Hash Tables

CMPT 225

Problem Examples

- What can we do if we want rapid access to individual data items?
  - Looking up data for a flight in an air traffic control system
  - Looking up the address of someone making a 911 call
  - Checking the spelling of words by looking up each one in a dictionary
- In each case speed is very important
  - But the data does not need to be maintained in order

Dictionary ADT

- Operations
  - Insert (key,value) pair
  - Lookup value for a key
  - Remove (key,value) pair
  - Modify (key,value) pair

- Dictionary ADT also known as
  - Associative Array
  - Map

Possible Solutions

- Balanced binary search tree
  - Binary search trees allow lookup and insertion in O(log*n*) time
    - Which is relatively fast
  - Binary search trees also maintain data in order, which may be not necessary for some problems
- Arrays
  - Allow insertion in constant time, but lookup requires linear time
  - But, if we know the index of a data item lookup can be performed in constant time

Thinking About Arrays

- Can we use an array to insert and retrieve data in constant time?
  - Yes – as long as we know an item's index
- Consider this (very) constrained problem domain:
  - A phone company wants to store data about its customers in Convenientville
  - The company has around 9,000 customers
  - Convenientville has a single area code (604-555?)

Living in Convenientville

- Create an array of size 10,000
  - Assign customers to array elements using their (four digit) phone number as the index
  - Only around 1,000 array elements are wasted
  - Customers can be looked up in constant time using their phone numbers
- Of course this is not a general solution
  - It relies on having conveniently numbered *key* values

Phone Numbers in General

- Let's consider storing information about Canadians given their phone numbers
  - Between 000-000-000 and 999-999-9999
- It's easy to convert phone numbers to integers
  - Just get rid of the "-"s
  - The keys range between 0 and 9,999,999,999
- Use Convenientville scheme to store data
  - But will this work?

A Really Big Array!

- If we use Canadian phone numbers as the index to an array how big is the array?
  - 9,999,999,999 (ten billion)
  - That's a really big array!
- Consider that the estimate of the current population of Canada is 33,476,688*
  - That means that we will use around 0.3% of the array
    - That's a lot of wasted space
    - And the array probably won't fit in main memory …
- *According to the 2011 Census

More Examples

- What if we had to store data by name?
  - We would need to convert strings to integer indexes
- Here is one way to encode strings as integers
  - Assign a value between 1 and 26 to each letter
  - a = 1, z = 26 (regardless of case)
  - Sum the letter values in the string

"dog" = 4 + 15 + 7 = 26

"god" = 7 + 15 + 4 = 26

Finding Unique String Values

- Ideally we would like to have a unique integer for each possible string
- This is relatively straightforward
  - As before, assign each letter a value between 1 and 26
  - And multiply the letter's value by 26^i, where **i** is the position of the letter in the word:
    - "dog" = $4*26^2 + 15*26^1 + 7*26^0 = 3{,}101$
    - "god" = $7*26^2 + 15*26^1 + 4*26^0 = 5{,}126$

Afhahgm Vsyu

- The proposed system generates a unique number for each string
  - However most strings are not meaningful
  - Given a string containing ten letters there are 2610 possible combinations of letters
    - That is, 141,167,095,653,376 different possible strings
- It is not practical to create an array large enough to store all possible strings
  - Just like the general telephone number problem

So What's The Problem?

- In an ideal world we would know which key values were to be recorded
  - The Convenientville example was very close to this ideal
- Most of the time this is not the case
  - Usually, key values are not known in advance
  - And, in many cases, the universe of possible key values is very large (e.g. names)
  - So it is not practical to reserve space for all possible key values

A Different Approach

- Don't determine the array size by the maximum possible number of keys
- Fix the array size based on the amount of data to be stored
  - Map the key value (phone number or name or some other data) to an array element
  - We still need to convert the key value to an integer index using a **hash function**
- This is the basic idea behind hash tables

Hash Tables

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Hash Tables

- A hash table consists of an *array* to store the data in
  - The table may contain complex types, or pointers to objects
  - One attribute of the object is designated as the table's key
- And a *hash function* that maps a key to an array index

