

# GAME PLAYING

## CHAPTER 6

# Outline

- ◇ Games
- ◇ Perfect play
  - minimax decisions
  - $\alpha$ - $\beta$  pruning
- ◇ Resource limits and approximate evaluation
- ◇ Games of chance
- ◇ Games of imperfect information

# Games vs. search problems

“Unpredictable” opponent  $\Rightarrow$  solution is a **strategy**  
specifying a move for every possible opponent reply

Time limits  $\Rightarrow$  unlikely to find goal, must approximate

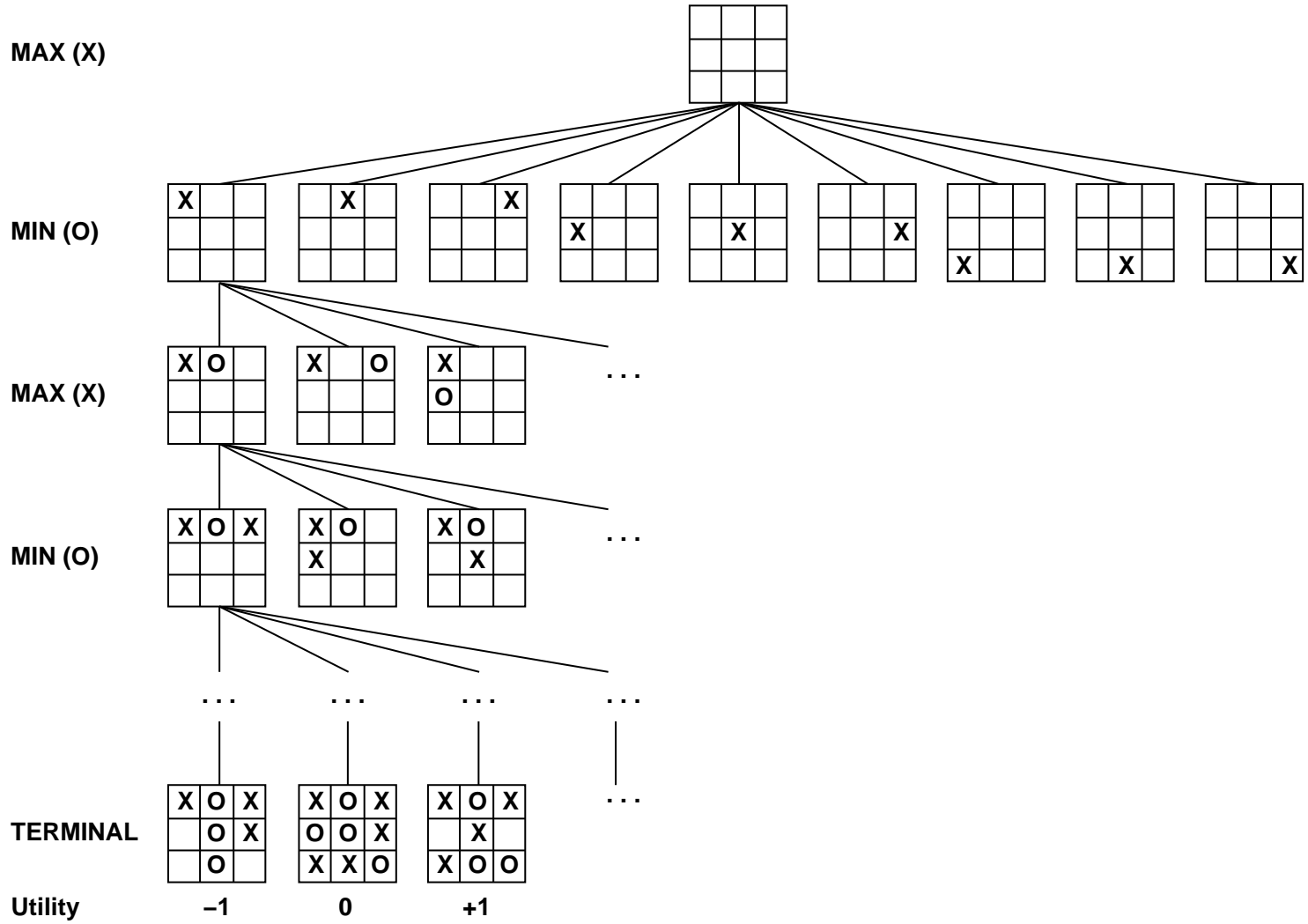
Plan of attack:

- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

# Types of games

	deterministic	chance
perfect information	chess, checkers, go, othello	backgammon monopoly
imperfect information	battleships, blind tictactoe	bridge, poker, scrabble nuclear war

