Assignment 2: Classification / Deep learning

Due October 25 at 11:59pm

This assignment is to be done individually.

Important Note: The university policy on academic dishonesty (cheating) will be taken very seriously in this course. You may not provide or use any solution, in whole or in part, to or by another student.

You are encouraged to discuss the concepts involved in the questions with other students. If you are in doubt as to what constitutes acceptable discussion, please ask! Further, please take advantage of office hours offered by the instructor and the TA if you are having difficulties with this assignment.

DO NOT:

- Give/receive code or proofs to/from other students
- Use Google to find solutions for assignment

DO:

- Meet with other students to discuss assignment (it is best not to take any notes during such meetings, and to re-work assignment on your own)
- Use online resources (e.g. Wikipedia) to understand the concepts needed to solve the assignment

---

1 Softmax for Multi-Class Classification (10 marks)

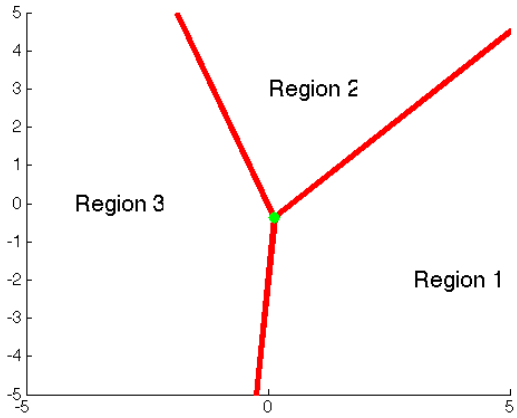
The *softmax function* is a multi-class generalization of the logistic sigmoid:

$$p(\mathcal{C}_k|\boldsymbol{x}) = \frac{\exp(a_k)}{\sum_j \exp(a_j)} \tag{1}$$

Consider a case where the *activation functions* a_j are linear functions of the input. Assume there are 3 classes ($\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$), and the input is $\boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2$

- $a_1 = 3x_1 + 1x_2 + 1$
- $a_2 = 1x_1 + 3x_2 + 2$
- $a_3 = -3x_1 + 1.5x_2 + 2$

The image below shows the 3 decision regions induced by these activation functions, their common point intersection point (in green) and decision boundaries (in red).



Answer the following questions. For 2 and 3, you may provide qualitative answers (i.e. no need to analyze limits).

1. (3 marks) What are the probabilities $p(\mathcal{C}_k|\boldsymbol{x})$ at the green point?
2. (3 marks) What happens to the probabilities along each of the red lines? What happens as we move along a red line (away from the green point)?
3. (4 marks) What happens to the probabilities as we move far away from the intersection point, staying in the middle of one region?

2 Generalized Linear Models for Classification (10 marks)

Consider a generalized linear model for classification over a two-dimensional input $\boldsymbol{x} = (x_1, x_2) \in \mathbb{R}^2$. Assume there are 2 classes.

Use the polynomial basis functions below:

label= $\phi_0(\boldsymbol{x}) = 1$

label= $\phi_1(\boldsymbol{x}) = x_1^2$

label= $\phi_2(\boldsymbol{x}) = x_2^2$

Suppose we learn weight vectors $\boldsymbol{w}^1 = (6, 2, 2)$ and $\boldsymbol{w}^2 = (8, 0, 0)$, so that

label= $y_1(\boldsymbol{x}) = 6\phi_0(\boldsymbol{x}) + 2\phi_1(\boldsymbol{x}) + 2\phi_2(\boldsymbol{x})$

label= $y_2(\boldsymbol{x}) = 8\phi_0(\boldsymbol{x}) + 0\phi_1(\boldsymbol{x}) + 0\phi_2(\boldsymbol{x})$

Answer the following questions.

1. (3 marks) Draw the function $y_1(\boldsymbol{x})$.
2. (3 marks) Draw the function $y_2(\boldsymbol{x})$.
3. (4 marks) Draw the decision regions that are given by the multi-class method that assigns a label to a point $\boldsymbol{x}$ by $\arg\max_k y_k(\boldsymbol{x})$.

If you use some code to draw the plots, please submit the code as well for this problem.

