Simon Fraser University

Spring 2010

CMPT 882

Instructor: Oliver Schulte

Assignment 1: Preference Relations. Decision Theory. Pareto Optimality. Game Types.

The due date for this assignment is Wednesday, February 3.

Instructions: The assignment is a mix of straightforward computations and sometimes challenging mathematical questions. Check the instructions in the syllabus. You may consult with any book or other non-human source that you like. If you work in a group, put down the name of all members of your group. *You must write out the solutions to the assignment on your own*. There are a total of 60 points as indicated in parentheses, y if you manage the bonus question as well. On your assignment, put down your **name**, the number of the assignment and the number of the course. Spelling and grammar count. Write in pen, *not* in pencil. Finally, please staple your assignment.

1. (4) Christian is considering computer brands. His three choices are: Apple, IBM and Toshiba. He is interested in three attributes of computers: Price, Speed, and Memory. With respect to each of these three dimensions, he has a rational preference ranking; the following options/dimensions table represents these preference rankings with three utility functions as follows.

	Dimensions		
Options	Price	Speed	Memory
Apple	1	5	6
Toshiba	20	15	20
IBM	20	9	20

a. For all options x, y, write down all the pairs x, y such that x strongly Pareto-dominates y.

b. For all options x, y, write down all the pairs x,y such that x weakly Pareto-dominates y. (If x both strongly and weakly Pareto-dominates y, be sure to note that fact.)

2. (4) When faced with a multi-dimensional choice problem, Rosi forms her preference relation $\geq$ as follows:

a. If option x weakly Pareto-dominates option y, then $x > y$.
b. otherwise she is indifferent, $x \sim y$.

Are Rosi's preferences rational guaranteed to be rational (in every choice problem)? If yes, explain briefly why. If not, explain why not.

3. (10) In class we stated a theorem saying that if a preference relation is rational (reflexive,total,transitive) and there only finitely many options, then it can be represented by a real-valued utility function. Show that the finiteness assumption is essential. That is, specify a choice problem with an infinite option set *O* and specify a rational preference relation ≥ on *O* such that there is no utility function u: O → R that represents the preference relation ≥.

4. (4) For the most part, we look at decision theory as a tool for helping us make better decisions ourselves. However, to the extent that other people follow decision-theoretic principles---consciously or not---we can use decision theory to explain and predict the behaviour of other people. Let's look at some real-life cases.

A striking and depressing phenomenon from social psychology is *bystander apathy*. If an experimenter falls down on the street pretending to have a heart attack, most people by far will go past him without stopping to help. (By the way, university students are more likely to help than the general population.) Similarly, a crowd will often just observe someone suffering or in one infamous case, even being killed, without doing anything. Let's consider a decision-theoretic model of this situation as it appears from the point of view of a single person deciding whether to help or not.

	States of the World	
Options	Someone else will help if I don't	Nobody else will help
Not help	4	2
Be the first to help	0	1

A common attempt to explain bystander apathy involves the idea that people have a strong preference for not standing out, especially in a crowd. There is independent evidence for this notion from the phenomenon of peer pressure. Suppose that the bystander most prefers for someone else to help. The worst outcome is to be the first to help if someone else would have helped anyway, because then the bystander stood out unnecessarily. The second worst is being the first to help if nobody else will help because the bystander hates standing out from the crowd. Not helping when nobody else will help is the second best outcome. The resulting preferences can be represented by a utility function as shown in the matrix above.

a. What is the maximin choice in this decision problem? (If there is more than one, put down both.)

b. What is the minimax regret choice in this decision problem? (If there is more than one, put down both.)

c. Does one option strictly dominate another? If so, which?

d. Does one option weakly dominate another? If so, which?

5. (6) Let's assume that the bystander is more concerned with seeing the victim saved than we imagined. The bystander still prefers that someone else save the victim, but if nobody else will, he'd rather stand out than see the victim die. The utility function below corresponds to these preferences.

	States of the World	
Options	Someone else will help if I don't	Nobody else will help
Not help	4	1
Be the first to help	0	2

a. What is the maximin choice in this decision problem? (If there is more than one, put down both.)

b. What is the minimax regret choice in this decision problem? (If there is more than one, put down both.)

c. Does one option strictly dominate another? If so, which?

d. Does one option weakly dominate another? If so, which?

e. What is the range of probabilities for "Someone else will help if I don't" such that "not help" has higher expected utility of "be the first to help". (Hint: Start by finding the "point of balance", the probability that makes the expected utilities for the two acts equal.)

6. (20) Consider a decision problem with two options and three states of the world as shown below.

	States of the World		
Options			
a			
b			

a. Can you fill in payoffs and specify a probability function such that option *a strictly* dominates option *b* **and** both options have the same expected utility? If yes, specify some payoffs and a probability assignment that meet this condition. If no, explain why not.

b. Can you fill in payoffs and specify a probability function such that

option *a weakly* dominates option *b* **and** both options
have the same expected utility? If yes, specify some payoffs and a
probability assignment that meet this condition. If no, explain why not.

c. Can you fill in payoffs and specify a probability function such that
option *a strictly* dominates option *b* **and** both options
have the minmax regret? If yes, specify some payoffs and a
probability assignment that meet this condition. If no, explain why not.

d. Can you fill in payoffs and specify a probability function such that
option *a weakly* dominates option *b* **and** both options
have the same minmax regret? If yes, specify some payoffs and a
probability assignment that meet this condition. If no, explain why not.

7. (3) What type of game (i.e., BoS, Chicken, etc. – see readings) does the following game matrix represent? (Hint: Remember that utility functions represent preferences, so you can compare the preferences defined in the game matrix with those in standard games. Also, you can always change a player's utility function *u* by adding constants or multiplying by a positive number. If you can transform one game matrix into another using this kind of positive linear transformation, then they represent the same game.)

	L	R
T	1,1	-2,2
B	2,-2	-1,-1

8. (3) What type of game (i.e., BoS, Chicken, etc. – see readings) does the following game matrix represent?

	L	R
T	2,0	-2,-2
B	-2,-2	0,2

9. (6) Army A has a single plane with which it can strike one of two possible targets. Army B has one anti-aircraft gun that it can assign to one of the targets. If Army A attacks a target, A destroys the target if Army B's anti-aircraft gun does not defend the target. If B's gun is defending the target that A attacks, the target is safe. Model this situation as a game in which A has two options "attack target 1" and "attack target 2", and B has two options "defend target 1" and "defend target 2". Army A does not care which target it destroys, but prefers destroying a target to not destroying one. Army B prefers both targets to be safe, but is indifferent between target 1 and target 2 being destroyed.

a. (3) Write down a game matrix that models this description.

b. (3) What type of game is this? (Coordination Game, Prisoner's Dilemma, Chicken, etc. If it's not one of the standard types mentioned in the book or in class, you wrote down the wrong game matrix.)