# Game tree (2-player, deterministic, turns)

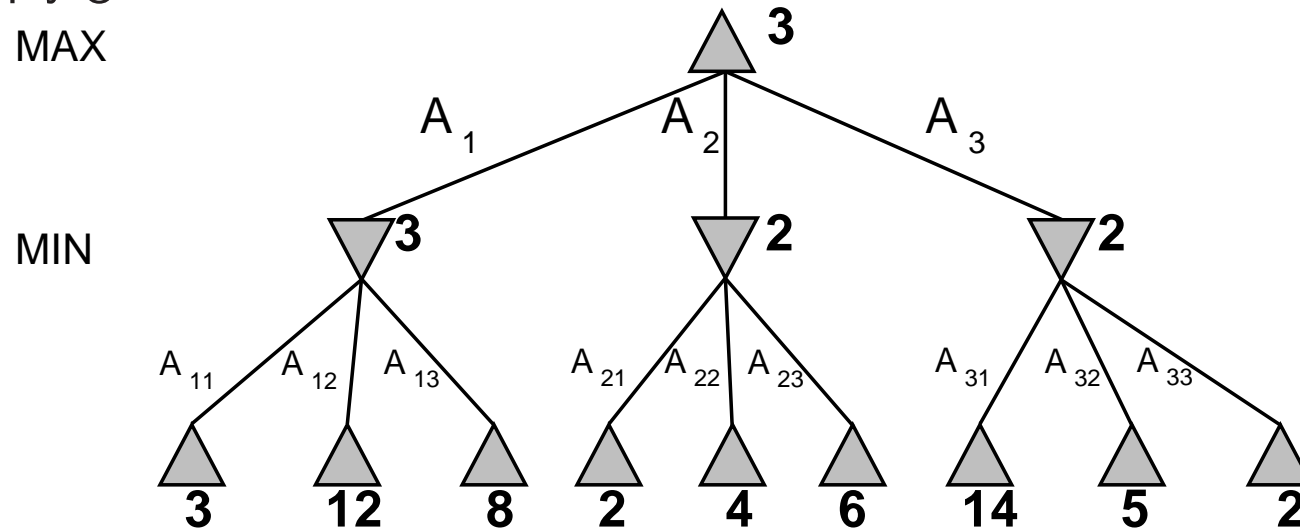


# Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest **minimax value**  
= best achievable payoff against best play

E.g., 2-ply game:



# Minimax algorithm

**function** MINIMAX-DECISION(*state*) **returns** *an action*

**inputs:** *state*, current state in game

**return** the *a* in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(*a*, *state*))

---

**function** MAX-VALUE(*state*) **returns** *a utility value*

**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

$v \leftarrow -\infty$

**for** *a, s* in SUCCESSORS(*state*) **do**  $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s))$

**return** *v*

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# Properties of minimax

Complete??



# Properties of minimax

Complete?? Only if tree is finite (chess has specific rules for this).

Optimal??

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Optimal?? Yes, against an optimal opponent. Otherwise??

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Time complexity??  $O(b^m)$

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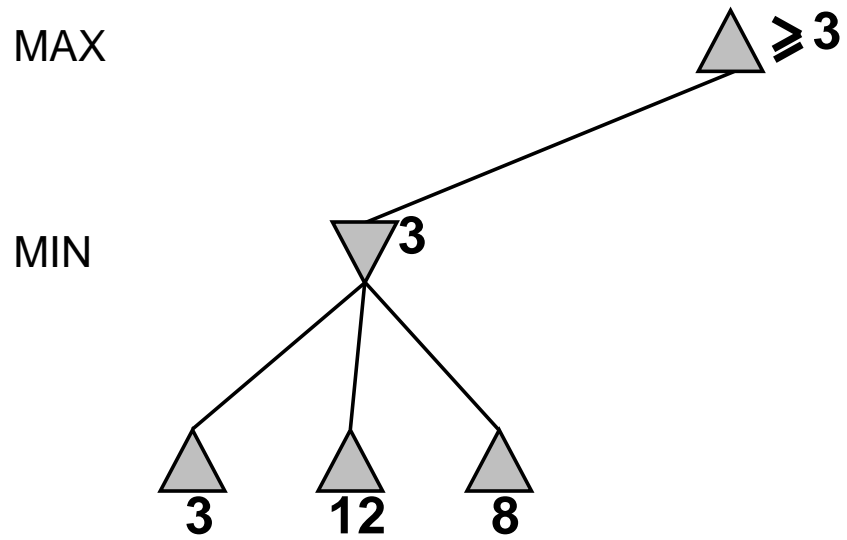
Time complexity??  $O(b^m)$

Space complexity??  $O(bm)$  (depth-first exploration)

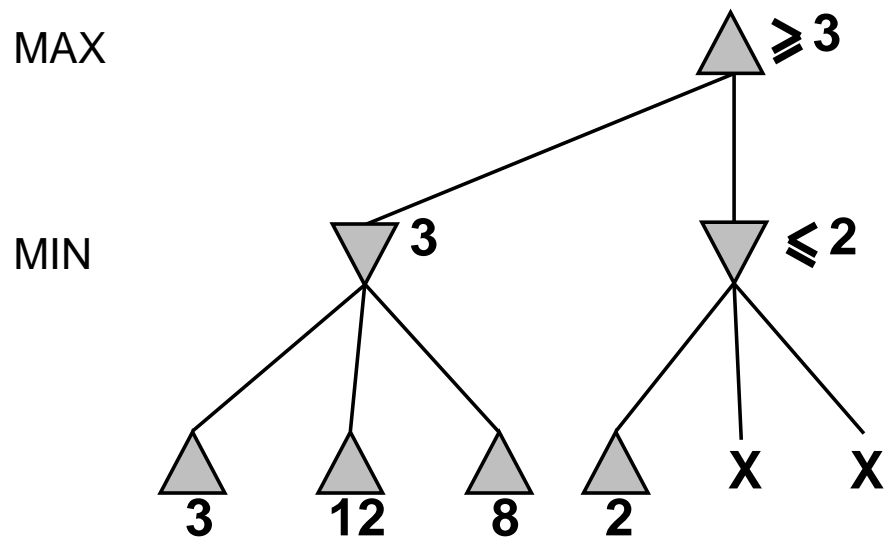
For chess,  $b \approx 35$ ,  $m \approx 100$  for “reasonable” games  
⇒ exact solution completely infeasible

But do we need to explore every path?