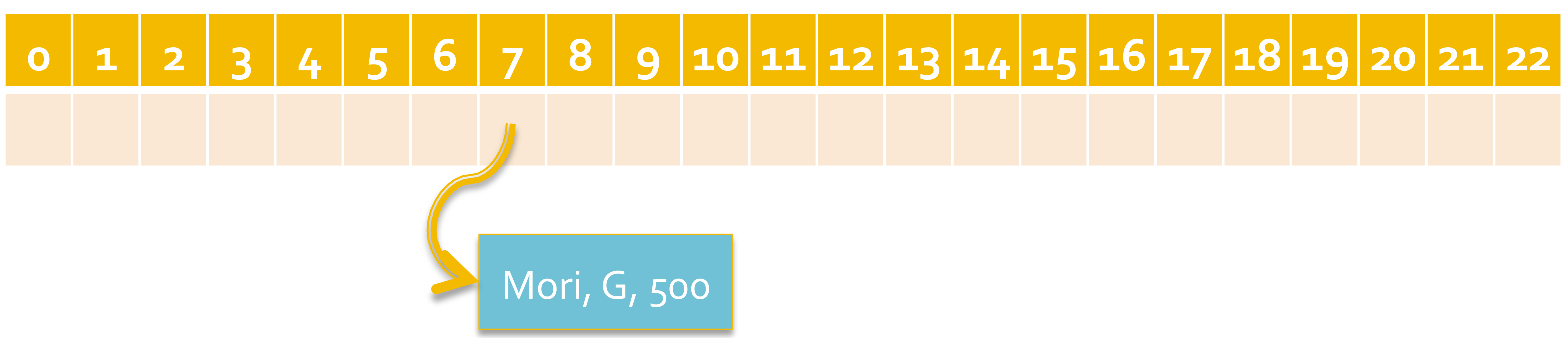
Hash Table Example

- Consider Customer data from A3
- Create array of pointers to Customer objects
  - This is the hash table
  - Customer *hash_table[H_SIZE];

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

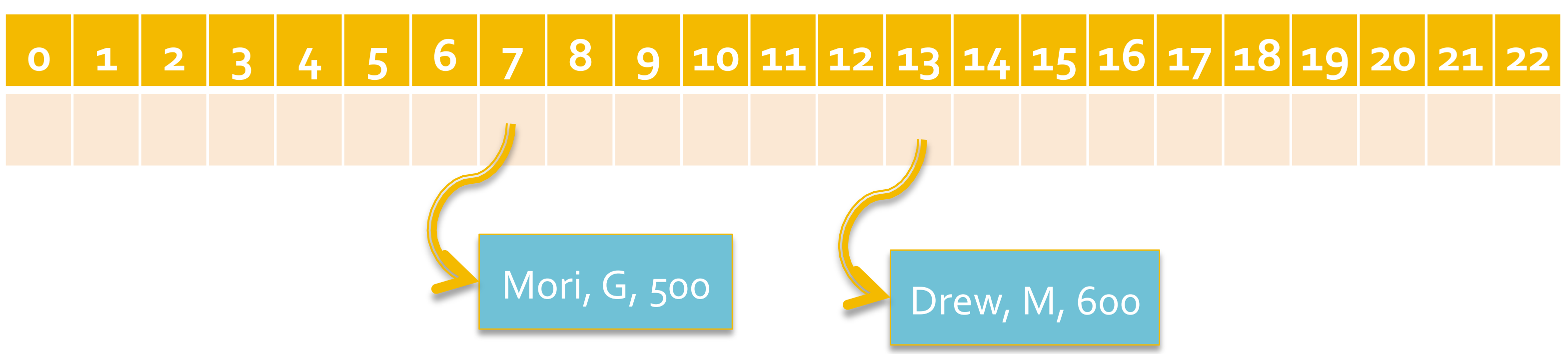
Hash Table Example

- Consider Customer data from A3
  - Say we wish to insert c = Customer (Mori, G.,500)
  - Where does it go?
  - Suppose we have a *hash function* h
    - h(c) = 7 (G is 7th letter in alphabet)



