CMPT 473
Software Testing, Reliability and Security

Scale
& Combinatorial Testing

Nick Sumner

Recall from last time...

- Consider our triangle classifier
  - Takes 3 integers for sides 1, 2, and 3

Characteristic	b1	b2	b3
Side 1 <?> 0	Side 1 > 0	Side 1 = 0	Side 1 < 0
Side 2 <?> 0	Side 2 > 0	Side 2 = 0	Side 2 < 0
Side 3 <?>0	Side 3 > 0	Side 3 = 0	Side 3 < 0

3 guiding questions...

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How many tests does this create?

What will this test well?
What won't this test well?

Recall from last time...

- Consider our triangle classifier
  - Takes 3 integers for sides 1, 2, and 3

Characteristic	b1	b2	b3	b4
Value of side 1	Side 1 > 1	Side 1 = 1	Side 1 = 0	Side 1 < 0
Value of side 2	Side 2 > 1	Side 2 = 1	Side 2 = 0	Side 2 < 0
Value of side 3	Side 3 > 1	Side 3 = 1	Side 3 = 0	Side 3 < 0

How many tests now?

What is the scale?

Suppose inputs or characteristics I_1, I_2, I_3, …, I_n

- How does the number of tests change?

What is the scale?

Suppose inputs or characteristics $I_1, I_2, I_3, \ldots, I_n$

- How does the number of tests change?
- $|D_1| * |D_2| * |D_3| * \ldots * |D_n| = k^n$
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Too many to maintain!

Too many to reasonably even create!