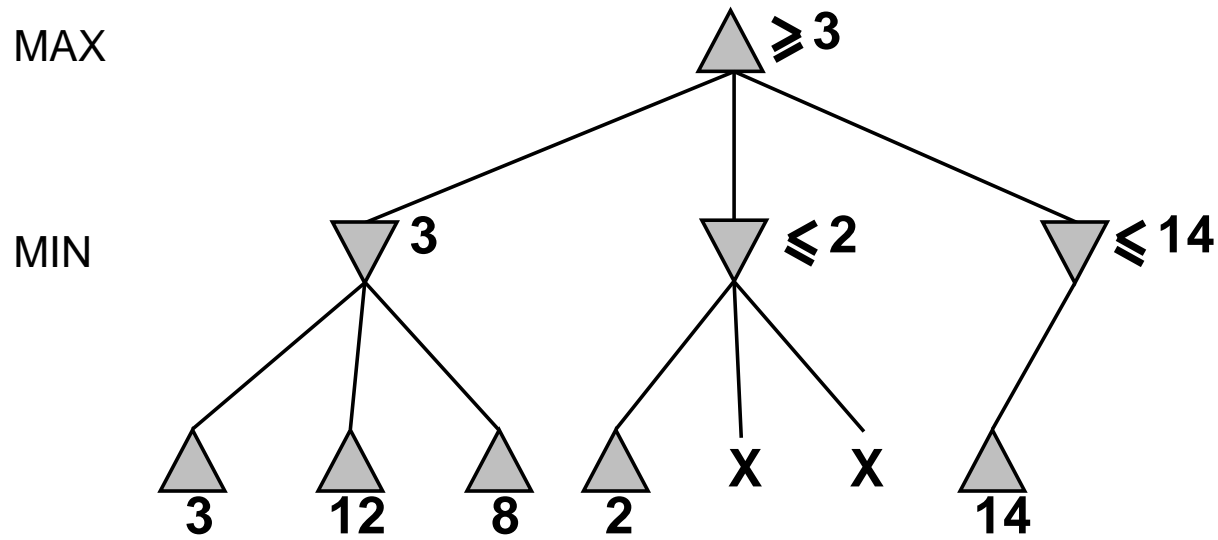
# $\alpha$ - $\beta$ pruning example



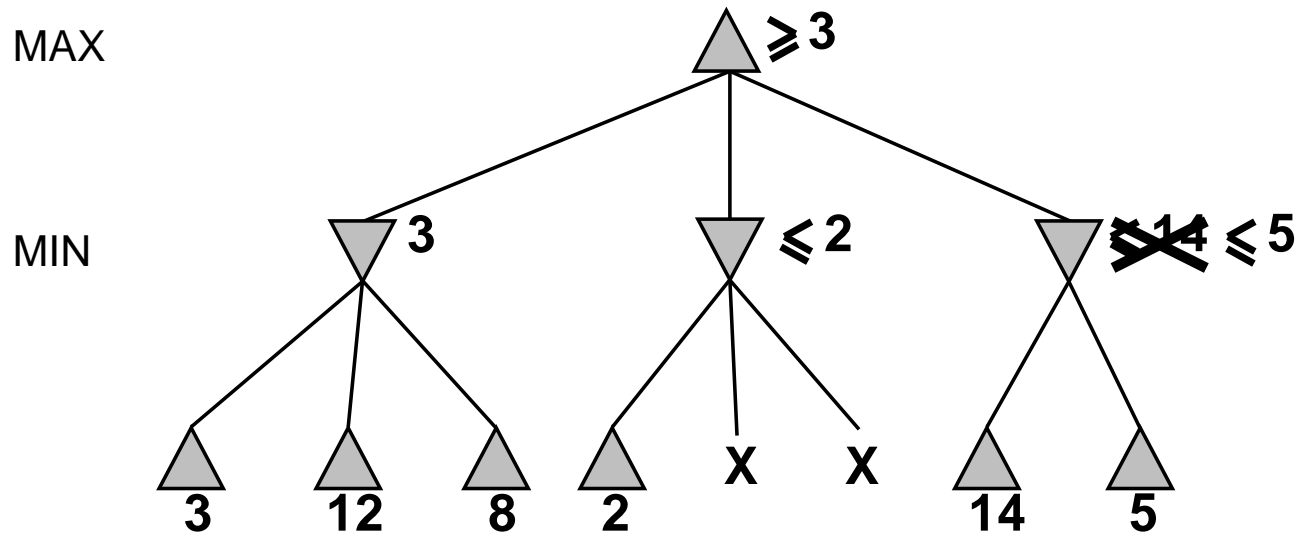
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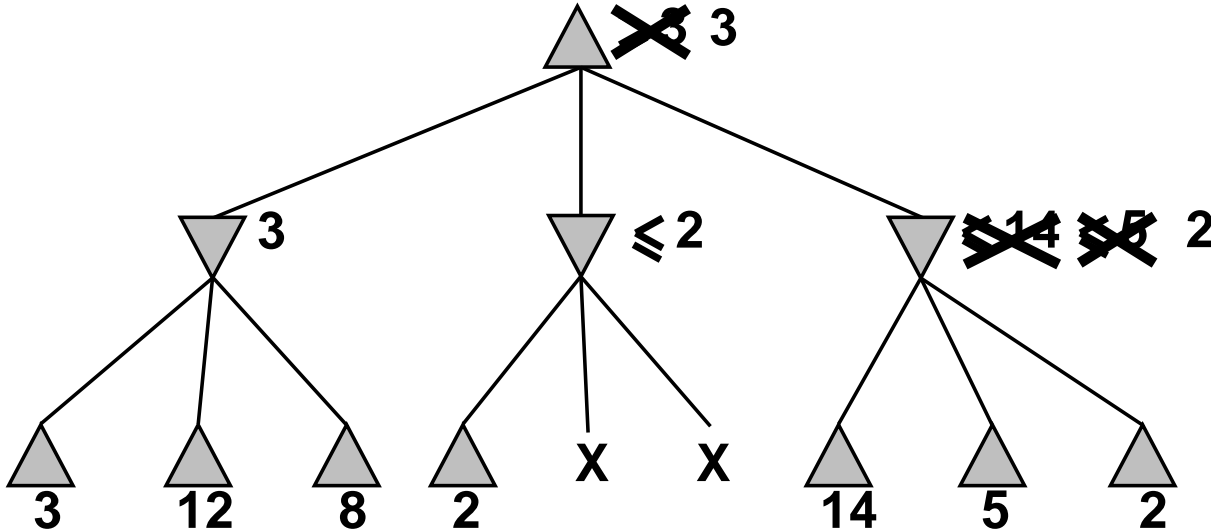




