Analytical Model of Sparse-Partial Wavelength Conversion in Wavelength-Routed WDM Networks

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***Abstract*— Wavelength conversion is one of the key techniques to improve the blocking performance in wavelength-routed WDM networks. Given that wavelength converters nowadays remain very expensive, how to make effective use of wavelength converters becomes an important issue. In this letter, we analyze the Sparse-Partial Wavelength Conversion network architecture and demonstrate that it can significantly save the number of wavelength converters, yet achieving excellent blocking performance. Theoretical and simulation results indicate that, the performance of a wavelength-routed WDM network with only 1-5% of wavelength conversion capability is very close to that with Full-Complete Wavelength Conversion capability.**

***Index Terms*— Wavelength Division Multiplexing.**

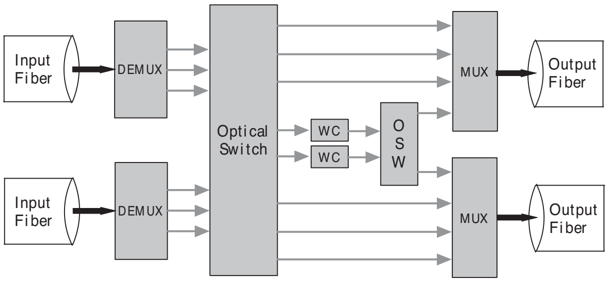

Fig. 1. A WCR with partial wavelength conversion.

I. INTRODUCTION

IN a wavelength-routed all-optical WDM network, two wavelength routers communicate with each other by setting up a lightpath in between. In a lightpath, same wavelengths are required to be allocated on all the fiber links, which is known as the wavelength continuity constraint. Due to the limited number of wavelength channels and the wavelength continuity constraint, some lightpath connection requests may not be satisfied, resulting in an undesired call blocking. To relax the continuity constraint and hence to reduce blocking, an effective approach is to use wavelength converters which can convert optical signals from one wavelength to another. A comprehensive introduction of wavelength conversion technologies can be found in [5]. In this letter, we only consider full-range wavelength conversion. A wavelength router equipped with wavelength converters is called a wavelength-convertible router (WCR). Because wavelength converters are very expensive, various WCR architectures have been proposed to save the cost.

In Complete Wavelength Conversion (CWC), each output port of the WCR is associated with a dedicated wavelength converter. The total converters required are thus the product of the number of fiber links and that of the wavelengths per link. Though being able to convert all the input wavelengths simultaneously, the cost of this ideal WCR can be prohibitively high, especially if all the routers in the network are WCRs (referred to as Full-Complete Wavelength Conversion, or FCWC). On the other hand, if only a part of the wavelength routers can do wavelength conversion, the network is called with Sparse Wavelength Conversion (SWC) [4]. The latter has received much attention recently, because it offers an effective and flexible solution for network carriers to upgrade their optical backbone gradually to support wavelength conversion. However, most of the previous works assume to use complete wavelength conversion, which is not practical and efficient.

Fig. 1 shows the architecture of a WCR with share-per-node Partial Wavelength Conversion (PWC) [3], where a pool of wavelength converters are shared by all the output ports. It requires much less number of wavelength converters, and it has been shown to be able to achieve very close performance compared with complete wavelength conversion [2] [3].

Recently, the Sparse-Partial Wavelength Conversion (SPWC) network architecture has been proposed aiming at combining the advantages of PWC and SWC [2]. In such networks, only part of wavelength routers are WCRs with PWC, while other wavelength routers have no wavelength conversion capability. This architecture has two important advantages: 1) it can significantly reduce the number of wavelength converters needed; 2) it is very flexible for the network carrier to migrate their network to support wavelength conversion, either by replacing the old wavelength routers with WCRs, or by adding more converters into WCRs.

In this letter, we describe an analytical model for calculating the overall blocking probability of an SPWC network and further reveal the effectiveness of SPWC.

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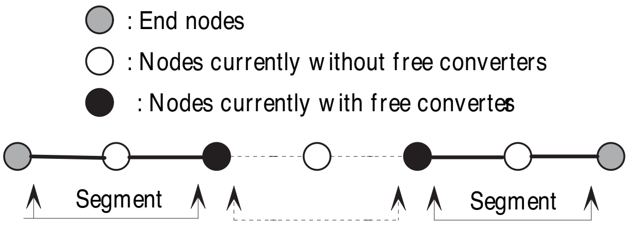

Fig. 2. A lightpath and its segments.

II. Network Model and Operations

We consider an arbitrary WDM network topology with N nodes and J links, respectively labeled from 1 to N and 1 to J. For the sake of simplicity, we assume bi-directional links, and each link can support W wavelengths in both directions. The fixed shortest path routing and random wavelength assignment RWA algorithm is adopted. The pre-determined shortest route between node pair a is denoted by R_a.