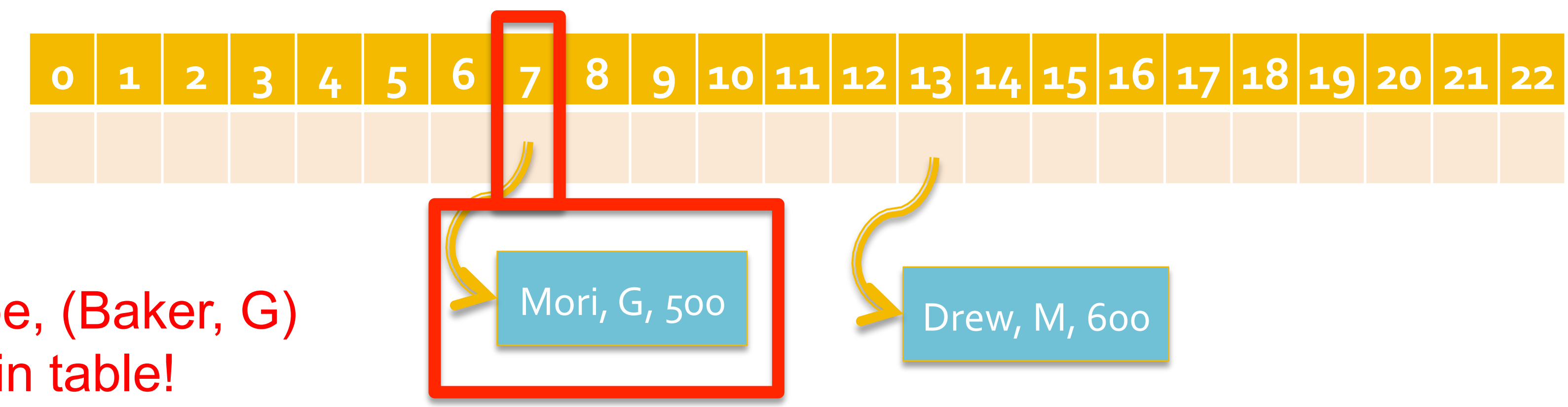
Hash Table Example

- Consider Customer data from A3
  - Say we wish to insert d = Customer (Drew, M.,600)
  - Where does it go?
    - h(d) = 13   (M is 13th letter in alphabet)



Hash Table Example

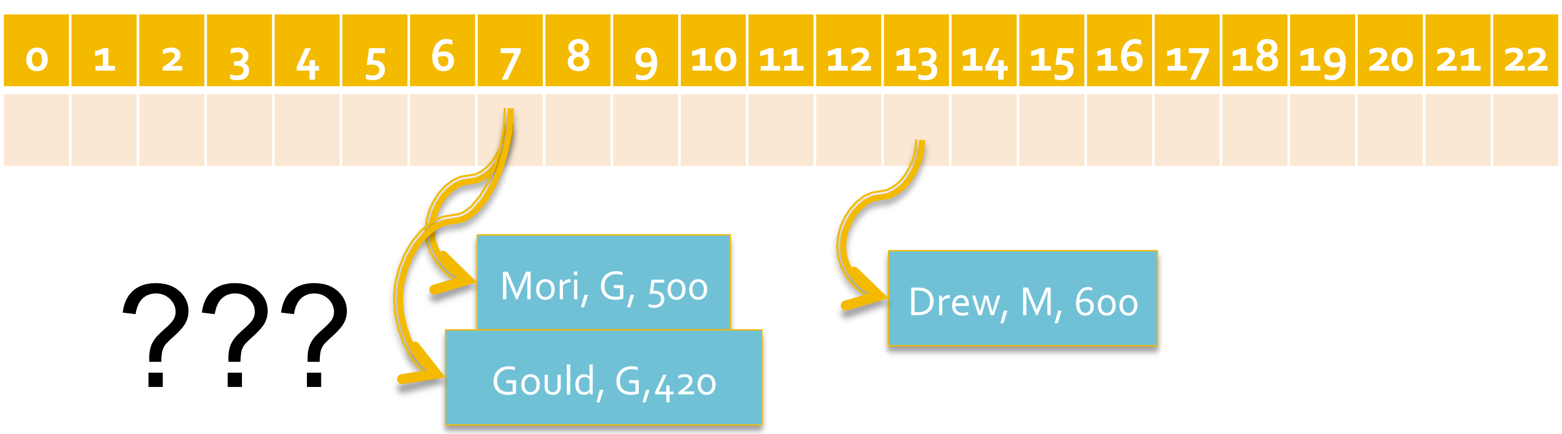
- Consider Customer data from A3
  - Say we wish to search for Customer c (Baker, G, 480)
  - Where could it be?
    - h(c) = 7   (G is 7th letter in alphabet)



Nope, (Baker, G) not in table!

Hash Table Example

- Consider Customer data from A3
  - Say we wish to insert e = Customer (Gould, G,420)
  - Where does it go?
    - h(e) = 7 (G is 7^{th} letter in alphabet)



Collisions

- A hash function may map two different keys to the same index
  - Referred to as a collision
  - Consider mapping phone numbers to an array of size 1,000 where h = phone mod 1,000
    - Both 604-555-1987 and 512-555-7987 map to the same index (6,045,551,987 mod 1,000 = 987)
- A good hash function can significantly reduce the number of collisions
- It is still necessary to have a policy to deal with any collisions that may occur

