1. Let \( \phi \) be the propositional formula
\[
((q \lor (r \land s)) \land p) \rightarrow ((p \lor q) \land (\neg r \lor s)).
\]
(a) Give two truth assignments \( \alpha, \beta \) for the atoms of \( \phi \), such that \( \alpha \models \phi \) and \( \beta \not\models \phi \). (Hint: Drawing the formula tree might help with this. Recall that \((A \rightarrow B)\) is an abbreviation for \((\neg A \lor B)\).)
(b) Draw the formula tree for \( \phi \), in the manner of Example 3 in the Propositional Logic section of the course notes. Label each node of the tree with either T or F, to indicate whether the corresponding sub-formula is satisfied by \( \beta \) or not.

2. Write a careful, detailed, semantic proof of each of the following properties:
(a) For every two propositional formulas \( A, B \),
\[ A \models B \text{ if and only if } (A \rightarrow B) \text{ is a tautology.} \]
(b) For every two propositional formulas \( A, B \),
\[ \neg(A \land B) \models (\neg A \lor \neg B) \]
(c) For all propositional formulas \( A, B \) and every set of propositional formulas \( \Gamma \),
\[ \text{If } \Gamma \models A \text{ and } \Gamma \cup \{A\} \models B, \text{ then } \Gamma \models B. \]

3. A homomorphism from a graph \( G \) to a graph \( H \) is a map from the vertices of \( G \) to the vertices of \( H \) that preserves edges. That is, if \((u, v)\) is an edge of \( G \), and \( u \) maps to vertex \( w \) of \( H \), and \( v \) maps to vertex \( x \) of \( H \), then \((w, x)\) must be an edge of \( H \).
(a) Write a problem definition, in the style used at the end of Section 2.1 (e.g., Example 1) of the Introduction section of the course notes, clearly giving the vocabularies involved. For this part, you do not need to give \( \phi \).
(b) Write the properties that formula \( \phi \) specifies, in terms of the vocabulary symbols you chose. The precise notation you use is not critical, but getting as close as you can to ESSENCE or FO (or some combination) based on what you know so far is preferred.