CMPT 882

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Trade-offs and Social Choice: Pareto-Optimality

Choice Without Uncertainty: Trade-offs

- In a choice situation with no uncertainty, the consequences of each option are known. It may seem that in that case choice is easy: choose the option that leads to the most preferred outcome.
- But making up your mind can be difficult when the available options have strengths and weaknesses that *trade off* against each other. In this lecture we will look at some basic principles for making a choice in that kind of situation. Let's start with a general model of trade-offs.

- As usual, we begin with a finite set of options O, call them $o_1, o_2, \ldots, o_n$, and an agent A.

- This time we add a set D of **dimensions** or **attributes** that describe features of the options; call them $d_1, d_2, \ldots, d_k$.

- We assume that the agent has rational preferences among the options **with respect to each dimension**. Thus we can assign a score or utility to each option for a given dimension.

Options	Dimensions d_1	d_2	....	d_k
o_1	5	7	....	9
o_2	100	58	....	6
o_n	80	2	....	-3

Example

Options	Dimensions Rent/mo	Roommates	Distance to SFU
Burnaby Apt.	costs \$800	3	5 min
Port Moody Apt.	costs \$800	3	10 min
Langley Apt.	costs \$600	4	20 min

$\Downarrow$

Options	Dimensions Rent/mo	Roommates	Distance to SFU
Burnaby Apt.	10	3	45
Port Moody Apt.	10	3	30
Langley Apt.	15	2	15

Strong Pareto-Dominance

Consider the following two options. Which would you choose?

	Dimensions		
Options	Rent/mo	Roommates	Distance to SFU
Burnaby Apt.	costs \$500	3	10 min
Langley Apt.	costs \$600	4	20 min

Definition

Let O be a set of options, D a set of dimensions, and let o_d be a rational preference relation among the options in O for each dimension d in D.

Option x **strongly Pareto-dominates** option y iff
for each dimension d in D, it is the case that $u(x)>u(y)$.

Weak Pareto-Dominance

Consider the following three options. Which would you choose?

Options	Dimensions Rent/mo	Roommates	Distance to SFU
N. Burnaby Apt.	\$750	3	5 min
Port Moody Apt.	\$800	4	20 min
S. Burnaby Apt.	\$750	3	15 min

Definition

Let *O* be a set of options, *D* a set of dimensions, and let o_d be a rational preference relation among the options in *O* for each dimension *d* in *D*.

Option *x* **weakly Pareto-dominates** option *y* iff

1. for <u>each</u> dimension *d* in *D,* it is the case that $u(x) \geq u(y)$ **and**
2. for <u>some</u> dimension *d* in *D,* it is the case that $u(x) > u(y)$.