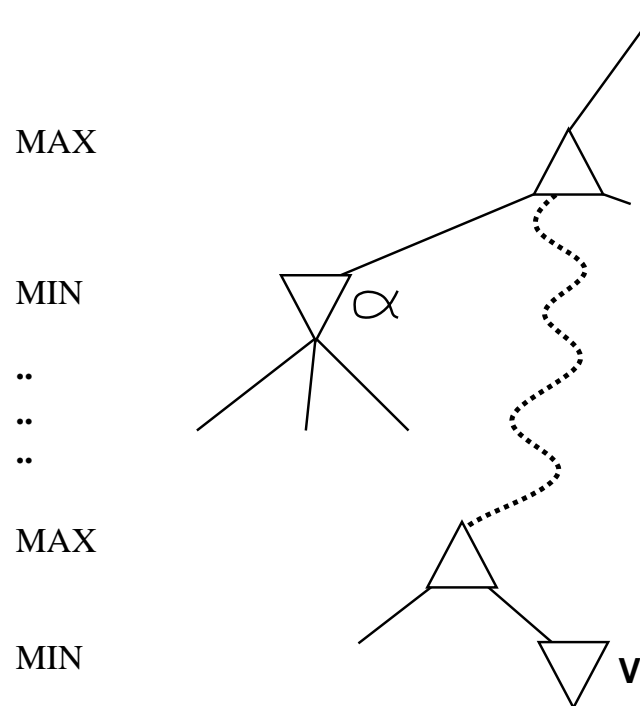
$\alpha$ - $\beta$  pruning example

MAX

MIN



## Why is it called $\alpha$ - $\beta$ ?



$\alpha$  is the best value (to MAX) found so far off the current path

If  $V$  is worse than  $\alpha$ , MAX will avoid it  $\Rightarrow$  prune that branch

Define  $\beta$  similarly for MIN

## The $\alpha$ - $\beta$ algorithm

**function** ALPHA-BETA-DECISION(*state*) **returns** an action  
**return** the *a* in ACTIONS(*state*) maximizing MIN-VALUE(RESULT(*a*, *state*))

---

**function** MAX-VALUE(*state*,  $\alpha$ ,  $\beta$ ) **returns** a utility value

**inputs:** *state*, current state in game

$\alpha$ , the value of the best alternative for MAX along the path to *state*

$\beta$ , the value of the best alternative for MIN along the path to *state*

**if** TERMINAL-TEST(*state*) **then return** UTILITY(*state*)

$v \leftarrow -\infty$

**for** *a*, *s* in SUCCESSORS(*state*) **do**

$v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, \alpha, \beta))$

**if**  $v \geq \beta$  **then return** *v*

$\alpha \leftarrow \text{MAX}(\alpha, v)$

**return** *v*

---

**function** MIN-VALUE(*state*,  $\alpha$ ,  $\beta$ ) **returns** a utility value

same as MAX-VALUE but with roles of  $\alpha$ ,  $\beta$  reversed

## Properties of $\alpha$ - $\beta$

Pruning **does not** affect final result

Good move ordering improves effectiveness of pruning

With “perfect ordering,” time complexity =  $O(b^{m/2})$   
 $\Rightarrow$  **doubles** solvable depth

A simple example of the value of reasoning about which computations are relevant (a form of **metareasoning**)

Unfortunately,  $35^{50}$  is still impossible!

## Resource limits

Standard approach:

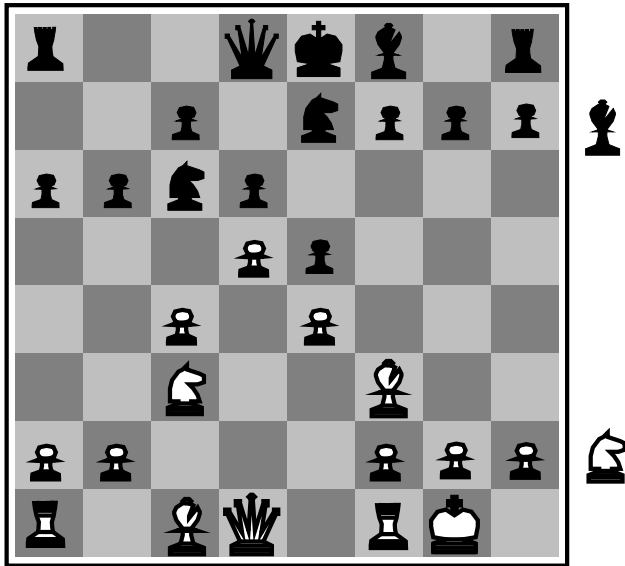
- Use CUTOFF-TEST instead of TERMINAL-TEST  
e.g., depth limit (perhaps add quiescence search)
- Use EVAL instead of UTILITY  
i.e., evaluation function that estimates desirability of position

