Final Exam

CMPT 882: Computational Game Theory

Simon Fraser University

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Instructor: Oliver Schulte

Student Name ____

Student ID ____

Instructions. This exam is worth 30% of your final mark in this course. Answer all questions. This exam consists of 15 pages. You have two hours to complete this exam. You can earn a total of 63 points on the exam. I indicate the worth of each question, which is also roughly the number of minutes that I expect you to spend on a given question. You should answer the questions in the space provided. If necessary, you may continue your answer on the back of the question sheet. Clarity, spelling and grammar count in evaluating your answer. If you are introducing symbols, or if you are using symbols introduced in the course in a special way, define what the symbols mean. If you are showing both your work and the answer, indicate clearly which part of what you have written is the answer to the question. *Write your answer in pen.* (Rough work may be in pencil.) **Write down your name and student number.**

In all cases show your reasoning in sufficient detail that I can see how you arrived at the answer.

General advice for computing Nash equilibria. 1) Unless the question states that you need only consider pure strategy equilibria, don't forget to consider mixed ones. 2) As discussed in class and in the book, iteratively eliminating strictly dominated strategies does not change the set of Nash equilibria. First eliminating strictly dominated strategies can substantially reduce the set of equilibria you need to find.

1. (10) Consider a decision problem with three options and three states of the world as shown below.

	States of the World		
Options			
a			
b			
c			

a. Can you fill in payoffs such that option *a strictly* dominates option *b* **and** both options have the same max regret? If yes, specify some payoffs and a probability assignment that meet this condition. If not, explain why not.

b. Can you fill in payoffs such that option *a weakly* dominates option *b* **and** both options have the same max regret? If yes, specify some payoffs and a probability assignment that meet this condition. If not, explain why not.

	States of the World		
Options			
a			
b			
c			

2. (15) Consider the following auction model: An object is up for sale, 5 players 1,..,5 are bidding. Each player *i* assigns a value v_i to the object (the value is private, not known to the others). Suppose that no player assigns the same value so we have $v_1 > v_2 > \ldots > v_5$. The auction mechanism used is a sealed-bid auction: the players simultaneously submit bids b_i (nonnegative numbers), and the object is given to the player with the lowest index among those who submit the highest bid. The winning player then pays the auctioneer the price of his bid, so his overall utility is $v_i - b_i$. To simplify the analysis, we allow continuous bids, so you can bid for instance $x/2$, $x/4$, x/n, any nonnegative real number. Analyze the *pure* strategy Nash equilibria of this game. In particular, which player(s) may obtain the object at Equilibrium, and at what price?

3. (24) Consider the zero-sum game below.

	A	B	C
A	γ, $-\gamma$	1,-1	-1,1
B	-1,1	γ, $-\gamma$	1,-1
C	1,-1	-1,1	γ, $-\gamma$

a) Find all the Nash equilibria of this game, for each value of γ with $0 \leq \gamma \leq 1$.

b) Find all the ESS of this game, for each value of γ with $0 \leq \gamma \leq 1$. If there are none for a certain value of γ, write "no ESS for …".

c) What are the minmax strategies for the row player, for each value of γ with $0 \leq \gamma \leq 1$?

d) What are the maxmin strategies for the row player, for each value of γ with $0 \leq \gamma \leq 1$?

e) What is the security level of the row player, as a function of γ with $0 \leq \gamma \leq 1$?

f) What are the minmax regret strategies for the row player, for each value of γ with $0 \leq \gamma \leq 1$?

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4. (16) In many market situations the goal of players is to estimate how much value other players assign to an object, which makes the assessment of values a game-theoretic problem. The great economist Keynes famously argued that the stock market operates that way: when you buy stock in a company, the main question is not what you think the company is intrinsically worth as a business, but what you think other people will want to pay for the stock. This may explain the herd behaviour seen in stock market rallies and crashes. A simple game-theoretic model of trying to guess the value of an object is the following game.

Each of 5 players chooses a number $i = 1,\ldots,4$. A prize of \$1 is split equally between all the players whose number is closest to 2/3 of the average number.

a. Apply iterated weak dominance to this game. What are the possible outcomes? Is there a Pareto-optimal outcome that is consistent with IWD?

b. What are the pure strategy Nash equilibria of this game?

5. (10) Consider the three-player game with the payoffs given as shown.

	L	R
T	0,0,3	0,0,0
B	1,0,0	0,0,0

Table A

	L	R
T	2,2,2	0,0,0
B	0,0,0	2,2,2

Table B

	L	R
T	0,0,0	0,0,0
B	0,1,0	0,0,3

Table C

Player 1 chooses one of the rows (T or B), player 2 chooses one of the columns (L or R), player 3 chooses one of the three tables (A,B,C).

a. Find the pure strategy equilibria. Which ones do not involve weakly dominated strategies (i.e., no weakly dominated strategy is part of the equilibrium)?

b. Find a correlated equilibrium that is *not* also a Nash equilibrium. That is, the correlated equilibrium should be neither a mixed nor a pure strategy Nash equilibrium in the game.

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6. (10) One of the great concerns of classical economics is modeling the result of competition. Let's apply equilibrium analysis to one such model.

