



# Classical Surface Reconstruction



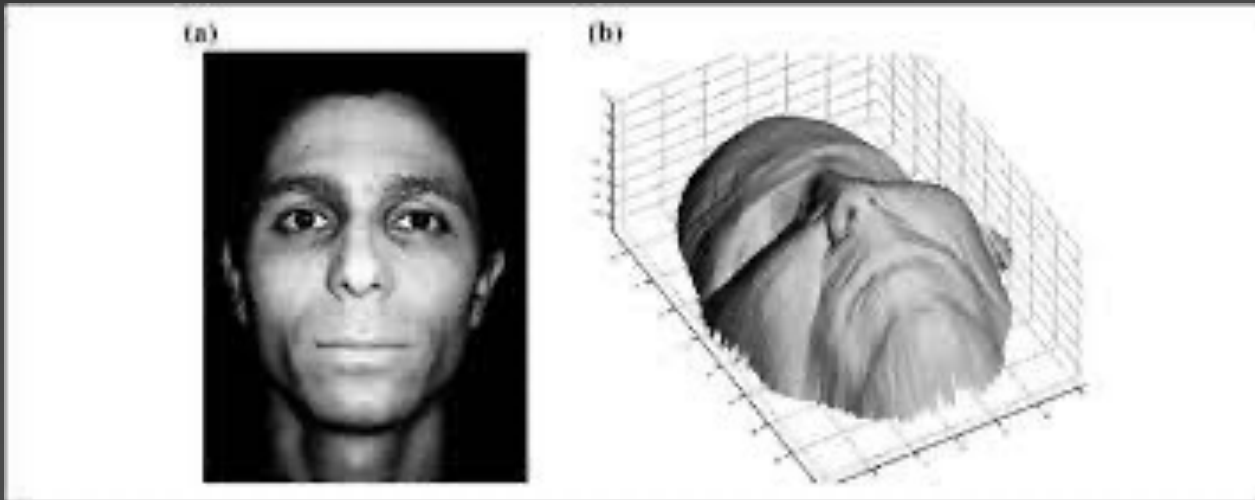
Richard Zhang

CMPT 464/764: Geometric Modeling in Computer Graphics

Lecture 8

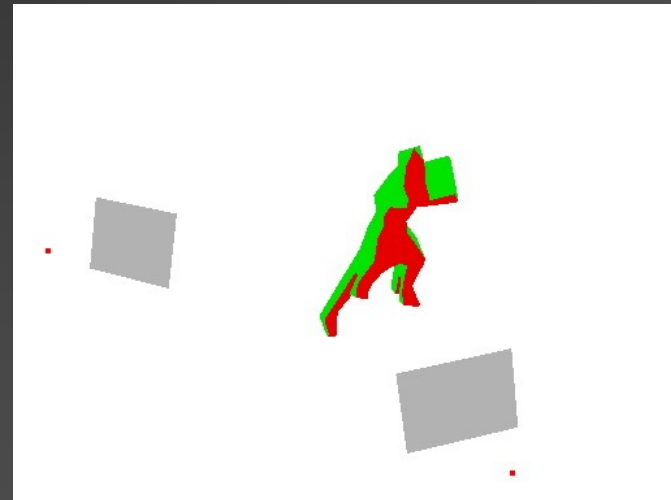
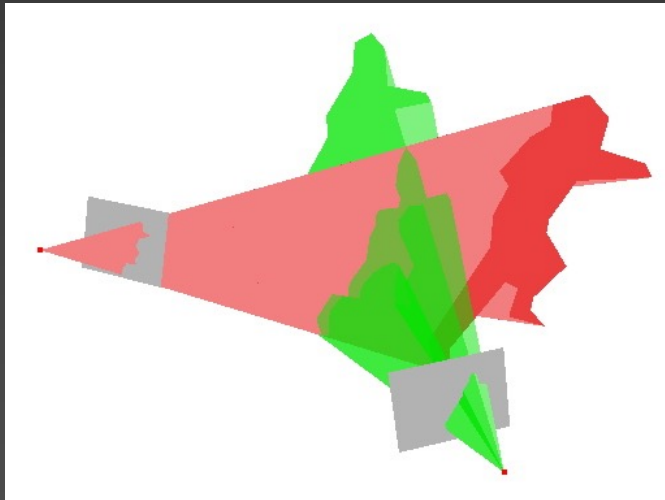
# Single-view input

