Simon Fraser University

Spring 2010

CMPT 882

Instructor: Oliver Schulte

Assignment 2: Analysis of Normal Form Games.

The due date for this assignment is Friday, March 5.
Note: to provide extra support, I've moved my office hours for this week from Friday March 5, 10 am to Wednesday, March 3, 10-12 am.

Instructions: Check the instructions in the syllabus. You may consult with any book or other non-human source that you like. If you work in a group, put down the name of all members of your group. *You must write out the solutions to the assignment on your own*. There are a total of 50 points as indicated in parentheses. On your assignment, put down your **name**, the number of the assignment and the number of the course. Spelling and grammar count. Write in pen, *not* in pencil. Finally, **please staple your assignment.**

In the following problems, apply equilibrium analysis to our standard games. For each game find a) all Nash equilibria. b) all trembling-hand perfect equilibria. c) If the game is symmetric, indicate whether the exists an ESS or not. If the game is not symmetric, put "not symmetric". d) If there is a correlated equilibrium that is *not* also a Nash equilibrium, specify at least one correlated equilibrium. Try to fit your examples into the space below. Correct solutions get full credit without argument, incorrect solutions may get partial credit if work is shown. A two-player game is symmetric if 1) both players have the same pure strategy set, and 2) $u_1(a_1,a_2) = u_2(a_2,a_1)$ for all pure strategies a_1,a_2.

Here's a sample solution for the Prisoner's Dilemma.

The Prisoner's Dilemma

	Column	
Row	T	B
T	0, 0	-2, 2
B	2, -2	-1, -1

Nash Equilibria	**Trembling-hand perfect**	**ESS**
(B,R)	yes	yes
There are no more equilibria		
Correlated Equilibrium	Only (B,R)	

1. (5) Analyze the equilibria for the pure coordination game

	Column	
Row	T	B
T	2, 2	0, 0
B	0, 0	2, 2

Nash Equilibria	**Trembling-hand perfect**	**ESS**
Correlated Equilibrium		

2. (5) Analyze the equilibria for the BoS

	Column	
Row	T	B
T	3, 1	0, 0
B	0, 0	1, 3

Nash Equilibria	**Trembling-hand perfect**	**ESS**
Correlated Equilibrium		

3. (5) Analyze the equilibria for Chicken

	Column	
Row	T	B
T	-15, -15	4, 0
B	0, 4	1, 1

Nash Equilibria	**Trembling-hand perfect**	**ESS**
Correlated Equilibrium		

4. (5) An issue that arises in technology industry is that an inferior standard may become entrenched even if a better one is available. A historical example is the use of VHS tapes vs. Beta. This illustrates the network effects: users prefer technology used by others. Let's consider a simple game-theoretic model of this situation.

	User 2	
User 1	Superior technology	Inferior technology
Superior technology	2, 2	0, 0
Inferior technology	0, 0	1, 1

Analyze the equilibria as before.

Nash Equilibria	**Trembling-hand perfect**	**ESS**
Correlated Equilibrium		

6. (8) Consider the following game matrix.

	L	C	R
T	1 , 3	3 , 2	1 , 2
M	2 , 2	2 , 0	0 , 0
B	2 , 1	1 , 2	0 , 0

a. (3) What is the result of applying iterated weak dominance in this game matrix? (Remember to eliminate all dominated strategies for either player in each round of eliminating dominated strategies.)

b. (5) Analyze the equilibria (the game is not symmetric)

Nash Equilibria	**Trembling-hand perfect**
Correlated Equilibrium	

7. (12) Each of two players announces a nonnegative integer a_i equal to at most 100. If $a_1 + a_2$ is less than or equal to 100, then each player *i* receives payoff of a_i. If $a_1 + a_2 > 100$ and $a_i < a_j$, then player *i* receives a_i and player *j* receives $100\text{-}a_i$. If $a_1 + a_2 > 100$, and $a_i = a_j$, then each player receives 50.

The following table illustrates some outcomes.

a_1	a_2	u_1	u_2
60	100	60	40
50	40	50	40

Apply iterated weak dominance to this game. What are the possible outcomes?

8. (10) Nash's existence theorem states in any finite game matrix, there is at least one Nash equilibrium (which may be a mixed one). Can you specify an *infinite* game matrix that does *not* have a Nash equilibrium (neither mixed nor pure)? You need to specify

1. for each player what their set of strategies is (e.g., "every player's strategy set is the set of even numbers", or "every player's strategy set is the set {i: i = 2n for some integer n}").
2. for every combination of strategies (one for each player), what the payoff is to each player.

3. an argument that there is no Nash equilibrium (mixed or pure) in your game. (If you correctly and completely describe a game without a Nash equilibrium, but don’t quite manage the argument, you will still get partial credit.)

Hint: there is a very simple solution.