A/B Testing & Bandit Based Solutions

Nick Sumner
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  - You maintain a web site and are considering a change
  - You hypothesize that the change improves outcomes in some way
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Why Snapchat's re-redesign will fail and how to fix it, TechCrunch, 2018.
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You should already have an intuition for attacking this. What should you do?
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- **Solutions**
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  – Alternatively, you can use **multi-armed bandits** to attack the problem
    – Key idea: run controlled experiments live on the deployed software

• Caveat: We **will not** dive into a full stats background for these
  – We **will** discuss some common pitfalls that arise from misunderstandings
When might you want to know?

• Exploring ideas to improve usability
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- Exploring ideas to improve usability
  - Or performance (throughput, latency, ...)


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  - Or performance (throughput, latency, ...)
- Establishing the effectiveness of promotion before campaigns
- Staged rollouts of major changes
  - Minimizing risk of: CD, fragmented configurations, ...
    e.g. rolling out apps to the Android store
Simple A/B Testing

• You have:
  – two solutions, A and B (e.g., A is old, B is new)
  – A hypothesis (e.g. A will improve conversion over B by at least 5%)
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\mu_1 < \mu_2
\]
Recalling T-tests

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  - distinguishing directed and undirected differences
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- **RECALL:**
  We never prove a hypothesis!
  We gather sufficient evidence to reject the null hypothesis and thus accept the alternative
Recalling T-tests

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t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}
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\( H_a: \mu_1 - \mu_2 > \Delta \) \quad t > t_{\alpha, v} \\
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Where \( H_0: \mu_1 - \mu_2 = \Delta \)

\( H_a: \mu_1 - \mu_2 > \Delta \) \hspace{1cm} t > t_{\alpha, v} \hspace{1cm} p = P[T \geq t | H_0] \\
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\end{align*} \)

\[ v^2 = \frac{(S_1^2/m + S_2^2/n)^2}{(S_1^2/m)^2/m-1 + (S_2^2/n)^2/n-1} \]

\[ p = P[T \geq t | H_0] \quad p = P[T \leq t | H_0] \quad p = P[|T| \geq t | H_0] \]
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\[ v = \frac{\left(\frac{S_1^2}{m} + \frac{S_2^2}{n}\right)}{(S_1^2/m)^2 + (S_2^2/n)^2} \]

\[ \frac{m-1}{n-1} \]

Where \( \alpha \) captures the level of confidence for a p-value
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\[ v = \left( \frac{S_1^2 + S_2^2}{S_1^2/m + S_2^2/n} \right) \left( \frac{S_1^2/m}{m - 1} + \frac{S_2^2/n}{n - 1} \right) \]

But subtle challenges arise in practice!

A p-value
Problem: Choosing and tagging populations

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- Possible factors in the results ought to be identified up front. Collecting them after the fact requires rerunning an experiment.
- Your sample ought to be representative.
### Problem: False positives and negatives

<table>
<thead>
<tr>
<th></th>
<th>Type I error</th>
<th>Type II error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P[\text{fail to reject } H_0 \mid H_0]$</td>
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Type I error: false positive
Type II error: false negative

There is *always* a risk of error
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Suppose you run 5 tests with \( p = 0.1 \), What is the likelihood of a false positive?
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Could you correct for this?
Problem: Choosing hypotheses

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  - Testing many things increases the likelihood of \textit{false positives}.
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- The more hypotheses you test, the greater your risk of false positives.
  - This can be mitigated, but you should choose hypotheses well up front.
Problem: Stopping criteria & confidence

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- This can also be expressed as “minimum detectable effect size”
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  - This can also be expressed as “minimum detectable effect size”
  - If variance and sample sizes can differ, this is challenging, so most just use available sample size calculators based on $\alpha$ and $\beta$. 
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- The illusion of significance
Problem: Novelty effects

- Users are used to seeing a blue “buy” button and ignore it, so you change it to red.
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- Users are used to seeing a blue “buy” button and ignore it, so you change it to red.
  - Sales skyrocket. Red is clearly better!
  - Until a week later when sales return to normal...
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• The novelty of the change for the sample may bias the underlying results of the study
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If the testing is important, you should be doing something obvious or consulting a statistician.
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- But what if even the notion of a predetermined campaign does not fit?
  - Sequential hypothesis testing & Bayesian approaches
  - Bandits
Sequential Hypothesis Testing

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  - Making components for computers
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Why might running a t-test be undesirable?
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  - ✓ x x ✓ x x x x ...