Hash Functions

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Hash Functions and Modulo

- A simple and effective hash function is:
  - Convert the key value to an integer, x
  - $h(x) = x \bmod tableSize$
- We want the keys to be distributed evenly over the underlying array
  - This can usually be achieved by choosing a prime number as the table size

Converting Strings to Integers

- A simple method of converting a string to an integer is to:
  - Assign the values 1 to 26 to each letter
  - Concatenate the binary values for each letter
    - Similar to the method previously discussed
- Using the string "cat" as an example:
  - c = 3 = 00011, a = 00001, t = 20 = 10100
  - So "cat" = 000110000110100 (or 3,124)
  - Note that $32^2 * 3 + 32^1 * 1 + 20 = 3{,}124$

Strings to Integers

- If each letter of a string is represented as a 32 bit number then for a length *n* string
  - value = $ch_0 * 32^{n-1} + \ldots + ch_{n-2} * 32^1 + ch_{n-1} * 32^0$
  - For large strings, this value will be very large
    - And may result in overflow
- This expression can be *factored*
  - $(\ldots(ch_0 * 32 + ch_1) * 32 + ch_2) * \ldots) * 32 + ch_{n-1}$
  - This technique is called *Horner's Rule*
  - This minimizes the number of arithmetic operations
  - Overflow can be prevented by applying the mod operator after each expression in parentheses

Hash Functions

- Should be fast and easy to calculate
  - Access to a hash table should be nearly instantaneous and in constant time
  - Most common hash functions require a single division on the representation of the key
  - Converting the key to a number should also be able to be performed quickly
- Should scatter data evenly through the hash table

Scattering Data

- A typical hash function usually results in some collisions
  - A *perfect* hash function avoids collisions entirely
    - Each search key value maps to a different index
    - Only possible when all of the search key values actually stored in the table are known
- The goal is to reduce the number and effect of collisions
- To achieve this the data should be distributed evenly over the table

Random Data

- Assume that every search key is equally likely (i.e. uniform distribution, random)
- A good hash function should scatter the search keys evenly
  - There should be an equal probability of an item being hashed to each location
  - For example, consider hashing 9 digit SFU ID numbers (x) on h = (last 2 digits of x) *mod* 40
  - Some of the 40 table locations are mapped to by 3 prefixes, others by only 2
  - A better hash function would be $h = x$ *mod* 101

Non Random Data

- Evenly scattering non random data can be more difficult than scattering random data
  - As an example of non random data consider a key: {*last name*, *first name*}
  - Some first and last names occur much more frequently than others
- While this is a complex subject there are two general principles
  - Use the entire search key in the hash function
  - If the hash function uses modulo arithmetic, the base should be prime

Collisions

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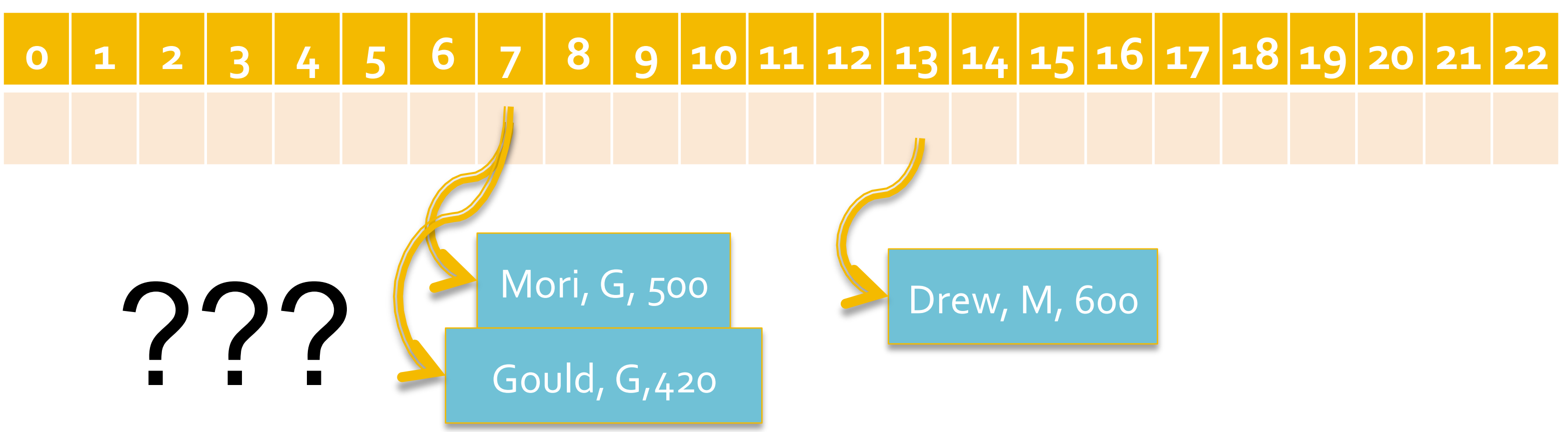
Dealing with Collisions