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  - Constraints

```
Pattern Size:
  Empty                                [Property Empty]
  Single character                     [Property NonEmpty]
  Many characters                      [Property NonEmpty]
  Longer than any line in the file     [Property NonEmpty]
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Quoting:
  Pattern is quoted                    [Property Quoted]
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  - Not test as thoroughly (sampling)

Why might this be okay?

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  - [property] to identify rules for useful tests
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- What else might we do?
  - Not test as thoroughly (sampling)
  - Identify related variables/domains & test together

Why might this lead to fewer tests?

Choosing Combinations

Several possible strategies to consider:

- All Combinations
  - Every combination of every block is tried

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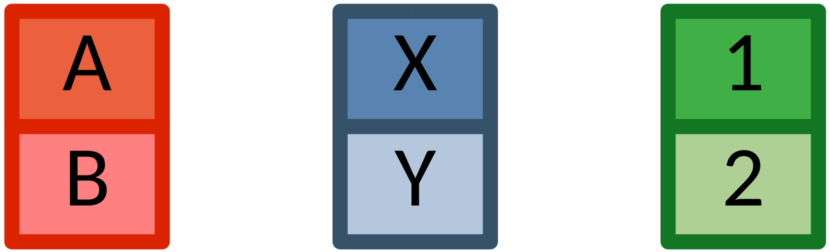

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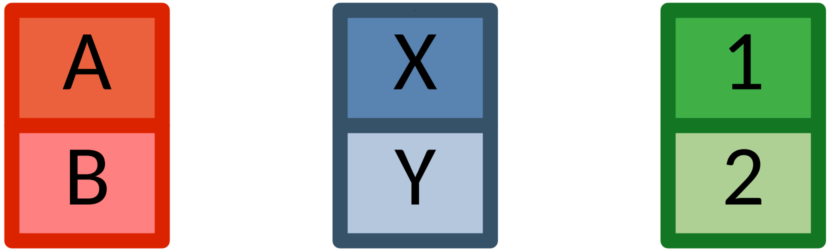

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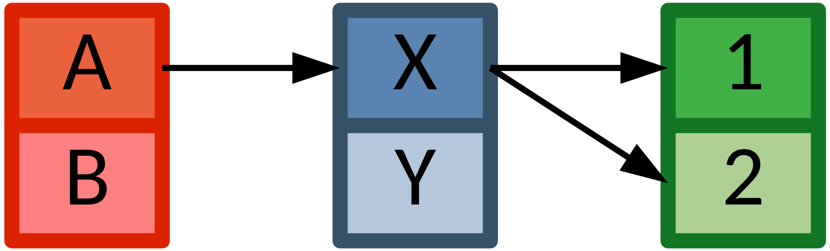

Adequate Tests:
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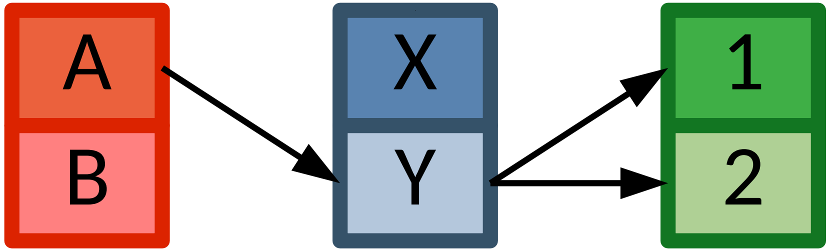

Adequate Tests:

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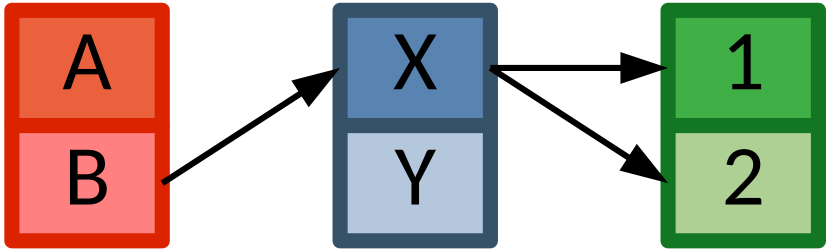

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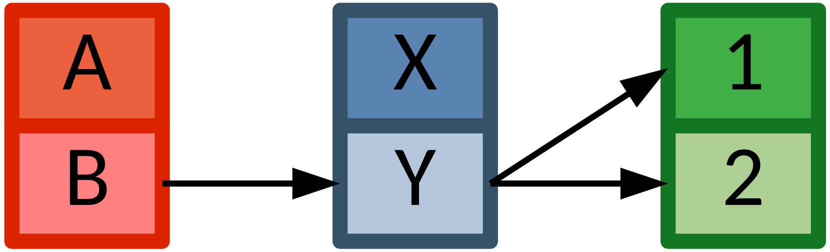

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BY1, BY2

Choosing Combinations

Several possible strategies to consider:

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But is it inherently bad?

Combinations – Each Choice

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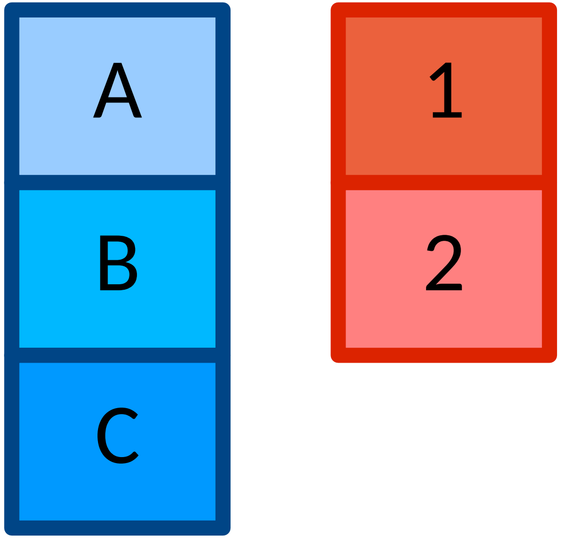

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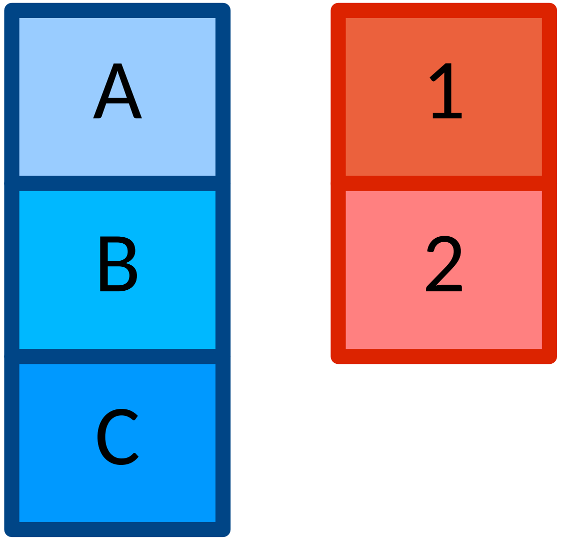

Adequate Tests:
(A,1), (B,2), (C,1)