- A prototypical computer vision problem: 3D geometry/surface reconstruction **from single or multiple view sensors (images)**
  - Classical “shape from shading” result



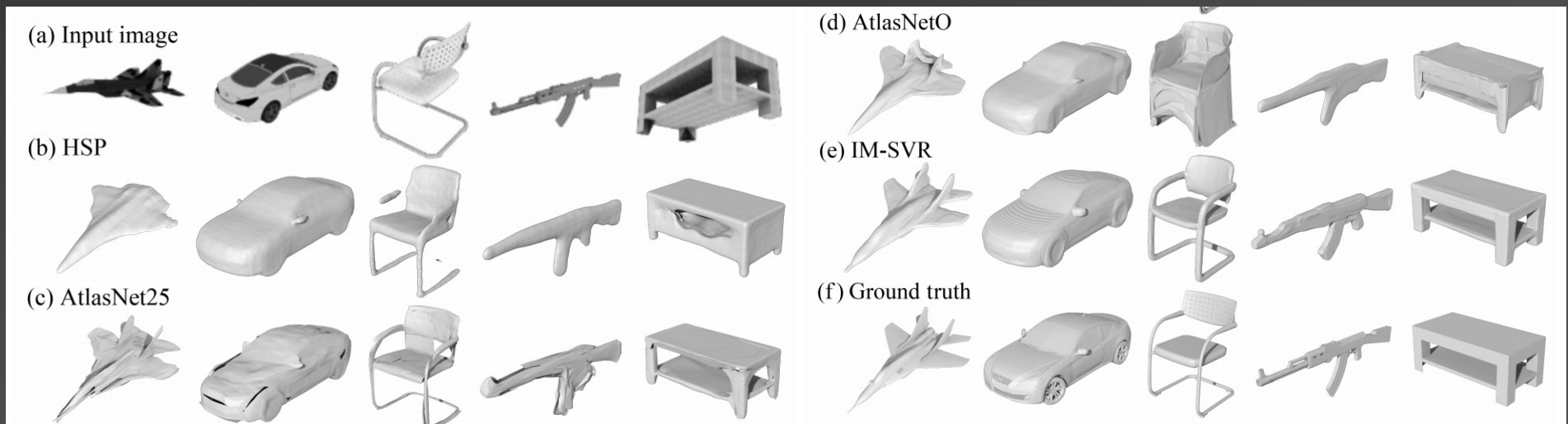
# Multi-view input

- A prototypical computer vision problem is 3D geometry/surface reconstruction from single or multiple view sensors (images)
  - **Visual hull:** shape from **multi-view silhouettes**



# Learning-based: single-view

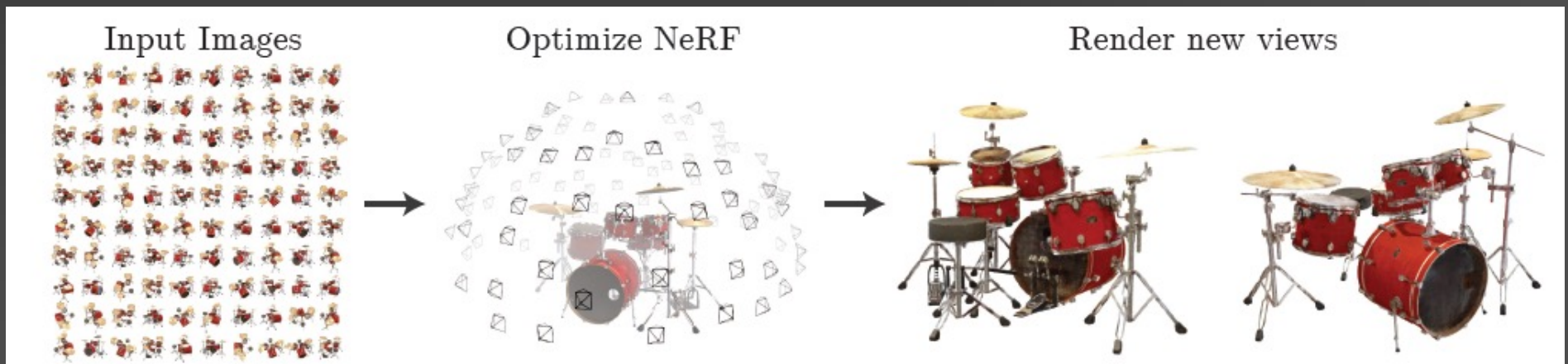
- A prototypical computer vision problem: 3D geometry/surface reconstruction from a single or multiple view sensors (images)
  - One of the most intensely studied problems in geometric deep learning