What new problem arises?
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  - ✓ x x ✓ x x x ...
  - What are the stopping criteria?
    When is there enough evidence to be convinced?

- NOTE: This problem is challenging and is an active area of research
  - We will only look at one approach
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  - $B < S_K$ $\Rightarrow$ reject $H_0$ and stop
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  - \( S_K < A \) ⇒ fail to reject H0 and stop
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  - $A < S_K < B \Rightarrow$ continue sampling
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  - $S_K < A \Rightarrow$ fail to reject $H_0$ and stop  
  - $A < S_K < B \Rightarrow$ continue sampling

- Done using Wald’s Sequential Probability Ratio Test

$$S_K = \log \prod_{i=1}^{K} \frac{p(X_i|H_A)}{p(X_i|H_0)}$$  
  a **likelihood ratio test**
Sequential Hypothesis Testing

- Given a sequence of observations $X_1X_2X_3...X_K$, we want $A, B, S_K$ such that
  - $A < B$ ⇒ reject $H_0$ and stop
  - $B < S_K$ ⇒ fail to reject $H_0$ and stop
  - $S_K < A$ ⇒ continue sampling
  - $A < S_K < B$ ⇒ continue sampling

- Done using Wald’s Sequential Probability Ratio Test

\[
S_K = \log \prod_{i=1}^{K} \frac{p(X_i|H_A)}{p(X_i|H_0)} \quad \text{a likelihood ratio test}
\]

\[
A = \log \frac{\beta}{1-\alpha} \quad B = \log \frac{1-\beta}{\alpha}
\]
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a likelihood ratio test  \hspace{1cm} A = \log \frac{\beta}{1-\alpha} \hspace{1cm} B = \log \frac{1-\beta}{\alpha}

\[ S_0 = 0 \]
\[ S_K = S_{K-1} + \log p(X_k|H_A) - \log p(X_k|H_0) \]
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- Caveat/risk:
  - May only be beneficial/useful for simple hypotheses. Otherwise it is complex.
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  $S_K = \log \prod_{i=1}^{K} \frac{p(X_i|H_A)}{p(X_i|H_0)}$  a **likelihood ratio test**

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- **Simpler approaches exist based on the Gambler’s Ruin** (w/ no H0 estimate)
Multi-Armed Bandits

- What if we don’t really care whether $H_0$ is false; we just want to make a good choice now?
Multi-Armed Bandits

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- Given options A, B, C, and D, which is the best to use based on evidence so far?
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---

So why might you prefer bandits over A/B tests (or vice versa)?
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- Many solutions. Two common ones:
  - $\epsilon$-greedy strategy
  - Thompson sampling
Multi-Armed Bandits

- Usual assumptions
  - Reward probabilities (like conversion rates) don’t change
Multi-Armed Bandits

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- While solutions can be robust when assumptions are violated, there can be better variants or better solutions
Multi-Armed Bandits: $\varepsilon$-Greedy Strategy

- $\varepsilon$-greedy strategy
  - Has the benefit of being dead simple
  - May be too sensitive to variance and perform worse than other approaches
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on_choice():
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  else:
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- **Can also vary/scale $\epsilon$ over time.**
  - Can be used to logarithmically bound regret by limiting future exploration (decay)

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- Feels a bit ad hoc. Why would you use it?

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Multi-Armed Bandits: Thompson Sampling

• Thompson sampling
  – Tends to behave well with delayed feedback
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initialize():
    for each arm i:
        failures[i] = 0
        successes[i] = 0

on_choice():
    for each arm i:
        sample from Beta(successes[i]+1, failures[i]+1)
        select argmax_i samples[i]
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PDF Beta(4,1)
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PDF Beta(4,1)  
PDF Beta(4,4)  
PDF Beta(2,4)
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Contextual Bandits

- What if the reward likelihood depends on
  - History
  - Environmental state
Contextual Bandits

- What if the reward likelihood depends on
  - History
  - Environmental state

- *Contextual* Bandits are able to take features at time $t$ into account
Other uses of bandits in software quality

- Fuzz testing
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- Auto configuration / optimization
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  - Hyperparameter tuning in machine learning
  - ... 
- Verification & cryptanalysis
- ...
Choosing a solution

- **A/B Testing**
  - Can be robust as long as the sample is representative

- **Bandits**
  - Allow you to take advantage of results as they find the solution
  - Can enable adaptation over time rather than one shot optimality
Summary: A/B Testing & Bandits

- Hypothesis testing can help you choose one version of something over another
- Sequential strategies can allow for early stopping & peeking
- Bandit based techniques allow for optimizing expected benefit while exploring options