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What does this look like for the triangle classifier?

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Are these tests *good*? Why?

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How many tests? Why?

Combinations – Each Choice

- How can we minimize the number of tests and still test each block?
- Each Choice
  - 1 value from each block is used in at least 1 test
  - # tests = maximum number of blocks

How many tests? Why?

Combinations – ???

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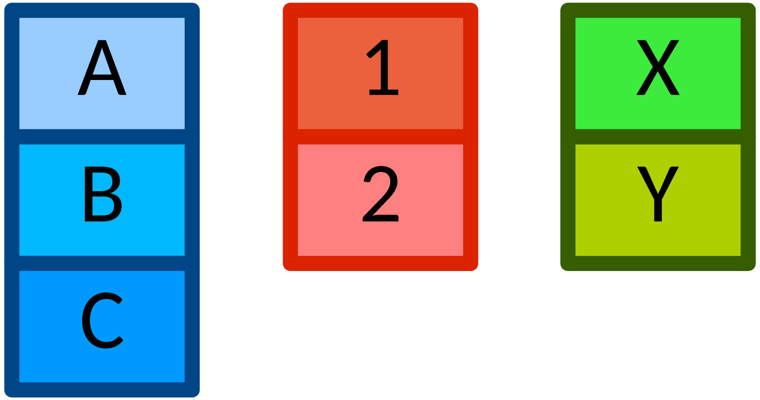

Adequate Tests:
(A,1,*), (A,2,*)
(B,1,*), (B,2,*)
(C,1,*), (C,2,*)

Combinations – ???

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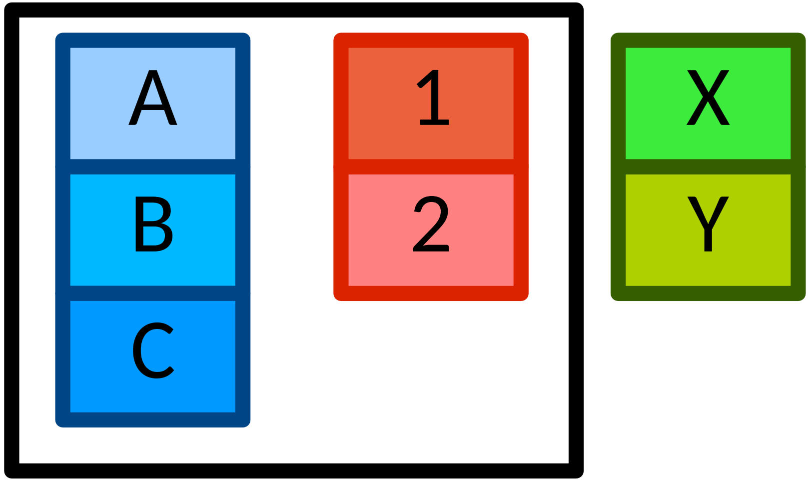
all combinations of two



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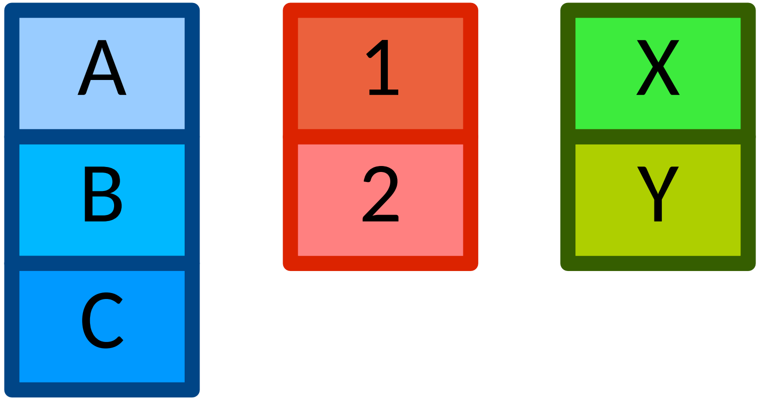

Adequate Tests:
(A,1,**X**), (A,2,**Y**)
(B,1,**Y**), (B,2,**X**)
(C,1,*), (C,2,*)