- A collision occurs when two different keys are mapped to the same index
  - Collisions may occur even when the hash function is good
- There are two main ways of dealing with collisions
  - Open addressing
  - Separate chaining

Hash Table Example

- Consider Customer data from A3
  - Say we wish to insert e = Customer (Gould, G,420)
  - Where does it go?
    - h(e) = 7   (G is 7th letter in alphabet)



Open Addressing

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Open Addressing

- Idea – when an insertion results in a collision look for an empty array element
  - Start at the index to which the hash function mapped the inserted item
  - Look for a free space in the array following a particular search pattern, known as *probing*
- There are three open addressing schemes
  - Linear probing
  - Quadratic probing
  - Double hashing

Open Addressing I – Linear Probing

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Linear Probing

- The hash table is searched sequentially
  - Starting with the original hash location
  - Search *h*(*search key*) + 1, then *h*(*search key*) + 2, and so on until an available location is found
  - If the sequence of probes reaches the last element of the array, wrap around to *arr*[0]

Linear Probing Example

- Hash table is size 23
- The hash function, *h* = *x mod* 23, where *x* is the search key value
- The search key values are shown in the table

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58									21	

Linear Probing Example

- Insert 81, h = 81 *mod* 23 = 12
- Which collides with 58 so use linear probing to find a free space
- First look at 12 + 1, which is free so insert the item at index 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81								21	

Linear Probing Example

- Insert 35, $h = 35 \ mod \ 23 = 12$
- Which collides with 58 so use linear probing to find a free space
- First look at 12 + 1, which is occupied so look at 12 + 2 and insert the item at index 14

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35							21	

Linear Probing Example

- Insert 60, $h = 60 \ mod \ 23 = 14$
- Note that even though the key doesn't hash to 12 it still collides with an item that did
- First look at 14 + 1, which is free

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35	60						21	

Linear Probing Example

- Insert 12, $h = 12 \ mod \ 23 = 12$
- The item will be inserted at index 16
- Notice that "primary clustering" is beginning to develop, making insertions less efficient

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35	60	12					21	

Searching

- Searching for an item is similar to insertion
- Find 59, $h = 59 \; mod \; 23 = 13$, index 13 does not contain 59, but is occupied
- Use linear probing to find 59 or an empty space
- Conclude that 59 is not in the table

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35	60	12					21	

Linear Probing

- The hash table is searched sequentially
  - Starting with the original hash location
  - Search *h*(*search key*) + 1, then *h*(*search key*) + 2, and so on until an available location is found
  - If the sequence of probes reaches the last element of the array, wrap around to *arr*[0]
- Linear probing leads to *primary clustering*
  - The table contains groups of consecutively occupied locations
  - These clusters tend to get larger as time goes on
    - Reducing the efficiency of the hash table

Open Addressing II – Quadratic Probing

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Quadratic Probing

- Quadratic probing is a refinement of linear probing that prevents primary clustering
  - For each successive probe, i, add i^2 to the original location index
    - 1st probe: $h(x)+1^2$, 2nd: $h(x)+2^2$, 3rd: $h(x)+3^2$, etc.

Quadratic Probing Example

- Hash table is size 23
- The hash function, $h = x \; mod \; 23$, where x is the search key value
- The search key values are shown in the table

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58									21	

Quadratic Probing Example

- Insert 81, $h = 81 \ mod\ 23 = 12$
- Which collides with 58 so use quadratic probing to find a free space
- First look at $12 + 1^2$, which is free so insert the item at index 13

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81								21	

Quadratic Probing Example

- Insert 35, h = 35 *mod* 23 = 12
- Which collides with 58
- First look at 12 + 1^2, which is occupied, then look at 12 + 2^2 = 16 and insert the item there

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81			35					21	

Quadratic Probing Example

- Insert 60, $h = 60 \ mod \ 23 = 14$
- The location is free, so insert the item

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	60		35					21	

Quadratic Probing Example

- Insert 12, $h = 12 \ mod \ 23 = 12$
- First check index $12 + 1^2$,
- Then $12 + 2^2 = 16$,
- Then $12 + 3^2 = 21$ (which is also occupied),
- Then $12 + 4^2 = 28$, wraps to index 5 which is free

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
					12	29			32			58	81	60		35					21	