Suppose we have 100 seconds, explore  $10^4$  nodes/second

$\Rightarrow 10^6$  nodes per move  $\approx 35^{8/2}$

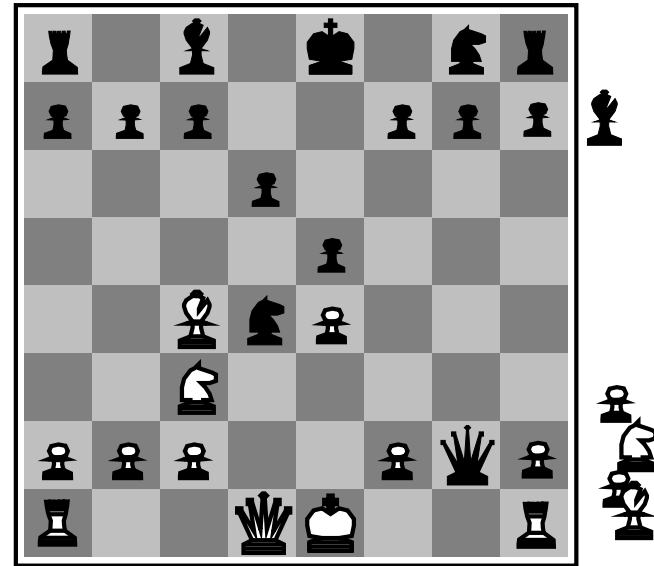
$\Rightarrow \alpha\text{-}\beta$  reaches depth 8  $\Rightarrow$  pretty good chess program

# Evaluation functions



**Black to move**

**White slightly better**



**White to move**

**Black winning**

For chess, typically **linear** weighted sum of **features**

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

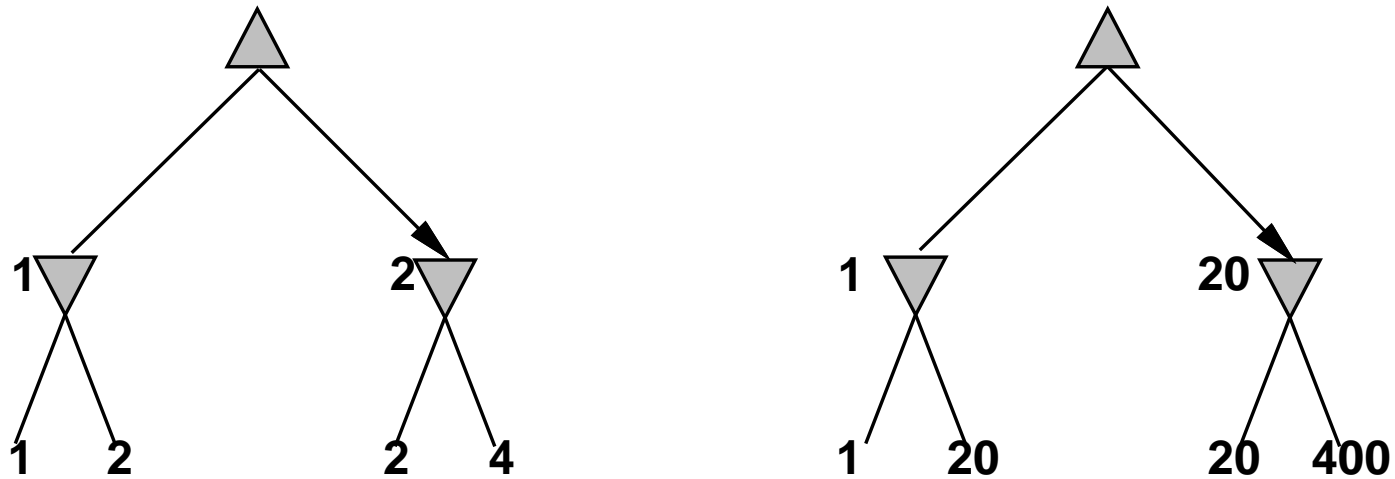
e.g.,  $w_1 = 9$  with

$f_1(s) = (\text{number of white queens}) - (\text{number of black queens}), \text{ etc.}$

# Digression: Exact values don't matter

MAX

MIN



Behaviour is preserved under any **monotonic** transformation of  $EVAL$

Only the order matters:

payoff in deterministic games acts as an **ordinal utility** function

## Deterministic games in practice

Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

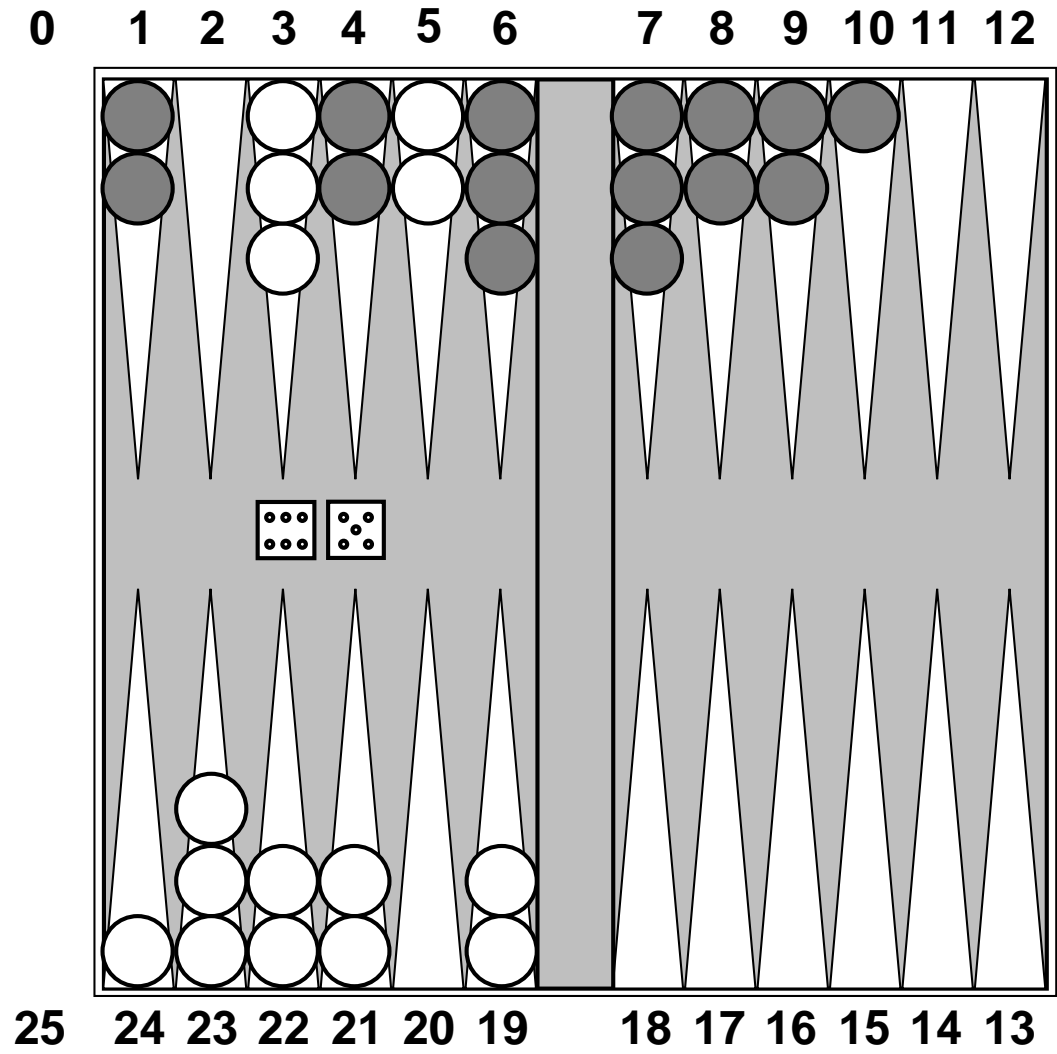
Chess: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello: human champions refuse to compete against computers, who are too good.

Go: human champions refuse to compete against computers, who are too bad. In go,  $b > 300$ , so most programs use pattern knowledge bases to suggest plausible moves.



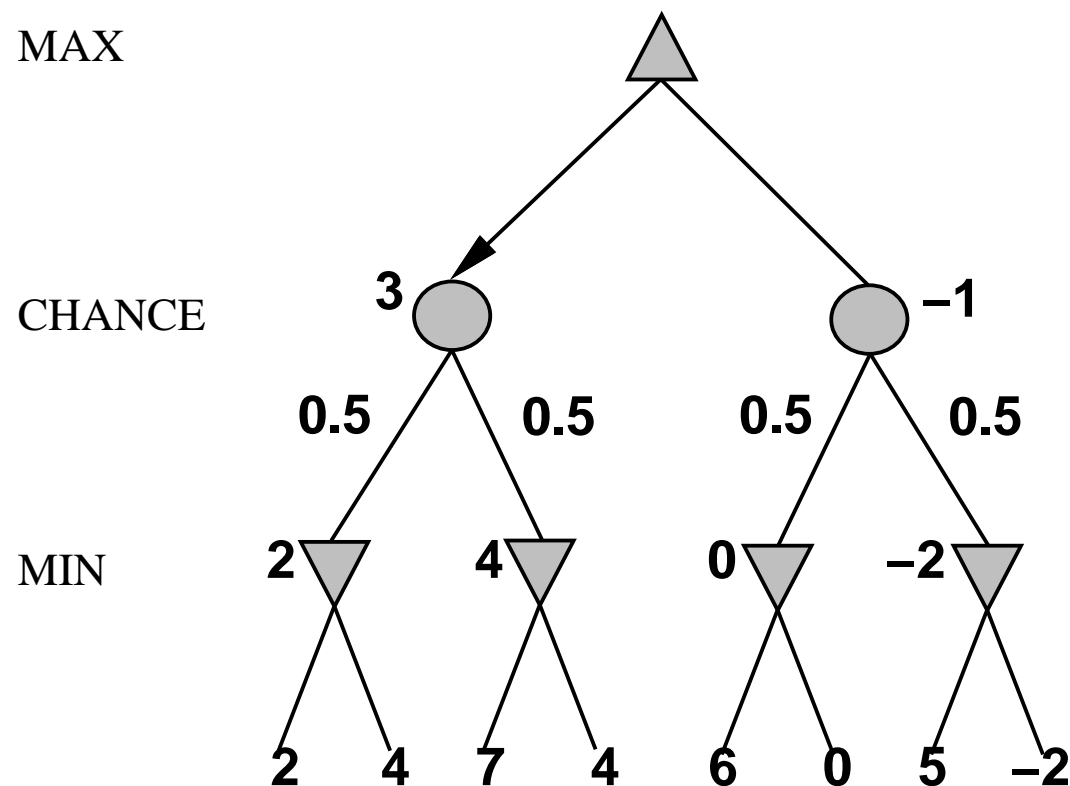
# Nondeterministic games: backgammon



# Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling

Simplified example with coin-flipping:



## Algorithm for nondeterministic games

EXPECTIMINIMAX gives perfect play

Just like MINIMAX, except we must also handle chance nodes:

...

**if** *state* is a MAX node **then**

**return** the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

**if** *state* is a MIN node **then**

**return** the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

**if** *state* is a chance node **then**

**return** average of EXPECTIMINIMAX-VALUE of SUCCESSORS(*state*)

...