Fill in X and Y to make sure all pairwise combos are tested!

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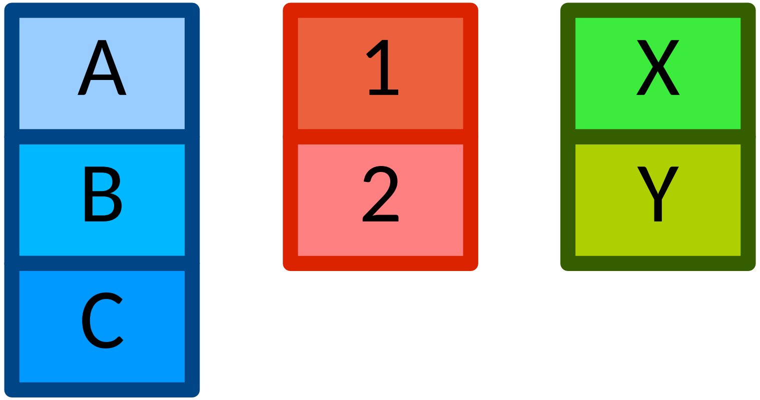

Adequate Tests:
(A,1,X), (A,2,Y)
(B,1,Y), (B,2,X)
(C,1,*), (C,2,*)

What should the last two be?

Combinations – ???

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Combinations – Pairwise

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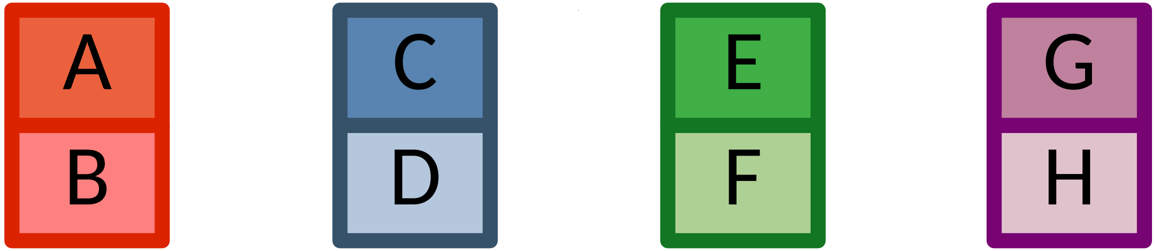

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How many tests?

Expected on the order of $|D_1| * |D_2| * \log(n)$

Combinations – ???

- Can we extend this further?

Combinations – T-wise

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What happens as T increases?

Combinations – T-wise

- Can we extend this further?
- T-wise
  - 1 value from each block for each group of T characteristics
  - #tests ≥ product of T largest domain partitionings
  - Bounded by $(\text{max number of blocks})^T$
  - More expensive than pairs & uncertain gains

T is often called the *test strength*