Exercise

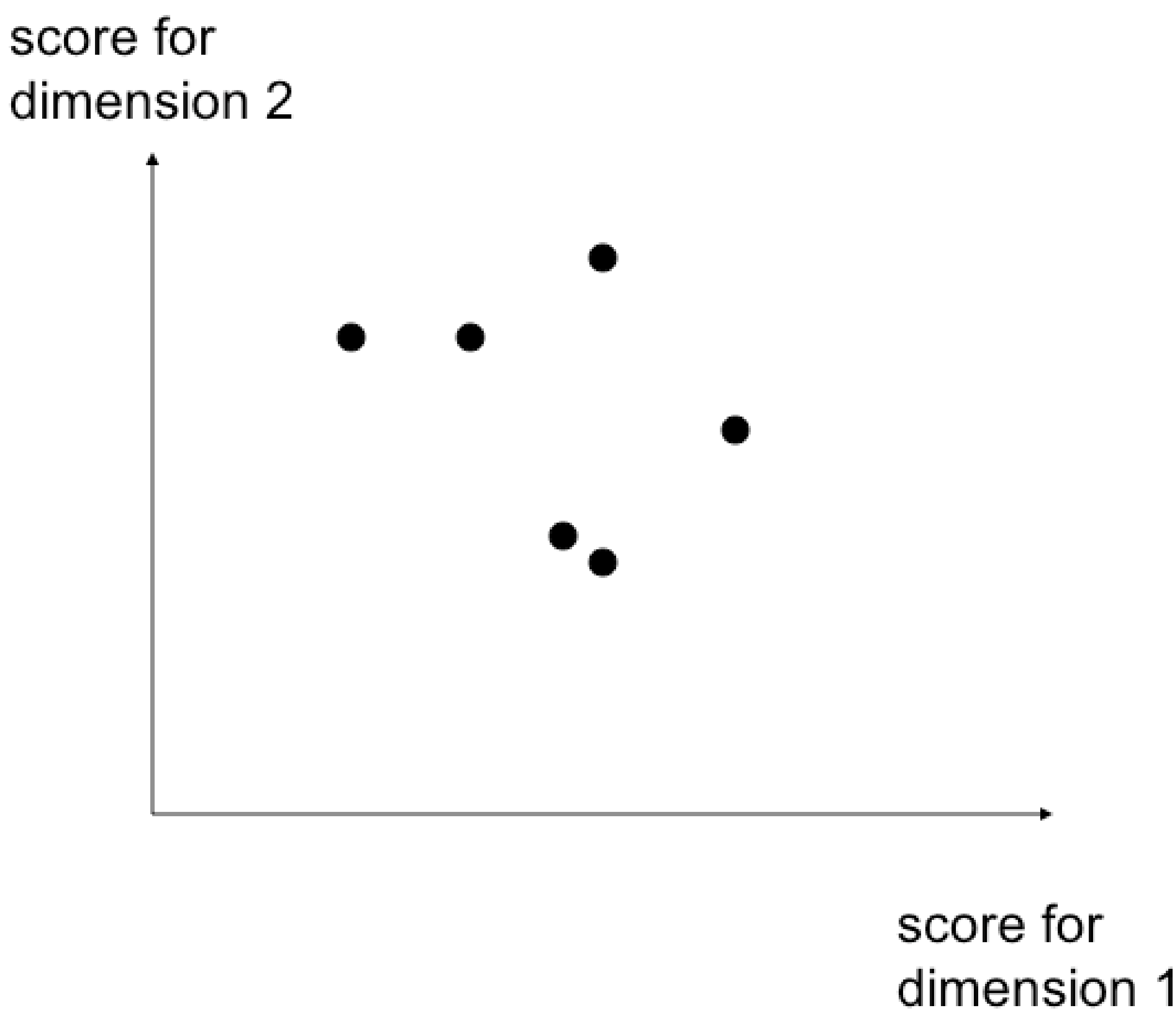
Keerthana is buying a cheese and maccaroni dinner. She has two options: Kraft dinner or the no-name product. Three attributes are relevant: Taste, price and brand name. Kraft dinner and the no-name product taste the same to Keerthana. She doesn’t care whether she is buying a brand name product. She prefers a cheaper product to a more expensive one.
Write down an options/dimensions matrix that models this choice situation. Does Kraft dinner strongly Pareto-dominate the no-name product, or vice versa? Does Kraft dinner weakly Pareto-dominate the no-name product, or vice versa?

Pareto-Optimality and the Pareto Frontier

Definition An option x is (strongly) Pareto-optimal if no other option y weakly Pareto-dominates x.

Definition The set of Pareto-optimal options is called the **Pareto frontier** of the decision problem.

In the case with 2 relevant dimensions, we can visualize the Pareto frontier as follows.