The SPWC network operates as follows: Upon arrival of a lightpath connection request, check the free wavelengths on each link along the pre-determined shortest path. If there exist common free wavelengths, choose one free wavelength randomly to set up the lightpath. Otherwise, we divide the route into several segments according to the WCRs that currently have free converters, as illustrated in Fig. 2. If any segment currently has no common free wavelength, the lightpath request has to be blocked; otherwise, use random wavelength assignment on each segment and allocate a wavelength converter if two consecutive segments choose different wavelengths.

III. Analytical Model

In this section, we propose an analytical model to calculate the overall blocking probability of a WDM network with SPWC architecture. We assume that lightpath requests for node pair a follows a Poisson process with rate A_a, and the connection holding times are exponentially distributed with a unit time.

The overall blocking probability B is defined as the ratio of the blocked traffic to offered traffic. That is,

$$B = \frac{\sum_a A_a B_{R_a}}{\sum_a A_a}. \quad (1)$$

To obtain the steady-state probability of the number of available wavelengths on each link, we use the reduced-load approximation method presented in [1]. Let random variable X_j represent the number of free wavelengths on link j, and $q_j(m_j)$ be the probability that m_j wavelengths are free on link j. We assume that X_j are independent, and the call requests arriving at link j follow a Poisson process with rate α_j. As such, the arriving and serving behavior on the link forms an $M/M/m/m$ system. Solving the Markov chain, we can have

$$q_j(m_j) = P(X_j = m_j) = \frac{\prod_{i=1}^{m_j}(W-i+1)}{\alpha_j^{m_j}} P(X_j = 0) \quad (2)$$

and

$$q_j(0) = P(X_j = 0) = [1 + \sum_{m_j=1}^{W} \frac{\prod_{i=1}^{m_j}(W-i+1)}{\alpha_j^{m_j}}]^{-1}. \quad (3)$$

Consider the traffic on link j, we can determine α_j by

$$\alpha_j(1-q_j(0)) = \sum_{\alpha,\ where\ link\ j\ belongs\ to\ R_a} A_a(1-B_{R_a}). \quad (4)$$

Suppose the number of WCRs in route R_a is D, excluding the two end nodes. Since a WCR has only a limited number of converters, it can have two states: 1) no free converter available; and 2) one or more free converters available. Hence, there are 2^D different conversion states for route R_a. For each conversion state X, given its number of WCRs with free converters, E_X $(0 \leq E_X \leq D)$, route R_a can be divided into $E_X + 1$ segments, represented by $S_0, S_1, \ldots, S_{E_X}$. Each segment should choose the same wavelength on its links. We introduce $U_{S_k}(i)$ to represent the probability that i wavelengths are common free on segment S_k. A lightpath will be setup on the route successfully if and only if each segment has at least one common free wavelength. Let $B_{R_a,X}$ denote the route blocking probability for conversion state X, we have:

$$B_{R_a,X} = 1 - \prod_{k=0}^{E_X}[1 - U_{S_k}(0)]. \quad (5)$$

The route blocking probability is thus given by

$$B_{R_a} = \sum_X [B_{R_a,X} P(X)] \quad (6)$$

where $P(X)$ is the probability of conversion state X. Consequently, the key of calculating B_{R_a} is how to derive $U_{S_k}(i)$ and $P(X)$.

Let us first show how to calculate $U_S(i)$. If S contains just one single link j, then $U_S(i)$ simply equals $q_j(i)$. If S is a two-hop segment composed by link j_1 and j_2, then the probability that j_1 has x free wavelengths is $q_{j_1}(x)$, and the probability that j_2 has y free wavelengths is $q_{j_2}(y)$. Given $q_{j_1}(x)$ and $q_{j_2}(y)$, the probability that there exist i common free wavelengths is $p(i|x,y)$, which can be calculated as :

$$p(i|x,y) = \begin{cases} \beta(x,y,i) & i \leq min(x,y);\ x+y-i \leq W; \\ & 1 \leq x,y \leq W \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

where

$$\beta(x,y,i) = \frac{\binom{y}{i}\binom{W-y}{x-i}}{\binom{W}{x}}. \quad (8)$$

As such, $U_S(i)$ for a two-hop segment can be derived as:

$$U_S(i) = \sum_{x=0}^{W}\sum_{y=0}^{W}[p(i|x,y) q_{j_1}(x) q_{j_2}(y)]. \quad (9)$$

The above analysis can be extended to determine $U_S(i)$ where the hop length of segment S, say h, is more than 2. Suppose the link set of segment S is $j_1, j_2, \ldots, j_h$. We use S' to