Combinations – Base Choice

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  - What if we know that certain values are important?

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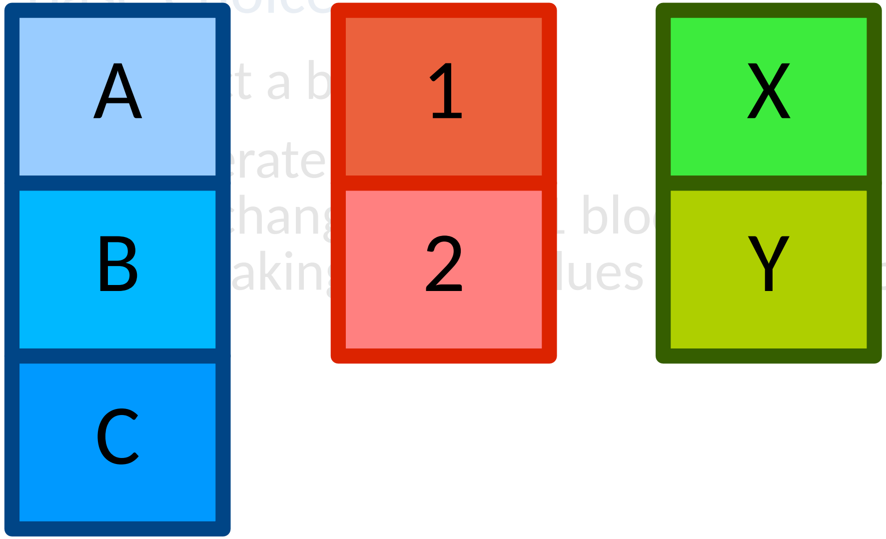

Base Test:

Adequate Tests:

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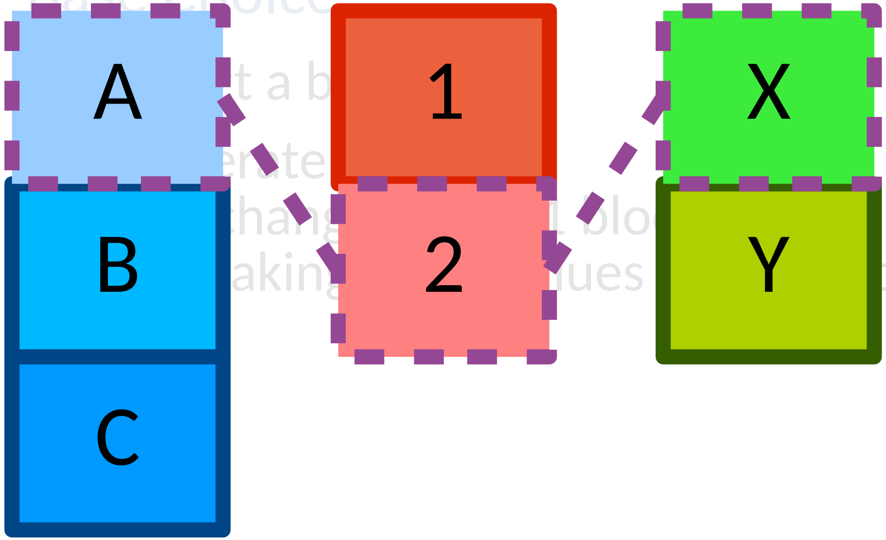

Base Test:
(A,2,X)

Adequate Tests:

Combinations – Base Choice

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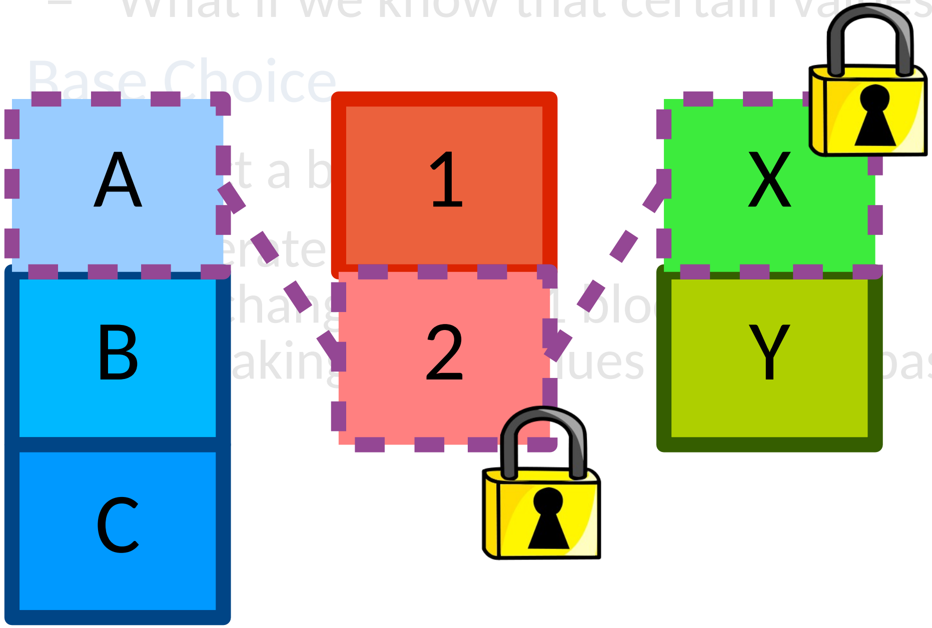

Base Test:
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Adequate Tests:
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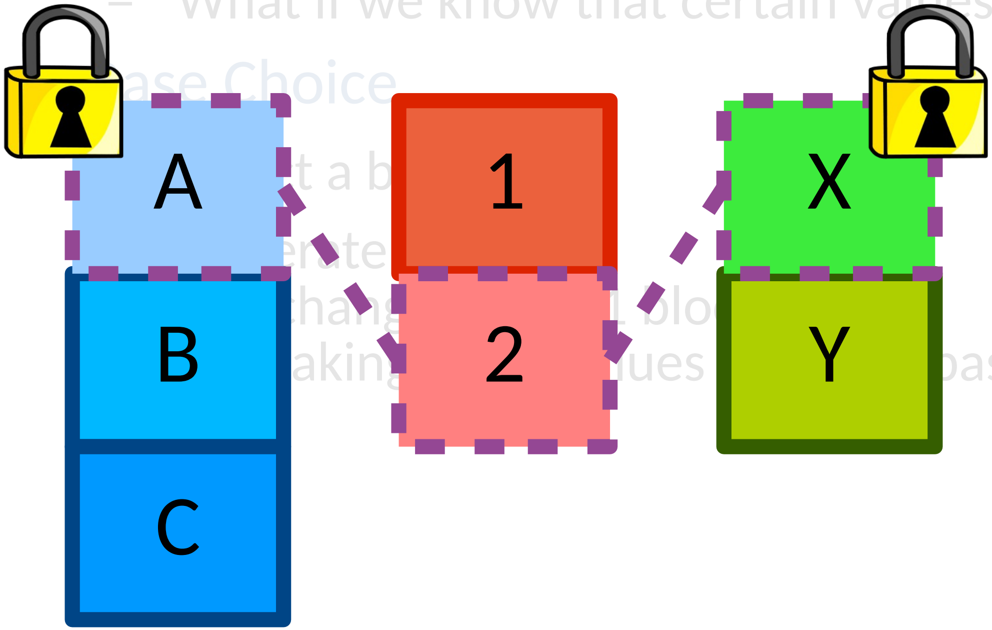

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(A,1,X)

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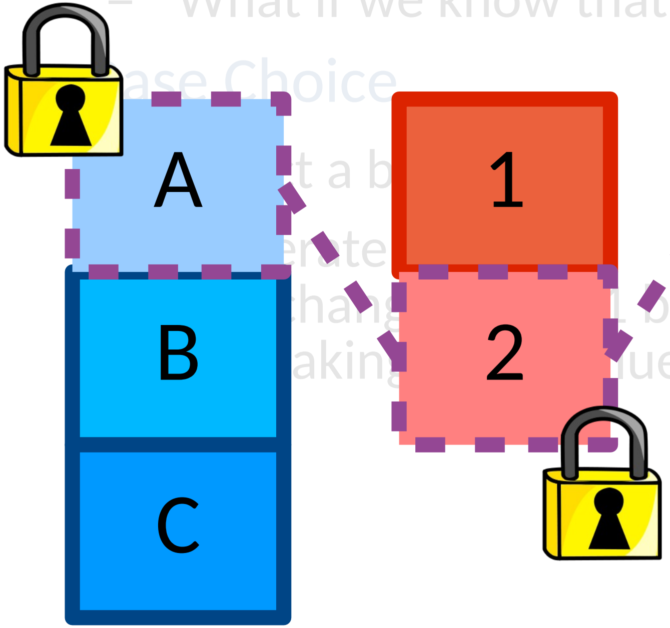

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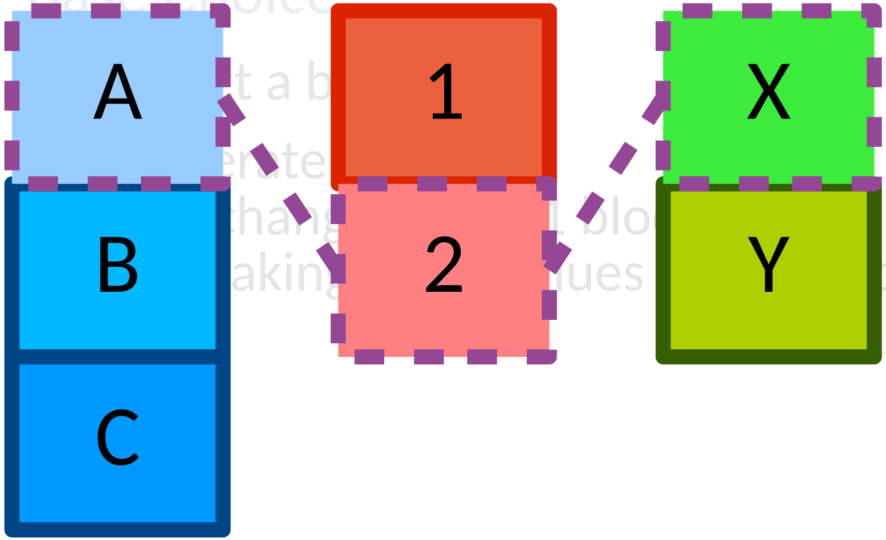

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  - # tests = 1 base + 1 per each other block