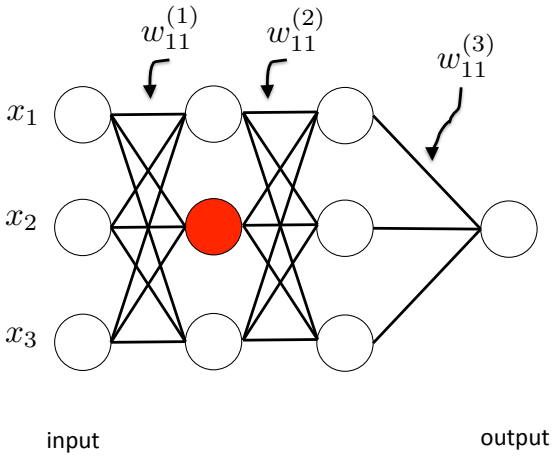
3 Error Backpropagation (30 marks)

We will derive error derivatives using back-propagation on the network below.

Notation: Please use notation following the examples of names for weights given in the figure. For activations/outputs, the red node would have activation $a_2^{(2)} = w_{21}^{(1)} x_1 + w_{22}^{(1)} x_2 + w_{23}^{(1)} x_3$ and output $z_2^{(2)} = h(a_2^{(2)})$.

Activation functions: Assume the activation functions $h(\cdot)$ for the hidden layers are logistics. For the final output node assume the activation function is an identity function $h(a) = a$.

Error function: Assume this network is doing regression, trained using the standard squared error so that $E_n(w) = \frac{1}{2}(y(\boldsymbol{x}_n, w) - t_n)^2$.



Consider the output layer.

- Calculate $\frac{\partial E_n(w)}{\partial a_1^{(4)}}$. Note that $a_1^{(4)}$ is the activation of the output node, and that $\frac{\partial E_n(w)}{\partial a_1^{(4)}} \equiv \delta_1^{(4)}$. (5 marks)
- Use this result to calculate $\frac{\partial E_n(w)}{\partial w_{12}^{(3)}}$. (5 marks)

Next, consider the penultimate layer of nodes.

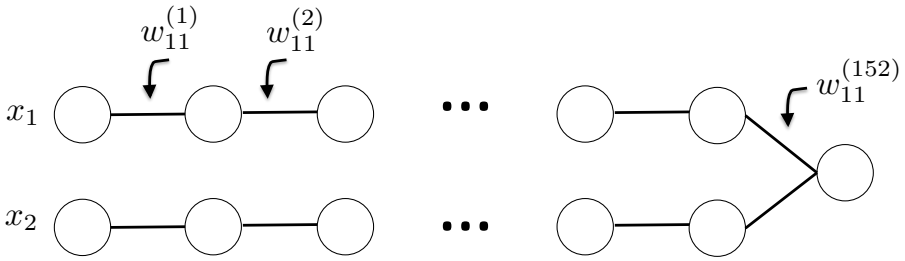
- Write an expression for $\frac{\partial E_n(w)}{\partial a_1^{(3)}}$. Use $\delta_1^{(4)}$ in this expression. (5 marks)
- Use this result to calculate $\frac{\partial E_n(w)}{\partial w_{11}^{(2)}}$. (5 marks)

Finally, consider the weights connecting from the inputs.

- Write an expression for $\frac{\partial E_n(w)}{\partial a_1^{(2)}}$. Use the set of $\delta_k^{(3)}$ in this expression. (5 marks)
- Use this result to calculate $\frac{\partial E_n(w)}{\partial w_{11}^{(1)}}$. (5 marks)

4 Vanishing Gradients (30 marks)

Consider the network below. Use the same notation as the previous question.



- Write an expression for $\frac{\partial E_n(w)}{\partial w_{11}^{(\ell)}}$ for all layers ℓ in the network. (6 marks)
- Suppose we use logistic sigmoid activation functions in this network. Derive the gradient of of logistic sigmoid function $h(a)$. Describe what would happen to the learning for weights early in the network for small ℓ if the sigmoid activations are close to the boundaries of their operating range, i.e. 0 and 1. Give at least two reasons for which the gradients of the weight could be small. (9 marks)
- Suppose we use rectified linear units (ReLU) in activation functions. When would the gradients be zero? (7 makrs)
- Suppose we modify the graph to have complete bipartite connections at each layer, still using ReLU activation functions. When would the gradients be zero? (8 marks)

5 Logistic Regression (40 marks)

In this question you will examine optimization for logistic regression.

1. Download the assignment 2 code and data from the website. Run the script `logistic_regression.py` in the lr directory. This code performs gradient descent to find $\boldsymbol{w}$ which minimizes negative log-likelihood (i.e. maximizes likelihood).

   Include the final output of Figures 2 and 3 (plot of separator path in slope-intercept space; plot of neg. log likelihood over epochs) in your report.

   Why are these plots oscillating? Briefly explain why in your report. (10 Marks)

2. Create a Python script `logistic_regression_mod.py` for the following.

   Modify `logistic_regression.py` to run gradient descent with the learning rates $\eta = 0.5, 0.3, 0.1, 0.05, 0.01$.