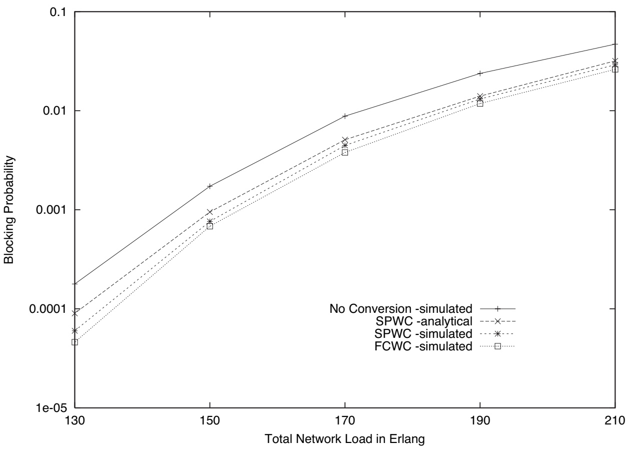

Fig. 3. Blocking Performance in NSFNET.

represent its sub-segment composed by links $j_1, j_2, \ldots, j_{h-1}$. By regarding segment S as the composition of sub-segment S' and link j_h, we can obtain $U_S(i)$ using the following recursive relation:

$$U_S(i) = \sum_{x=0}^{W} \sum_{y=0}^{W} [p(i|x,y) U_{S'}(x) q_{j_h}(y)]. \tag{10}$$

Now let us see how to calculate $P(X)$. Assume p_n is the probability that the nth WCR in route R_a has no free wavelength converter, and define

$$Y(n) = \begin{cases} 1 - p_n & \text{the } n\text{th WCR has free converters} \\ p_n & \text{the } n\text{th WCR has no free converter} \end{cases}$$

It follows that:

$$P(X) = \prod_{n=1}^{D} Y(n). \tag{11}$$

Hence the only unknown variable is p_n. Note that, for any accepted lightpath request bypassing the nth WCR, either (1) a common free wavelength exists on all links along that route; or (2) there is no common free wavelength, but each segment has its own common free wavelength. Only in situation (2) should wavelength converters be allocated in the WCRs. We refer to the lightpaths which cause wavelength conversion as "conversion traffic," and let T_n denote the total "conversion traffic" bypassing the nth WCR. According to our wavelength assignment algorithm described in Section II, an accepted lightpath uses wavelength conversion if and only if there is no common free wavelength on all links; only the lightpaths which use wavelength conversion are considered as conversion traffic. In addition, the probability that there is no common free wavelength on all links of R_a is $U_{R_a}(0)$, if we consider route R_a as a single segment. Thus T_n can be calculated as:

$$T_n = \sum_{\{R_a | routes\ bypassing\ the\ nth\ WCR\}} [A_a(1 - B_{R_a}) U_{R_a}(0)]. \tag{12}$$

We approximately consider that the conversion traffic arrives to the nth WCR following a Poisson process with rate T_n. Hence, it forms an $M/M/Z_n/Z_n$ system, where Z_n is the number of converters inside the nth WCR; and p_n, the probability that the nth WCR has no free wavelength converter is thus:

$$p_n = [1 + \sum_{j=1}^{Z_n} \frac{\prod_{i=1}^{j}(Z_n - i + 1)}{T_n^j}]^{-1}. \tag{13}$$

The numerical algorithm used to solve the above fixed-point non-linear equations is as follows:

1) Initialize B_{R_a} to 0 for all routes, and $q_j(0)$ to 0 for all links.
2) Determine α_j using (4) for all links.
3) Determine $q_j(m_j)$ using (2) and (3) for all links.
4) Determine B_{R_a} for all routes using (5) - (13). If the new values of B_{R_a} are converged to old ones, the iteration is terminated and we can go to Step 5). Otherwise go to Step 2) for next iteration.
5) Finally, determine the overall blocking probability B using (1).

IV. NUMERICAL RESULTS

In this section, we examine the performance of the SPWC architecture through numerical simulations. As in many previous studies, we assume that the traffic is uniformly distributed among node pairs. The lightpath requests arrive according to a Poisson process and the holding time is exponentially distributed. We assume each fiber link can support 40 wavelengths. In our simulations, every single data is obtained by conducting 30 independent replications of the same simulation and then calculating the mean results. The confidence level of the simulations is 95% and the relative error is within 5%.

For a typical network topology, the 14-node NSFNET [2], the blocking probability as a function of traffic load is shown in Fig. 3. For FCWC architecture, we need a total number of 1,600 converters. For SPWC, we only install 50 converters in the whole network using the converter placement algorithm described in [2]. From the figure, we notice that the analytical model slightly overestimates the blocking probability. The main reason is that the link traffic is modeled by Poisson distribution, which is a conservative assumption. It is obvious that both FCWC and SPWC can decrease the blocking probability by a large margin; and the performance of SPWC is very close to that of FCWC, which shows the superiority of SPWC architecture.

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