Example

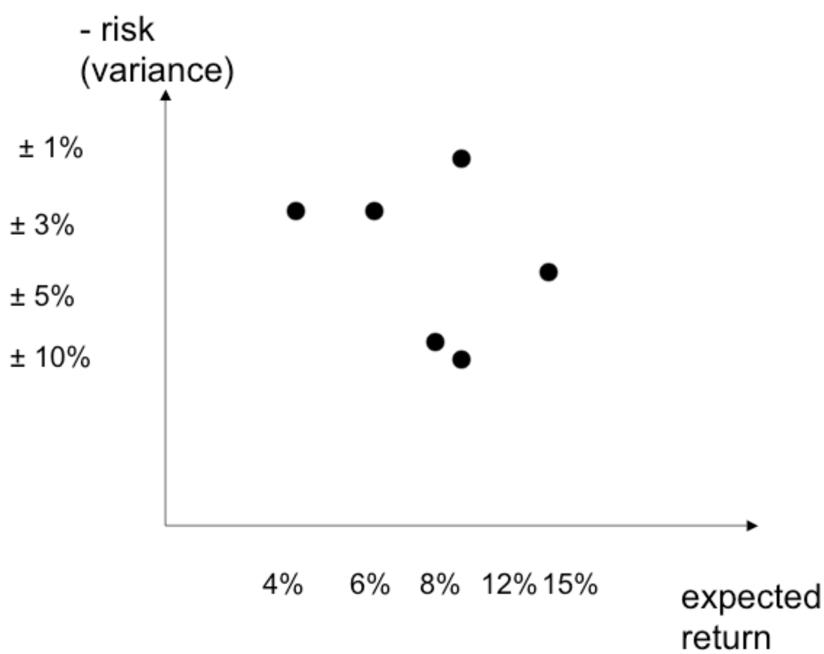

rank of job
lep.
nanager
ıssistant
nanager
10
20
30
40
50
salary (in
$10,000
units)

Mean-Variance Analysis

An important example of Pareto-optimality is in investment decisions (portfolio selection). An investment has two important characteristics: 1) The expected or average return (e.g., 3%), 2) the risk. Mathematically, the risk can be defined as the variance; we'll think of it as an interval around the expected return.

For example, if the expected return is 5% and the risk interval is ±10%, the possible returns range from -5% to +15%.

A typical investment approach is to first estimate the risk tolerance of the investor ("conservative", "aggressive"), and then find a maximum return investment for that risk level.



Social Choice

Mathematically, the situation of an agent deciding on trade-offs and a society reconciling different interests is the same (!)

Options	People ☺	☻	☹
o_1	5	3	5
o_2	100	4	20
....	....	....	....
o_n	80	2	-3

Instead of a set of dimensions D, we have a set P of members of society. We assume that each person p in P has a rational preference ordering o_p among the available options.

Strong Pareto-Dominance in Societies

Definition

Let P be a set of members of society. For each person p in P, let o_p be a rational preference ordering among a set of options O, and let u_p be the utility function for person p.

- An option x **strongly Pareto-dominates** another option y iff for all members of the society p, it is the case that $u_p(x) > u_p(y)$.

- An option x **weakly Pareto-dominates** another option y iff

1. for <u>each</u> member of the society p, it is the case that $u_p(x) \geq u_p(y)$ **and**
2. for <u>some</u> member p of the society, it is the case that $u_p(x) > u_p(y)$.

Example

Options	People ☺	☻	☹
Highland Pub	0	-5	2
White Spot	10	7	8
Earl's	15	7	10

Earl's and White Spot strongly Pareto-dominate the Pub. Earl's weakly Pareto-dominates White Spot.

Exercise

Consider the Prisoner's Dilemma drawn below.

Player 1	Player 2 Cooperate	Defect
Cooperate	3, 3	0, 4
Defect	4, 0	1, 1

Which of the four possible outcomes (CC, CD, DC, DD) strongly Pareto-dominate each other? Which weakly Pareto-dominate each other?

Social Choice and Voting Theory

- Social Choice theory investigates rules and algorithms for combining the preferences of several agents into a choice for the group.
- Pareto-optimality can be seen as a basic principle for social choice, much like choosing undominated options is for single-agent choice.
- What do you think are good rules for arriving at a social choice?
- How about majority vote?

The Condorcet Paradox (1794)

Options	People ☺	☻	☹
Highland Pub	3	1	2
White Spot	2	3	1
Earl’s	1	2	3

Suppose that our society makes its collective decisions by majority rule. If we write $\geq_S$ for the society's preference relation, majority rule means that x is strictly preferred to y by the society iff a majority of the people (strictly) prefer x to y. In the example above, majority rule will lead to intransitive preferences:

Highland Pub $>_S$ White Spot $>_S$ Earl's $>_S$ Highland Pub

(verify for yourself that's how the majority would vote)

Moral: Majority rule can lead a society into **irrational** decisions - even if each member of the society themselves *is* rational!

- If majority rule has problems, are there better or at least alternative rules? This is the topic of social choice theory, and also of voting theory.