Quadratic Probe Chains

- Note that after some time a sequence of probes repeats itself
  - e.g. 12, 13, 16, 21, 28(5), 37(14), 48(2), 61(15), 76(7), 93(1), 112(20), 133(18), 156(18), 181(20)
- This generally does not cause problems if
  - The data are not significantly skewed,
  - The hash table is large enough (around 2 * the number of items), and
  - The hash function scatters the data evenly across the table

Quadratic Probing

- Quadratic probing is a refinement of linear probing that prevents primary clustering
- Results in *secondary clustering*
  - The same sequence of probes is used when two different values hash to the same location
  - This delays the collision resolution for those values
- Analysis suggests that secondary clustering is not a significant problem

Open Addressing III – Double Hashing

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Double Hashing

- In both linear and quadratic probing the probe sequence is independent of the key
- Double hashing produces *key dependent* probe sequences
  - In this scheme a second hash function, h_2, determines the probe sequence
- The second hash function must follow these guidelines
  - $h_2(key) \neq 0$
  - $h_2 \neq h_1$
  - A typical h_2 is $p - (key \bmod p)$ where p is prime

Double Hashing Example

- Hash table is size 23
- The hash function, $h = x \ mod \ 23$, where x is the search key value
- The second hash function, $h_2 = 5 - (key \ mod \ 5)$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58									21	

Double Hashing Example

- Insert 81, $h = 81 \; mod \; 23 = 12$
- Which collides with 58 so use h_2 to find the probe sequence value
- $h_2 = 5 - (81 \; mod \; 5) = 4$, so insert at $12 + 4 = 16$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58				81					21	

Double Hashing Example

- Insert 35, $h = 35 \; mod \; 23 = 12$
- Which collides with 58 so use h_2 to find a free space
- $h_2 = 5 - (35 \; mod \; 5) = 5$, so insert at 12 + 5 = 17

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58				81	35				21	

Double Hashing Example

- Insert 60, $h = 60 \mathit{mod}\ 23 = 14$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58		60		81	35				21	

Double Hashing Example

- Insert 83, h = 83 *mod* 23 = 14
- h_2 = 5 – (83 *mod* 5) = 2, so insert at 14 + 2 = 16, which is occupied
- The second probe increments the insertion point by 2 again, so insert at 16 + 2 = 18

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58		60		81	35	83			21	

Deletions and Open Addressing

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Deletion Example

- Linear probing, $h(x) = x \mod 23$
- Suppose I want to delete 60
- Any problems?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35		12					21	

Deletions and Open Addressing

- Deletions add complexity to hash tables
  - It is easy to find and delete a particular item
  - But what happens when you want to search for some other item?
  - The recently empty space may make a probe sequence terminate prematurely
- One solution is to mark a table location as either empty, occupied or deleted
  - Locations in the deleted state can be re-used as items are inserted

Deletion Example

- Linear probing, *h(x)* = x *mod* 23
- Suppose I want to delete 60

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35	x	12					21	

Deletion Example

- Linear probing, $h(x) = x \bmod 23$
- Search for 12

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35	x	12					21	

Deletion Example

- Linear probing, *h(x)* = x *mod* 23
- Insert 15

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
						29			32			58	81	35	15	12					21	

Separate Chaining

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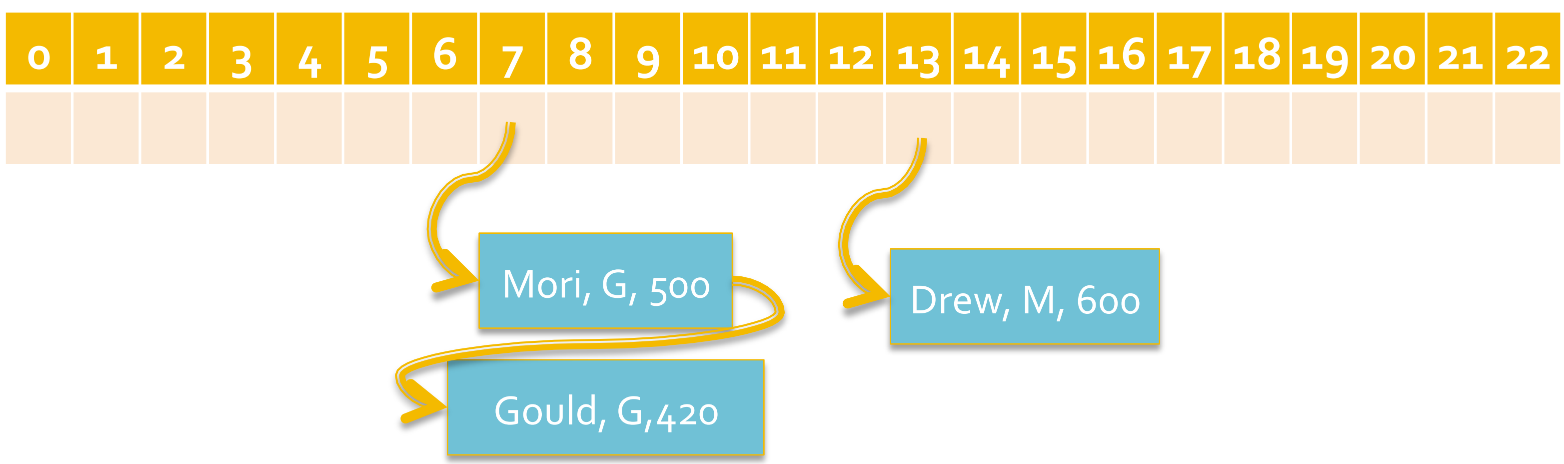
Separate Chaining

