

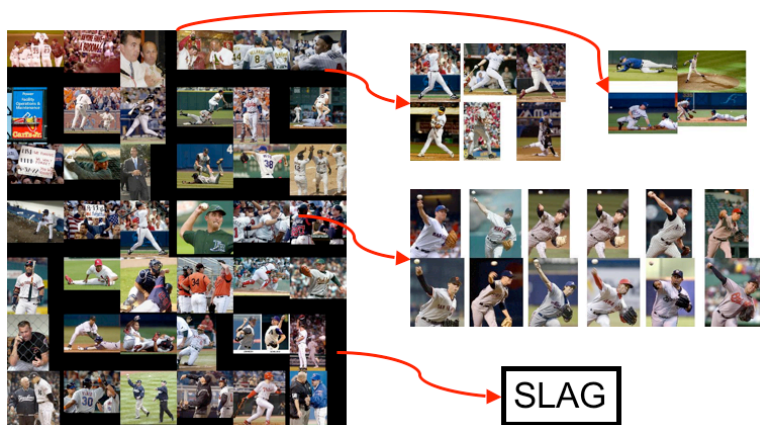
Stock Price Prediction

S&P/TSX COMPOSITE
as of 4-Apr-2008



- Problems in which t_i is continuous are called **regression**
- E.g. t_i is stock price, x_i contains company profit, debt, cash flow, gross sales, number of spam emails sent, ...

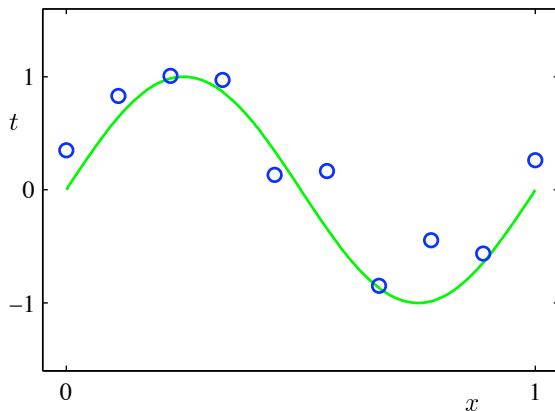
Clustering Images



Wang et al., CVPR 2006

- Only x_i is defined: **unsupervised learning**
- E.g. x_i describes image, find groups of similar images

An Example - Polynomial Curve Fitting



- Suppose we are given training set of N observations (x_1, \dots, x_N) and (t_1, \dots, t_N) , $x_i, t_i \in \mathbb{R}$
- Regression problem, estimate $y(x)$ from these data

Polynomial Curve Fitting

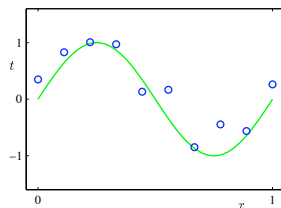
- What form is $y(x)$?
 - Let's try polynomials of degree M :

$$y(x, \mathbf{w}) = w_0 + w_1x + w_2x^2 + \dots + w_Mx^M$$

- This is the **hypothesis space**.
- How do we measure success?
 - Sum of squared errors:

$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\}^2$$

- Among functions in the class, choose that which minimizes this error



Polynomial Curve Fitting

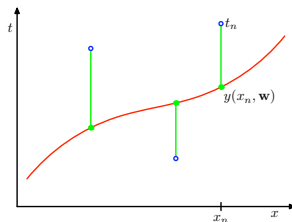
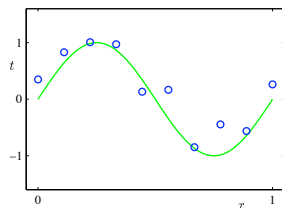
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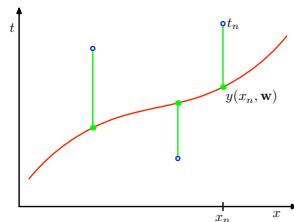
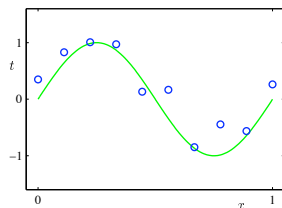
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Polynomial Curve Fitting

- Error function

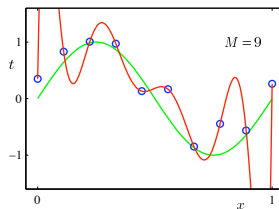
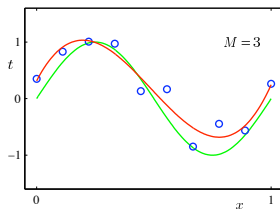
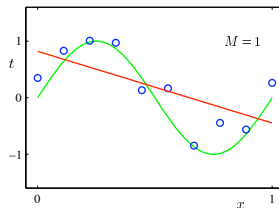
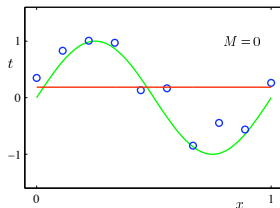
$$E(\mathbf{w}) = \frac{1}{2} \sum_{n=1}^N \{y(x_n, \mathbf{w}) - t_n\}^2$$

- Best coefficients

$$\mathbf{w}^* = \arg \min_{\mathbf{w}} E(\mathbf{w})$$

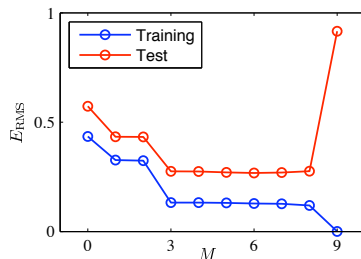
- Found using pseudo-inverse (more later)

Which Degree of Polynomial?



- A model selection problem
- $M = 9 \rightarrow E(\mathbf{w}^*) = 0$: This is over-fitting

Generalization



- Generalization is the holy grail of ML
 - Want good performance for new data
- Measure generalization using a separate set
 - Use root-mean-squared (RMS) error: $E_{RMS} = \sqrt{2E(\mathbf{w}^*)/N}$