How many tests?

$$1 + \sum |D_i - 1|$$

Base Choices

Which test to use as a base is crucial

Why? What if we choose poorly?

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How might we select a base test?

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  - Etc.

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- Guided by:
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  - Etc.
- Decision must be well understood & well maintained

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M base tests:

$$M * (1 + \sum |D_i - 1|)$$

How are they related?

All Combinations

Each Choice

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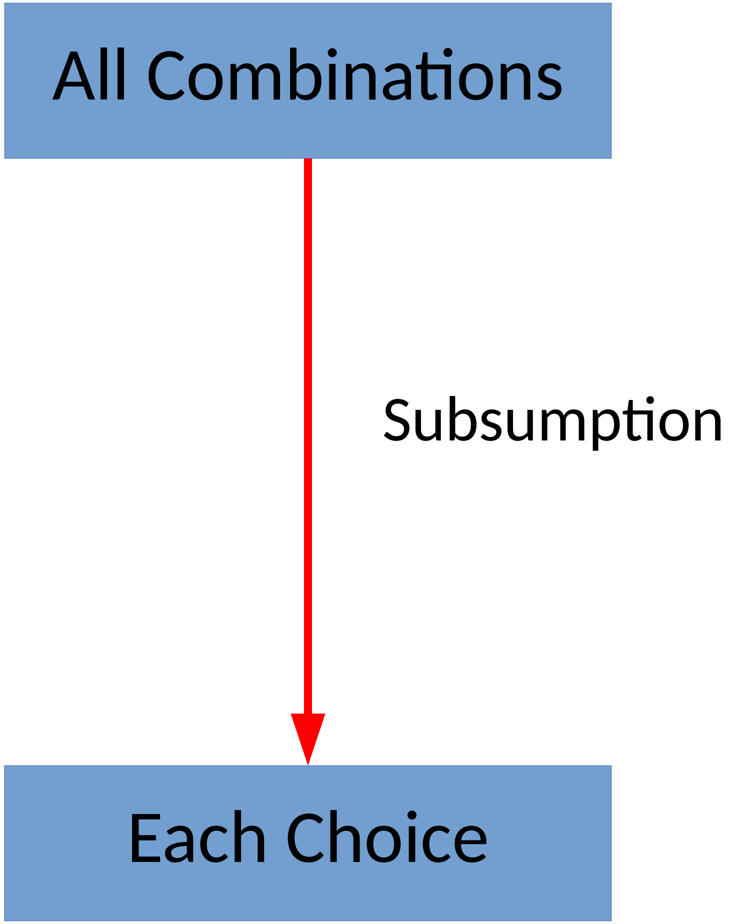

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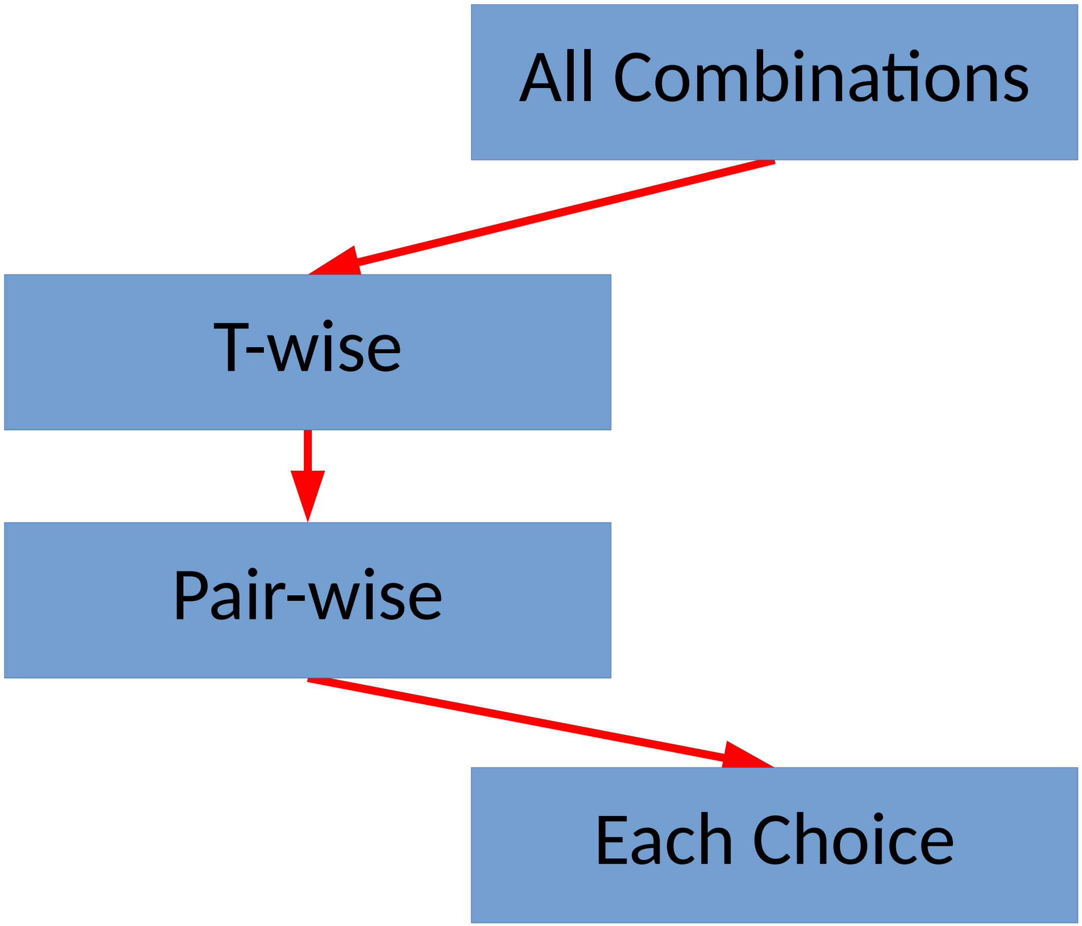

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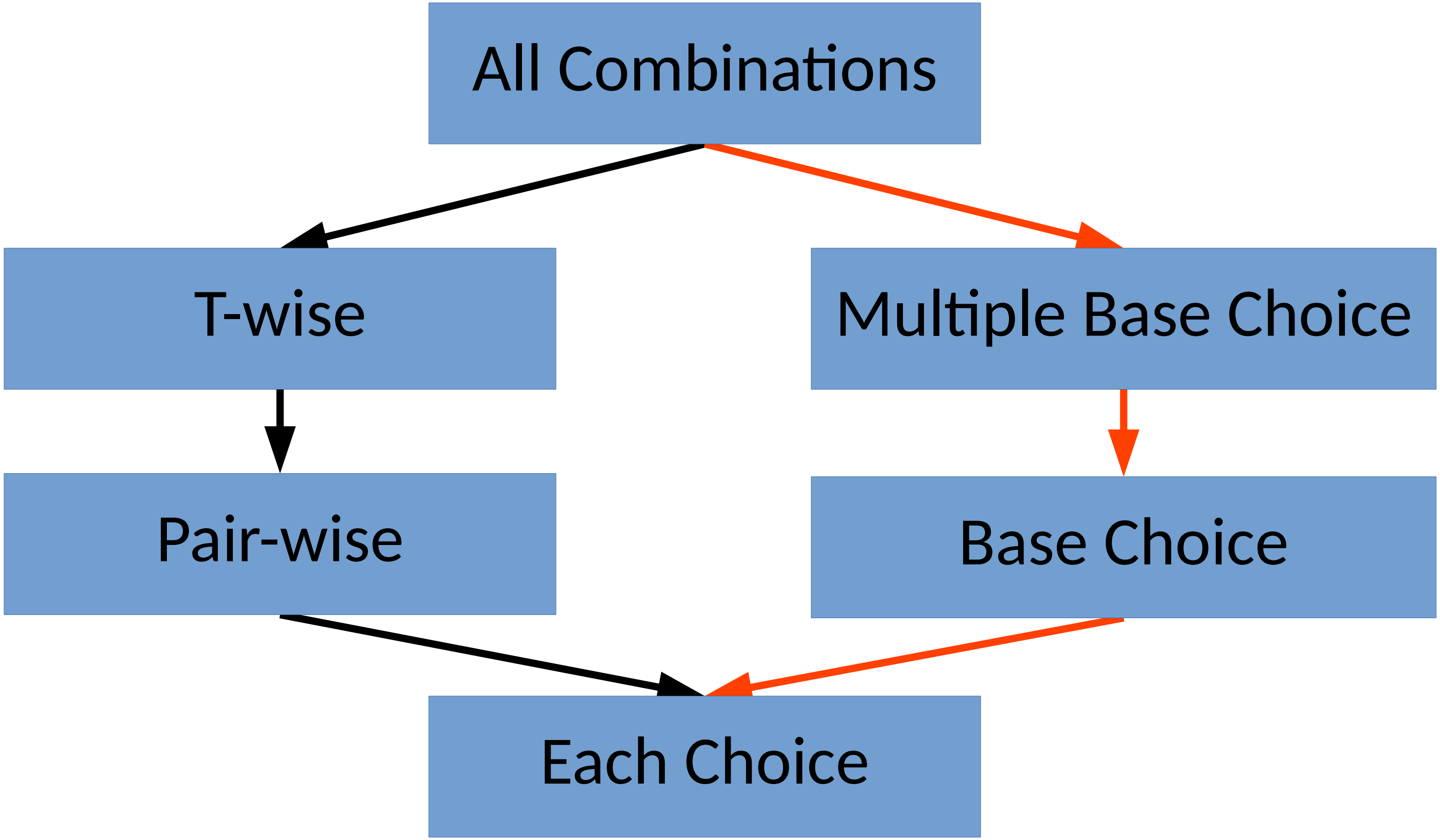

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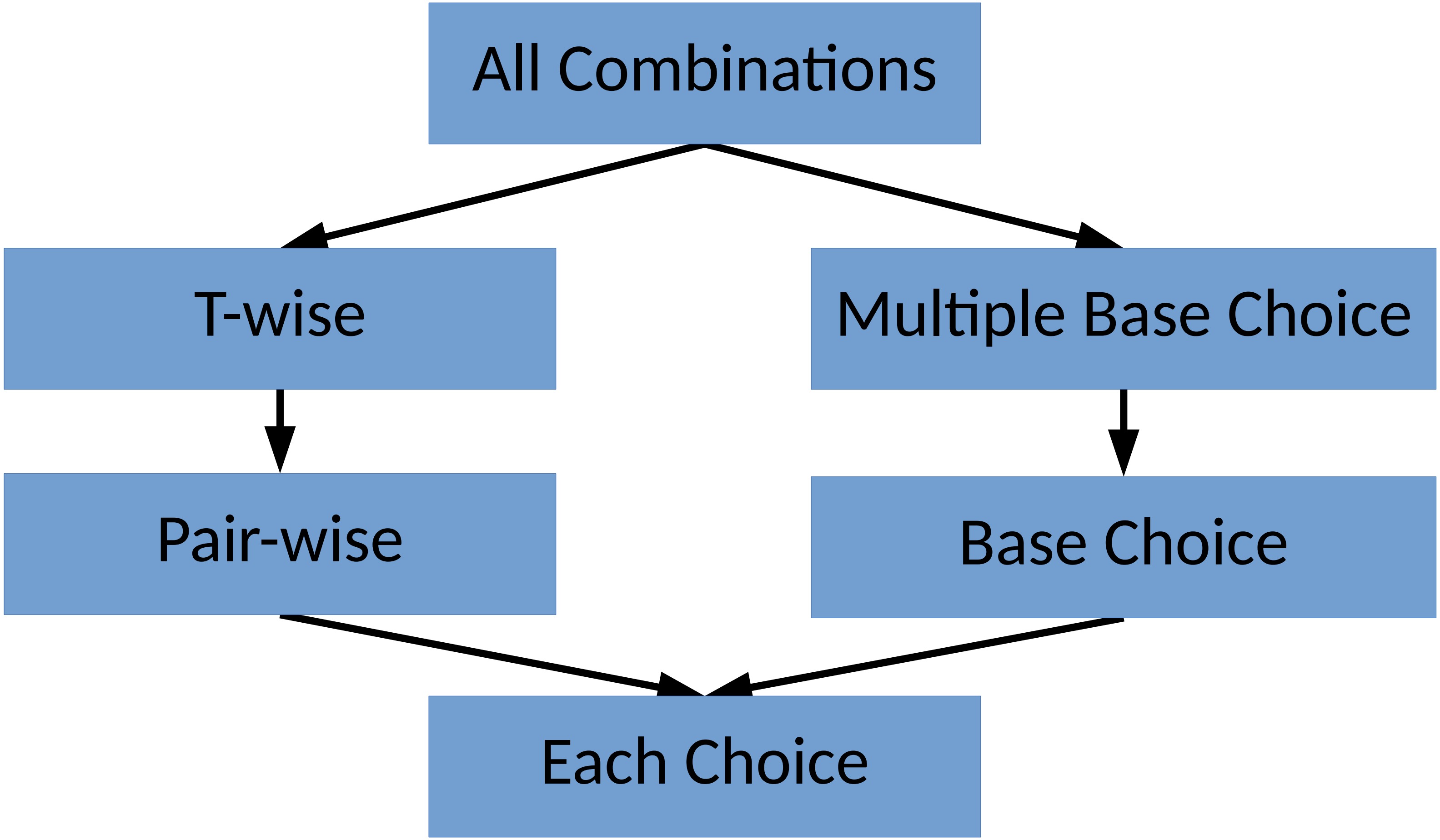