- Separate chaining takes a different approach to collisions
- Each entry in the hash table is a pointer to a linked list
  - If a collision occurs the new item is added to the end of the list at the appropriate location
- Performance degrades less rapidly using separate chaining

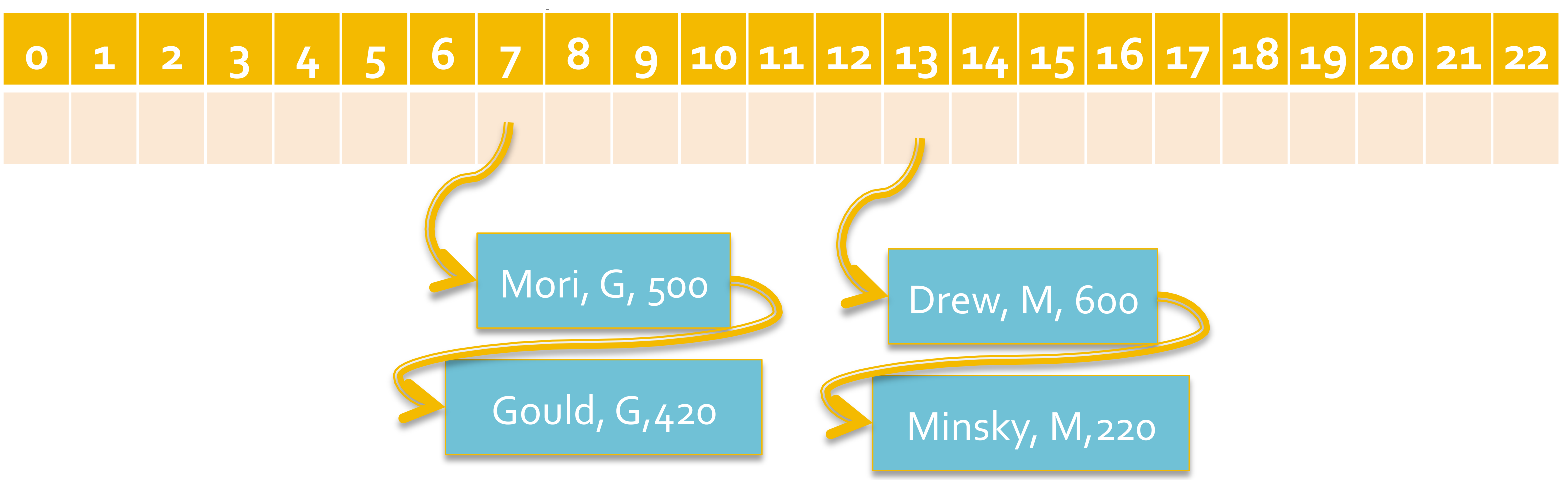
Separate Chaining Example

- Consider Customer data from A3
  - Say we wish to insert e = Customer (Gould, G,420)
  - Where does it go?
    - h(e) = 7   (G is 7th letter in alphabet)