## Nondeterministic games in practice

Dice rolls increase  $b$ : 21 possible rolls with 2 dice

Backgammon  $\approx$  20 legal moves (can be 6,000 with 1-1 roll)

$$\text{depth } 4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$$

As depth increases, probability of reaching a given node shrinks

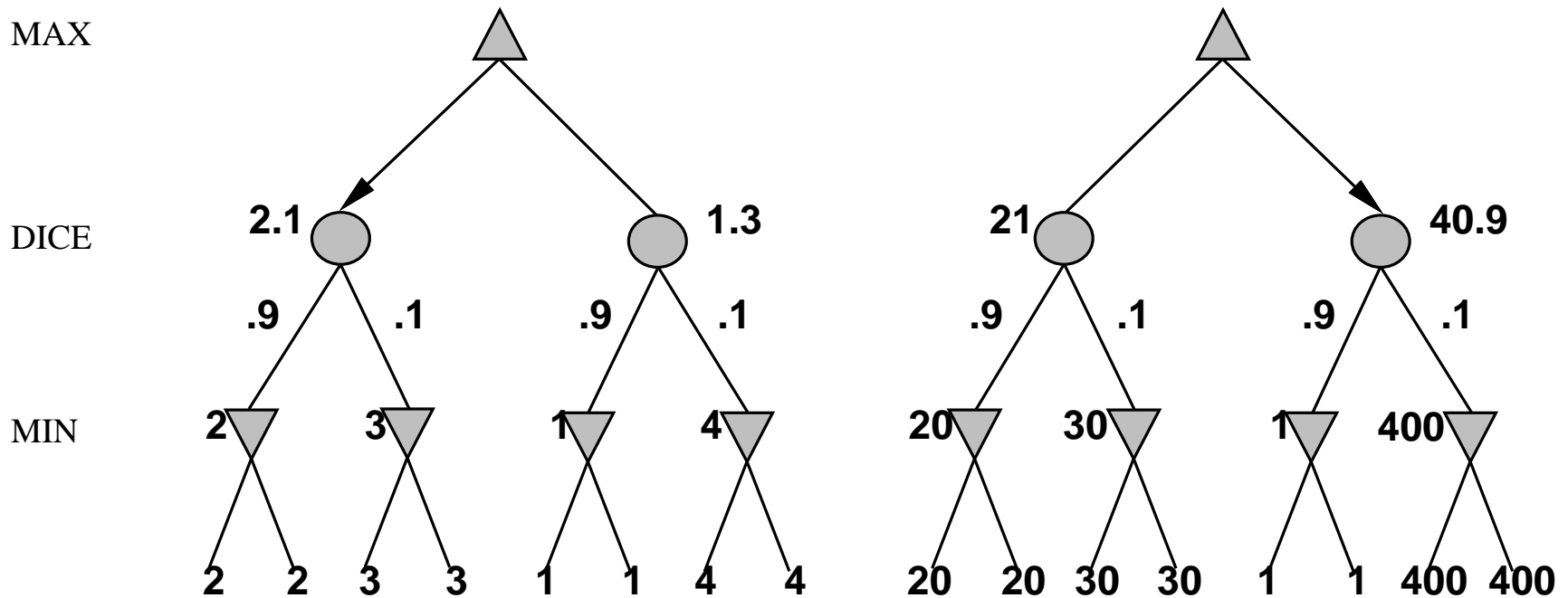
$\Rightarrow$  value of lookahead is diminished

$\alpha$ - $\beta$  pruning is much less effective

TDGAMMON uses depth-2 search + very good EVAL

$\approx$  world-champion level

# Digression: Exact values DO matter



Behaviour is preserved only by **positive linear** transformation of  $EVAL$

Hence  $EVAL$  should be proportional to the expected payoff

# Games of imperfect information

E.g., card games, where opponent's initial cards are unknown

Typically we can calculate a probability for each possible deal

Seems just like having one big dice roll at the beginning of the game\*

**Idea:** compute the minimax value of each action in each deal,  
then choose the action with highest expected value over all deals\*

Special case: if an action is optimal for all deals, it's optimal.\*

GIB, current best bridge program, approximates this idea by

- 1) generating 100 deals consistent with bidding information
- 2) picking the action that wins most tricks on average