   Include in your report a single plot comparing negative log-likelihood versus epoch for these different learning rates.

   Compare these results. What are the relative advantages of the different rates? (15 Marks)

3. Create a Python script `logistic_regression_sgd.py` for the following.

   Modify this code to do stochastic gradient descent. Use the parameters $\eta = 0.5, 0.3, 0.1, 0.05, 0.01$.

   Include in your report a new plot comparing negative log-likelihood versus iteration using stochastic gradient descent.

   Is stochastic gradient descent faster than gradient descent? Explain using your plots. (15 Marks)

6 Fine-Tuning a Pre-Trained Network (30 marks)

In this question you will experiment with fine-tuning a pre-trained network. This is a standard workflow in adapting existing deep networks to a new task.

We will utilize PyTorch (`https://pytorch.org`) a machine learning library for python.

The provided code builds upon ResNet20, a state of the art deep network for image classification. The model has been trained for CIFAR-100 image classification with 100 output classes.

The ResNet20 model has been adapted to solve a (simpler) different task, classifying an image as one of 10 classes on CIFAR10 dataset.

The code `imagenet_finetune.py` does the following:

- Constructs a deep network. This network starts with ResNet20 up to its average pooling layer. Then, a new linear classifier .
- Initializes the weights of the ResNet20 portion with the parameters from training on CIFAR-10.
- Performs training on only the new layers using CIFAR-10 dataset – all other weights are fixed to their values learned on ImageNet.

The code and data can be found on the course website. For convenience, Anaconda (`https://www.anaconda.com`) environment config files with the latest stable release of PyTorch and torchvision are provided for Python 3.6 for Linux users. To set up the virtual environment, install Anaconda and run the following command

```
conda env create -f cmpt726-pytorch-python36.yml
```

To activate the virtual environment, run the following command

```
source activate cmpt726-pytorch-python36
```

If you wish to download and install PyTorch by yourself, please use PyTorch (v 1.2.0), torchvision (v 0.4.0), and their dependencies. Windows users please note that PyTorch only support Python 3.

What to do:

Start by running the code provided. It may be slow to train since the code runs on a CPU. You can try figuring out how to change the code to train on a GPU if you have a good GPU and it could accelerate training. Try to do one of the following tasks:

- Write a Python function to be used at the end of training that generates HTML output showing each test image and its classification scores. You could produce an HTML table output for example. (You can convert the HTML output to PDF or use screen shots.)
- Run validation of the model every few training epochs on validation or test set of the dataset and save the model with the best validation error. Report the best validation error and the corresponding training epoch in your report. (Do not submit saved models for this task.)

- Try applying $L2$ regularization to the coefficients in the small networks we added.
- Try running this code on one of the datasets in torchvision.datasets (`https://pytorch.org/docs/stable/torchvision/datasets.html`) except CIFAR100. You may need to change some layers in the network. Try creating a custom dataloader that loads data from your own dataset and run the code using your dataloader. (Hints: Your own dataset should not come from torchvision.datasets. A standard approach is to implement your own torch.utils.data.Dataset and wrap it with torch.utils.data.DataLoader)
- Try modifying the structure of the new layers that were added on top of ResNet20.
- Try adding data augmentation for the training data using torchvision.transforms and then implementing your custom image transformation methods not available in torchvision.transforms, like gaussian blur.
- The current code is inefficient because it recomputes the output of ResNet20 every time a training/validation example is seen, even though those layers aren't being trained. Change this by saving the output of ResNet20 and using these as input rather than the dataloader currently used.
- The current code does not train the other layers in ResNet20. After training the new layers for a while (until good values have been obtained), turn on training for the other ResNet20 layers to see if better performance can be achieved.

Describe what task you do in details for this task in your report for this assignment. If you have any figures or tables to show, put them in the report as well. We also request you to submit code for this problem.

Submitting Your Assignment

The assignment must be submitted online at `https://courses.cs.sfu.ca`. You must submit three files:

1. An assignment report in **PDF format**, called `report.pdf`. This report must contain the solutions to questions 1-4 as well as the figures / explanations requested for 5-6. Attach your code to the end of the PDF report. Insert a page break between the code for each problem. For Problem 6, only attach the code below the line "Do not modify the code above this line".

2. A .zip file of all your code, called `code.zip`. Put the code for each problem in separate directories. For example, code for Problem 5 should be put under a directory named P5. For Problem 6, include a REAME.md file with the command to run your code as well.