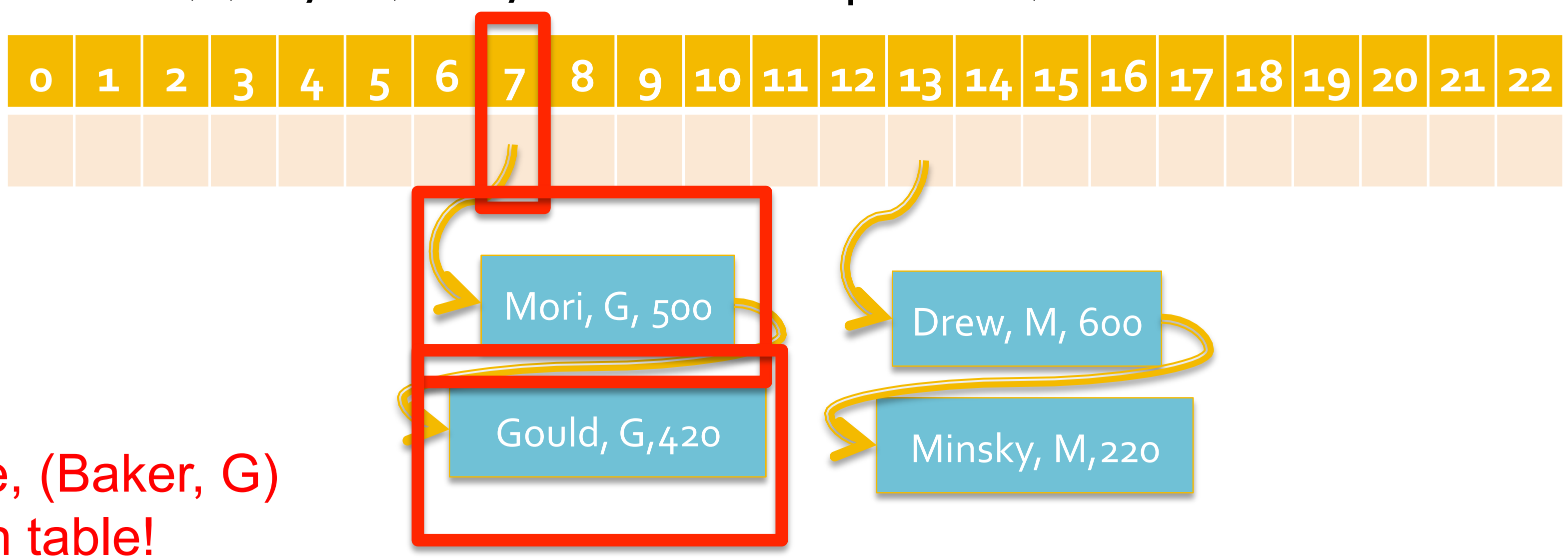
Separate Chaining Example

- Consider Customer data from A3
  - Say we wish to insert e = Customer (Minsky, M, 220)
  - Where does it go?



Separate Chaining Example

- Consider Customer data from A3
  - Say we wish to find e = Customer (Baker, G)
  - Where could it be?
    - h(e) = 7 (G is 7^{th} letter in alphabet)



Nope, (Baker, G)
not in table!

Efficiency

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Hash Table Efficiency

- When analyzing the efficiency of hashing it is necessary to consider load factor, α
  - α = *number of items / table size*
  - As the table fills, α increases, and the chance of a collision occurring also increases
  - So performance decreases as α increases
  - Unsuccessful searches require more comparisons than successful searches
- It is important to base the table size on the largest possible number of items
  - The table size should be selected so that α does not exceed 2/3

Average Comparisons

- Linear probing
  - When $\alpha = 2/3$ unsuccessful searches require 5 comparisons, and
  - Successful searches require 2 comparisons
- Quadratic probing and double hashing
  - When $\alpha = 2/3$ unsuccessful searches require 3 comparisons
  - Successful searches require 2 comparisons
- Separate chaining
  - The lists have to be traversed until the target is found
  - α comparisons for an unsuccessful search
  - $1 + \alpha / 2$ comparisons for a successful search

Hash Table Discussion

- If α is less than 0.5 open addressing and separate chaining give similar performance
  - As α increases, separate chaining performs better than open addressing
  - However, separate chaining increases storage overhead for the linked list pointers
- It is important to note that in the worst case hash table performance can be poor
  - That is, if the hash function does not evenly distribute data across the table

Summary

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Summary

- Hash tables
  - Store data in array
  - Position in array determined by hash function
- Hash functions can map different items to same position (collision)
  - Resolve via linear/quadratic probing, double hashing, or open chaining
- Performance of hash table can be very fast (constant time)
  - Actual performance depends on load factor and hash function

Objectives

- Understand the basic structure of a hash table and its associated hash function
  - Understand what makes a good (and a bad) hash function
- Understand how to deal with collisions
  - Open addressing
  - Separate chaining
- Be able to implement a hash table
- Understand how occupancy affects the efficiency of hash tables

Readings

- Carrano: Ch. 12