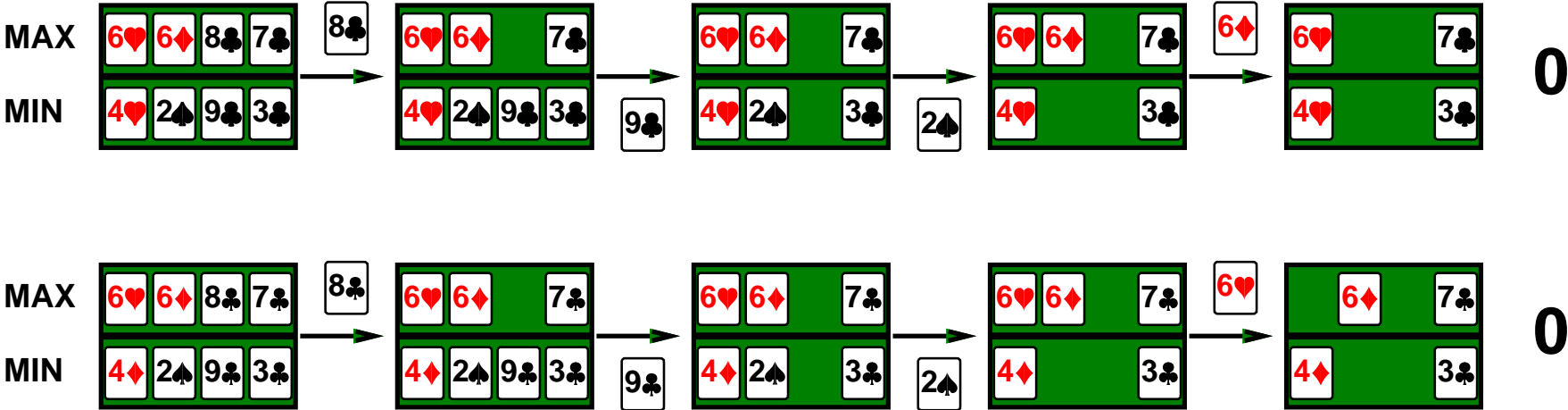
# Example

Four-card bridge/whist/hearts hand, MAX to play first



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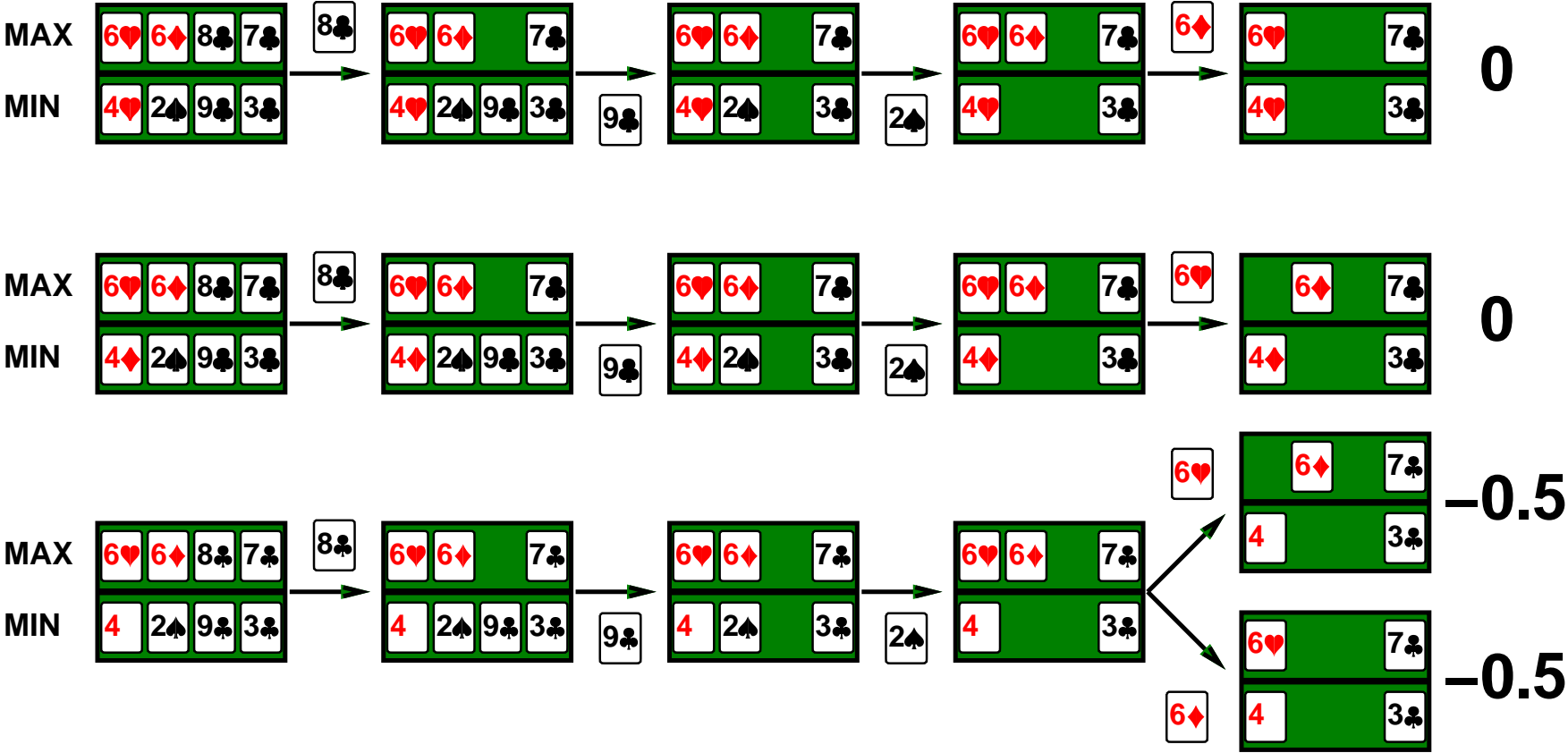
Four-card bridge/whist/hearts hand, MAX to play first





# Example

Four-card bridge/whist/hearts hand, MAX to play first



## Commonsense example

Road A leads to a small heap of gold pieces

Road B leads to a fork:

take the left fork and you'll find a mound of jewels;

take the right fork and you'll be run over by a bus.

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Road B leads to a fork:

take the left fork and you'll be run over by a bus;

take the right fork and you'll find a mound of jewels.

Road A leads to a small heap of gold pieces

Road B leads to a fork:

guess correctly and you'll find a mound of jewels;

guess incorrectly and you'll be run over by a bus.

## Proper analysis

\* Intuition that the value of an action is the average of its values in all actual states is **WRONG**

With partial observability, value of an action depends on the **information state** or **belief state** the agent is in

Can generate and search a tree of information states

Leads to rational behaviors such as

- ◇ Acting to obtain information
- ◇ Signalling to one's partner
- ◇ Acting randomly to minimize information disclosure

## Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- ◇ perfection is unattainable  $\Rightarrow$  must approximate
- ◇ good idea to think about what to think about
- ◇ uncertainty constrains the assignment of values to states
- ◇ optimal decisions depend on information state, not real state

Games are to AI as grand prix racing is to automobile design