(in) in C and update S with (in⨅S.in).
If the result changes, reprocess all callers of (foo,C)
```
Context Sensitivity - IFDS

- In some cases, context sensitive analysis can be reduced to special forms of graph reachability.
• In some cases, context sensitive analysis can be reduced to special forms of graph reachability.
  – Set of dataflow facts D is finite
  – Transfer functions are distributive \[ f(x \sqcap y) = f(x) \sqcap f(y) \]
  – Domain and range of transfer functions is \( \mathcal{P}(D) \)
  – Lattice ordering is set containment
Context Sensitivity - IFDS

- In some cases, context sensitive analysis can be reduced to special forms of graph reachability.
  - Set of dataflow facts D is finite
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  - Domain and range of transfer functions is \( \mathcal{P}(D) \)
  - Lattice ordering is set containment

  (Interprocedural Finite Distributive Subsets)
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

\[
a = 7 \\
b = a \\
c = d
\]
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```
a = 7
b = a
c = d
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

\[
\begin{align*}
a &= 7 \\
b &= a \\
c &= d
\end{align*}
\]
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```
a = 7
b = a
c = d
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```
  a = 7  
b = a  
c = d  
```

```
  a  b  c  d

a is defined, so make it unreachable
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```plaintext
a = 7
b = a
c = d
```

![Diagram showing variable reachability]

- c is unchanged, so propagate its reachability
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```
a = 7
b = a
c = d
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```
  a = 7
  b = a
  c = d
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```
a = 7
b = a
c = d
c and d are reachable here. They are undefined at this point.
```
Context Sensitivity - IFDS

Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1

    if a < 9:
        r = return p(x)
        x = r
        call p(x+10)

    y = 0
    return a

    y = 1
    return a
```

```python
x = 7
call p(x)
```

```python
r = return p(x)
x = r
call p(x+10)
```

```python
z = return p(x+10)
```

```python
p(a)
if a < 9
    y = 0
    return a
else:
    y = 1
    return a
```
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
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def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
```

```
x = 7
call p(x)
```

```
r = return p(x)
```

```
x = r
```

```
if a < 9
y = 0
y = 1

return a
```

```
z = return p(x+10)
```
Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1

    r = return p(x)
    x = r
    call p(x+10)
    z = return p(x+10)

    if a < 9
        p(a)
        y = 0
        y = 1
    return a
```

string: \((1)_{2}\)
Consider an undefined variable analysis...

def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1

string: (1)₂ unreachable
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
    return a

r = p(x)
call p(x)

x = r
call p(x+10)

z = p(x+10)
```

string: $(1)_1$
Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1

r = return p(x)
    x = r
    call p(x+10)

z = return p(x+10)
```

String: $(1)_1(2)_2$
Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
    return a

x = 7
r = p(x)
new line
x = r
new line
z = p(x+10)
```

string: \((1)_1(2)_2\) reachable
Context Sensitivity - IFDS

- Consider an undefined variable analysis...

```python
def main():
    x = 7
    r = p(x)
    x = r
    z = p(x+10)

def p(a):
    if a < 9:
        y = 0
    else:
        y = 1
    x = 7
    call p(x)
    r = return p(x)
    x = r
    call p(x+10)
    z = return p(x+10)
```

- A fact f holds before a node if f is **CFL-Reachable** in a language of matched parentheses
Context Sensitivity - IFDS

[Reps, POPL 1995]
Context Sensitivity - IFDS

• Does constant propagation fit our definition of IFDS?
Context Sensitivity - IFDS

- Does constant propagation fit our definition of IFDS?
- Can you think of ways that it could be made to fit into IFDS?
Dataflow Configurations

Can be configured in many ways:
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- Forward / Backward (e.g. reaching vs liveness)
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- May / Must (∪ vs ∩ in lattice when paths ∩)

Dataflow Configurations

Can be configured in many ways:

- Forward / Backward (e.g. reaching vs liveness)
- May / Must ($\cup$ vs $\cap$ in lattice when paths $\prod$)
- Sensitivity {Path? Flow? Context?}
Dataflow Configurations

Can be configured in many ways:

- Forward / Backward (e.g. reaching vs liveness)
- May / Must (\(\cup\) vs \(\cap\) in lattice when paths \(\prod\))
- Sensitivity {Path? Flow? Context?}

The configuration is ultimately driven by the property/problem of interest
Static Analysis

● We've already seen a few static analyses:
  – Call graph construction
  – Points-to graph construction (What are MAY/MUST?)
  – Static slicing
Static Analysis

• We've already seen a few static analyses:
  – Call graph construction
  – Points-to graph construction (What are MAY/MUST?)
  – Static slicing
• The choices for approximation are why these analyses are imprecise.
Other (Traditionally) Static Approaches

- Type based analyses
- Bounded state exploration
- Symbolic execution
- Model checking

Many of these have been integrated into dynamic analyses, as we shall see over the semester.
Static Analysis Summary

- Considers all possible executions
Static Analysis Summary

- Considers all possible executions
- Approximates program behavior to fight undecidability
Static Analysis Summary

- Considers all possible executions
- Approximates program behavior to fight undecidability
- Can answer queries like:
  - Must my program always ...?
  - May my program ever ...?
Static Analysis Summary

• Considers all possible executions
• Approximates program behavior to fight undecidability
• Can answer queries like:
  – **Must** my program always ...?
  – **May** my program ever ...?
• Dataflow analysis is one common form of static analysis