- We have two forms, $i=1,2$.
- Each firm produces the same good, in possible quantities normalized to be between 0 and 1. Thus the option space for each firm is $A = [0,1]$.
- If both firms produce 0, the price is (scaled to be) 1. The price of the good decreases linearly with the amount produced by the firms; say it is $1-a_1-a_2$. Hence the profit of each firm, given the production of the other, is given by $u_1(a_1,a_2) = a_1(1-a_1-a_2)$ and $u_2(a_1,a_2) = a_2(1-a_1-a_2)$.

a. Find the Nash equilibria of this game.

b. Which outcomes are consistent with IWD?

7. (10) Suppose that three players share a pie by using the following procedure. First player 1 proposes a division, then players 2 and 3 simultaneously respond either "yes" or "no". If players 2 and 3 both say "yes" then the division is implemented; otherwise no player receives anything. Each player prefers more of the pie to less. To formalize the problem, say that the players divide 1 unit of an infinitely divisible good. So a division is a list of three real numbers (d_1,d_2,d_3) such that $d_1+d_2+d_3 = 1$ and $0 \leq d_i$ for each i; here d_i stands for the amount allocated to player i. So the list (1/2,1/2,0) represents that players 1 and 2 each get half the pie, and player 3 gets nothing.
Find the subgame perfect Nash equilibria in pure strategies (you don't need to consider mixed strategies).

8. (10) *Model the following parlor game, inspired by Poker, with a* ***game tree****, and find the* ***Nash equilibria*** *of this game.* Don’t forget to consider mixed strategy equilibria. First player 1 receives a card that is either H (hi) or L (lo) with equal probabilities. Player 2 does not see the card. Player 1 may announce that her card is L, in which case she must pay $1 to player 2, or may claim that her card is H, in which case player 2 may choose to concede or to insist on seeing player 1’s card. If player 2 concedes then he must pay $1 to player 1. If he insists on seeing player 1’s card, then player 1 must pay him $4 if her card is L and he must pay her $4 if her card is H. To represent the initial random allocation of a card to player 1 in a game tree, you can use any of the methods presented in class or in one of the textbooks. Hint: Iteratively eliminating strictly dominated strategies does not change the set of Nash equilibria in the game.

9. (8) Describe the subgame-perfect equilibria in the dollar auction when three players each have a budget of $2.90. Bidding proceeds in increments of 10 cents.

10. (20) Consider a 4-round centipede game as shown below.

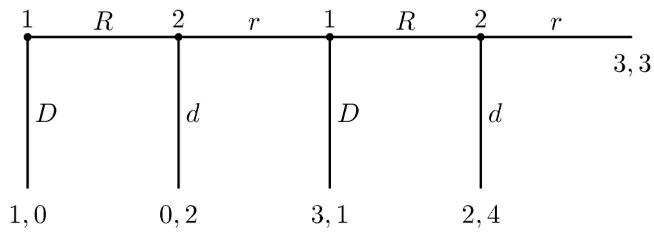

Figure 1

a. (4) Find the normal form of this game. (You can use the reduced normal form if you prefer, but make sure the meaning of your strategies is clear.)

b. (16) Apply equilibrium analysis to this game: Find a) all Nash equilibria. b) all subgame-perfect equilibria. c) If the game is symmetric, indicate whether the**re** exists an ESS or not. If the game is not symmetric, put "not symmetric". Try to fit your answers into the space below, with some justifications to follow.

Nash Equilibria	**Subgame-perfect**	**ESS**

11. (17) Consider the game shown in Figure 1.

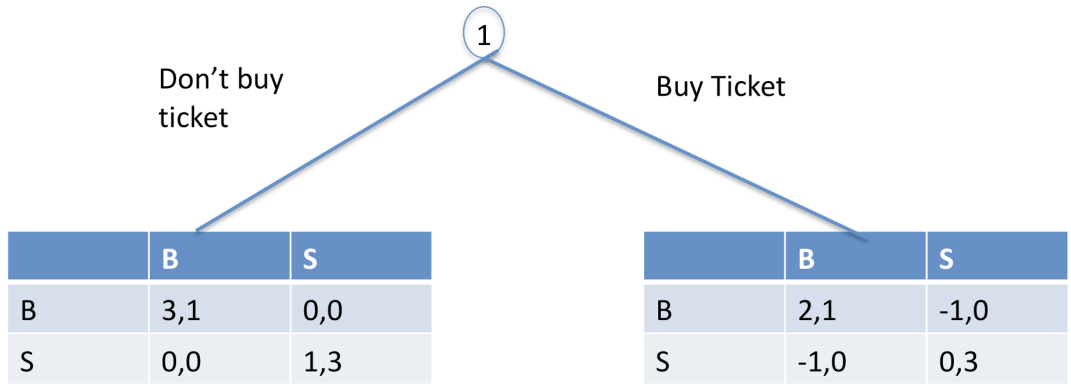

Figure 2: Player 1 can choose to buy a ticket at the price of 1 payoff unit. After player 1 has made the choice, the players play a simultaneous move battle of the sexes as shown in the game tree.

a. (4) Find the normal (matrix) form of this game. (The reduced normal form is okay.)

b. (4) What is the result of playing IWD?

c. (9) Find all pure strategy Nash equilibria. (You don't need to consider mixed strategies). Try to fit your answers into the space below, with some justifications to follow. Which of these equilibria involve weakly dominated strategies? (I.e., which are eliminated by IWD?)

Nash Equilibria	**Involves weakly dominated strategies?**

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