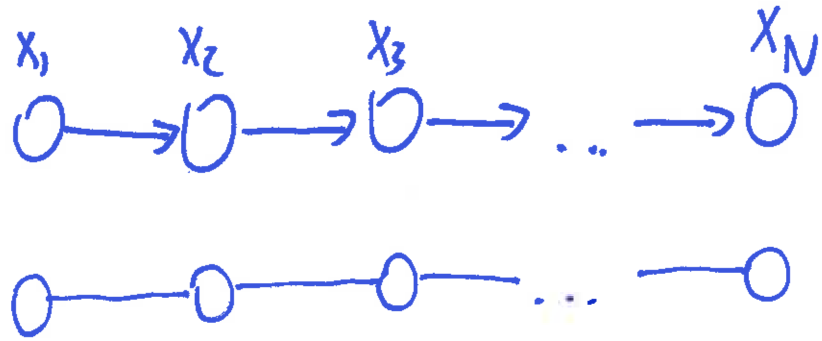



$x_n \in \{1, 2, \ldots, K\}$

$O(K^N)$

$$P(x_1) = \sum_{x_N} \cdots \sum_{x_3} \sum_{x_2} P(x_1, x_2, \ldots, x_N)$$

$$= \frac{1}{Z} \sum_{x_N} \cdots \sum_{x_3} \sum_{x_2} \psi_{12}(x_1, x_2)\, \psi_{23}(x_2, x_3) \cdots \psi_{N-1,N}(x_{N-1}, x_N)$$

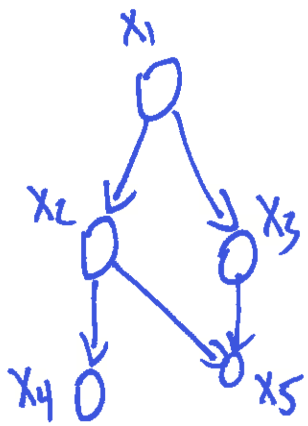
$$4 \times (2+3) = 4 \times 2 + 4 \times 3$$

$O(K^2)$ $\displaystyle\sum_{i=1}^{K} \sum_{j=1}^{K} a_i b_j = (a_1 + a_2 + \ldots + a_K)(b_1 + b_2 + \ldots + b_K)$ $O(K)$

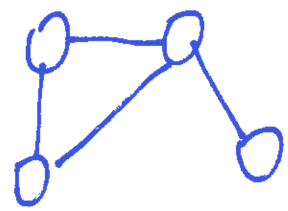
$$= \frac{1}{Z} \sum_{x_2} \cdots \left[\sum_{x_{N-2}} \psi_{N-3,N-2}(x_{N-3}, x_{N-2}) \left[\sum_{x_{N-1}} \psi_{N-2,N-1}(x_{N-2}, x_{N-1}) \left[\sum_{x_N} \psi_{N-1,N}(x_{N-1}, x_N)\right]\right]\right]$$

$O(NK^2)$

$$P(x_{11} \mid y_{12} = +1)$$

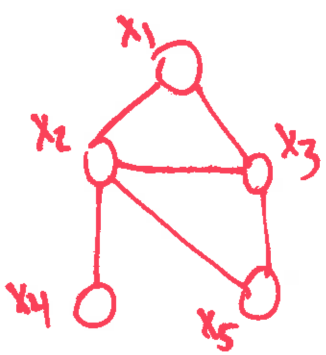



Bayesian Network



Markov Random Field

$$P(x_1, x_2, x_3, x_4, x_5) = P(x_1)\,P(x_2 \mid x_1)\,P(x_3 \mid x_1)\,P(x_4 \mid x_2)\,P(x_5 \mid x_2, x_3)$$




$$= \frac{1}{Z}\,\psi_{12}(x_1, x_2)\,\psi_{13}(x_1, x_3)\,\psi_{24}(x_2, x_4)\,\psi_{25}(x_2, x_5)\,\psi_{35}(x_3, x_5)$$

$\psi_{123}(x_1, x_2, x_3)$

$\psi_{235}(x_2, x_3, x_5)$

$$\sum_{x_N} \psi_{N-1,N}(x_{N-1}, x_N)$$

$$\psi_{N-1,N}(x_{N-1}, x_N) = \begin{array}{cc} & x_{N-1} \\ x_N & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$$

$$\sum_{x_N} \psi_{N-1,N}(x_{N-1}, x_N) = \begin{array}{c} x_{N-1} \\ \begin{bmatrix} a+c & b+d \end{bmatrix} \end{array}$$

$$\psi_{N-2,N-1}(x_{N-2}, x_{N-1}) \left[ \sum_{x_N} \psi_{N-1,N}(x_{N-1}, x_N) \right]$$

$$\times$$

$$\psi_{N-2,N-1}(x_{N-2}, x_{N-1}) = \begin{array}{cc} & x_{N-2} \\ x_{N-1} & \begin{bmatrix} s & t \\ u & v \end{bmatrix} \end{array}$$

$$\begin{array}{cc} & x_{N-1} \\ x_{N-2} & \begin{bmatrix} s \cdot (a+c) & u \cdot (b+d) \\ t \cdot (a+c) & v \cdot (b+d) \end{bmatrix} \end{array}$$