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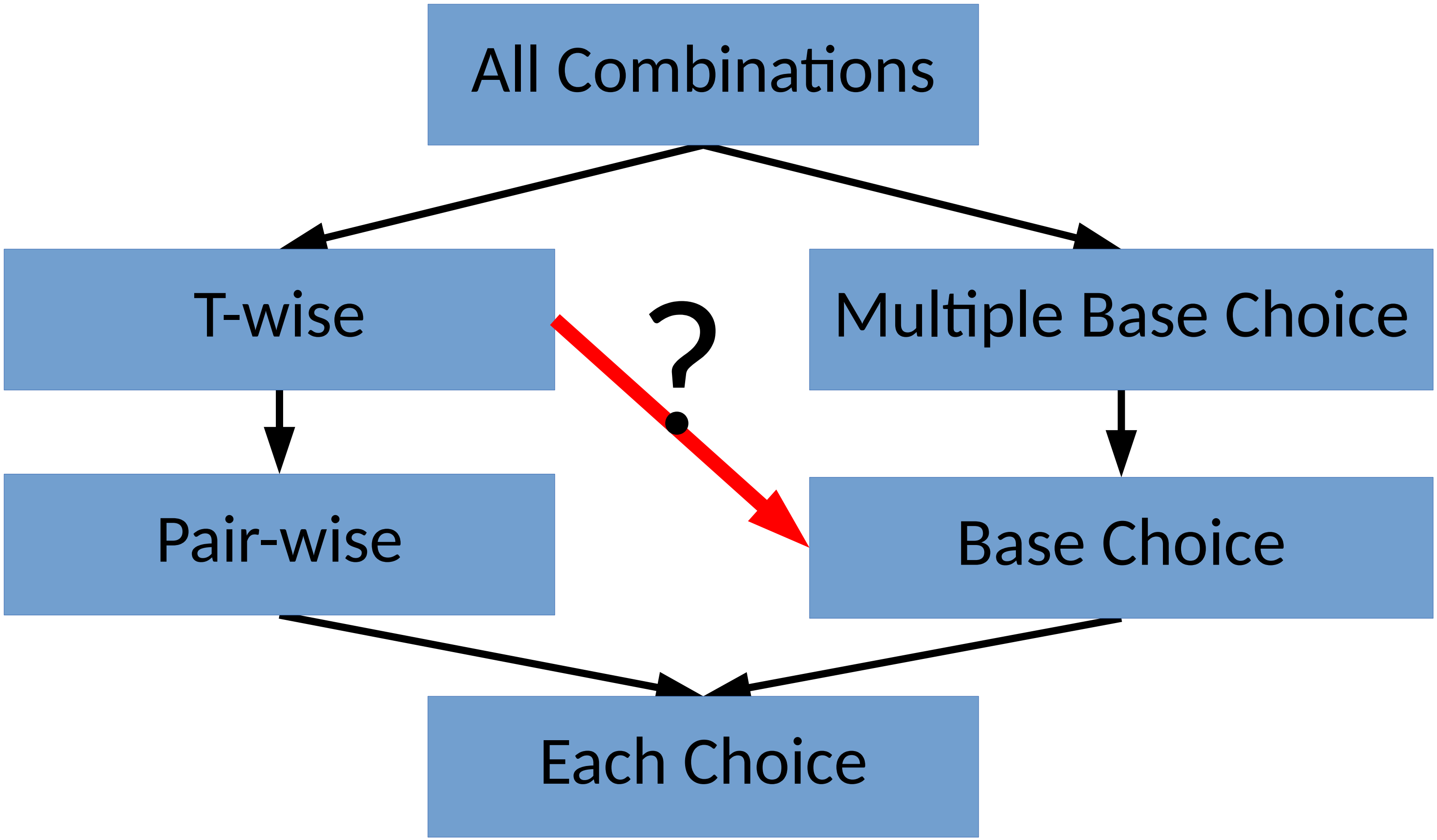

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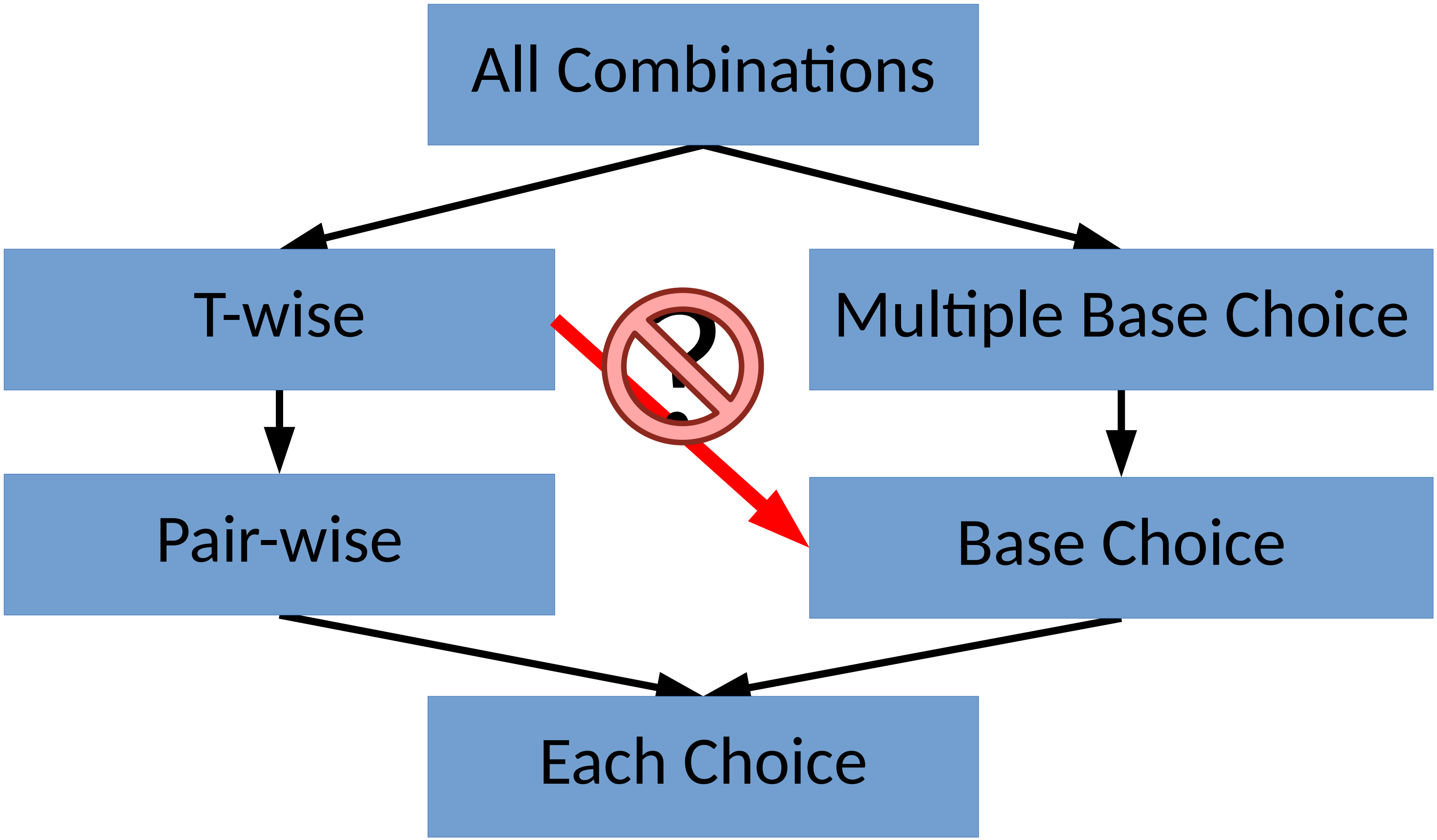

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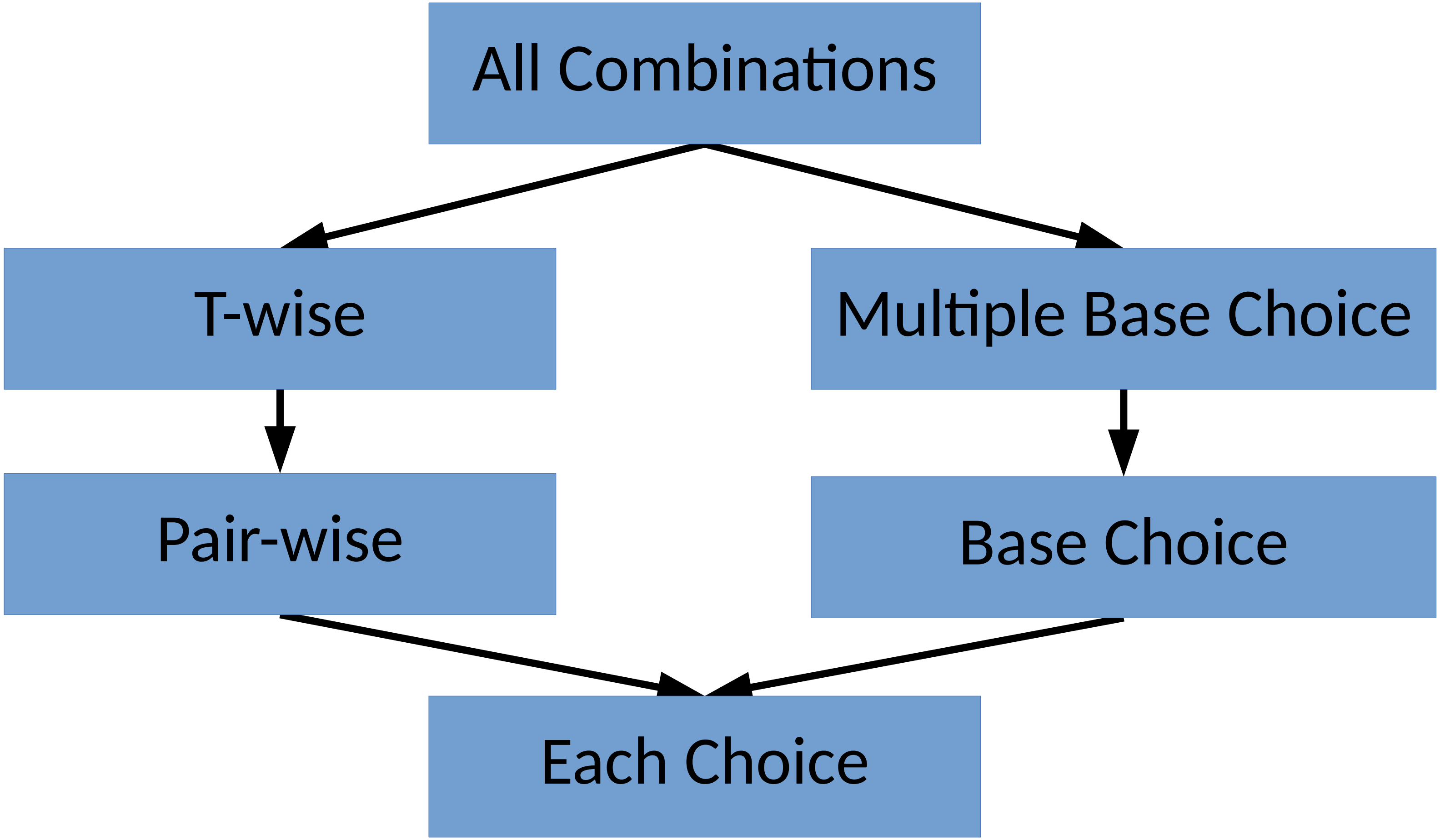

Remembering the constraints

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- Constraints, and [error]s can reduce the # of tests further
  - No need to test invalid constraints
  - No need to test more than one [error]

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  - Pairwise interactions are only truly tested when independent of others
  - The selected representative problem persists
  - Simple random testing seems to be as effective
- Care must be taken, while there is tooling & some industry adoption, it cannot be adopted blindly

Summary

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- Combinatorial testing strategies can reduce the cost of input space partitioning
- Care must be taken to control the loss of testing power in the process