[IM-Net: Chen and Zhang 2019]

# Learning-based: multi-view

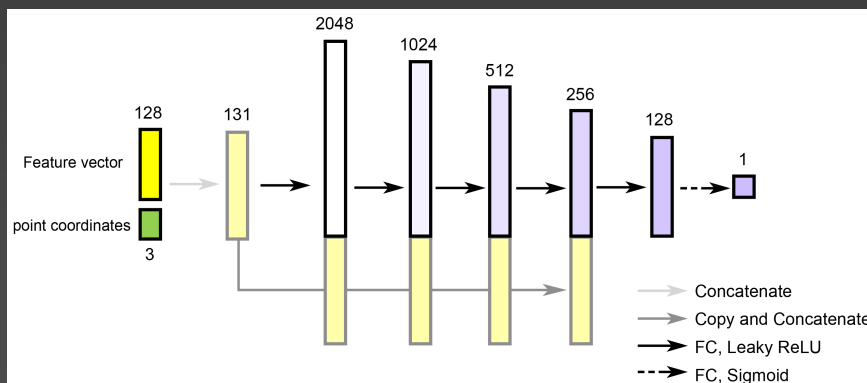
- A prototypical computer vision problem is 3D geometry/surface reconstruction from a single or multiple view sensors (images)
  - NeRF (2020): Neural Radiance Field, from multi-view images
  - **Novel view synthesis** (NVS): need **many images and long training**



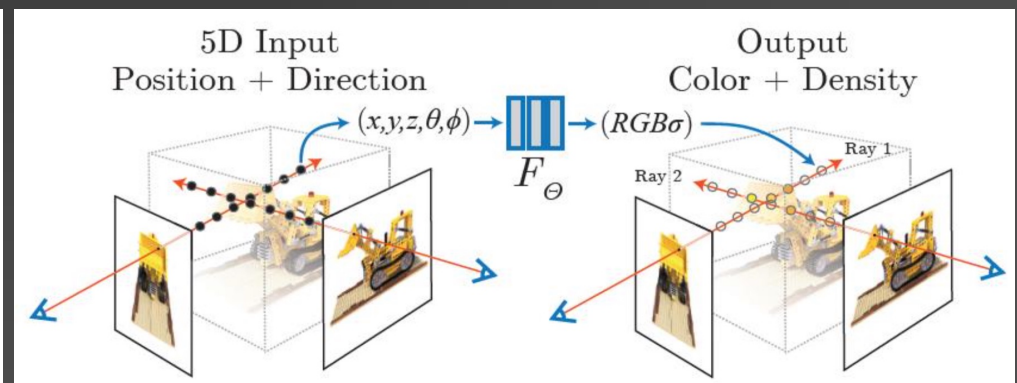
[Middenhall et al. 2020]

# Learning-based: multi-view

- A prototypical computer vision problem is 3D geometry/surface reconstruction from a single or multiple view sensors (images)
  - NeRF (2020): Neural Radiance Fields from multi-view images
  - **Connections between IM-Net and NeRF**




[IM-Net: Chen and Zhang 2019]



[Middenhall et al. 2020]

# See: Account from “NeRF Explosion”

Frank Dellaert   Publications   Teaching   Talks   Blog Posts



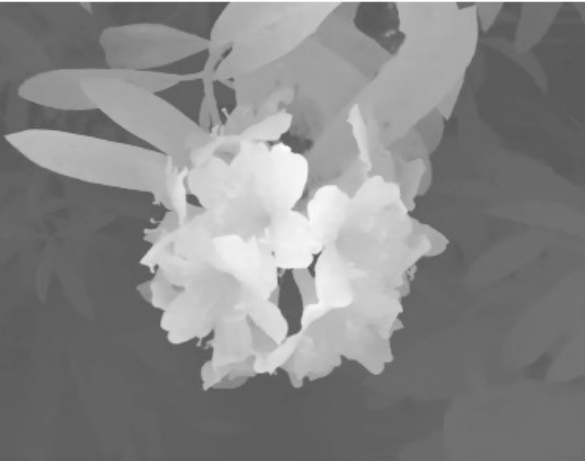

**Frank Dellaert**  
Professor, Robotics & Computer Vision

- Atlanta, GA
- Georgia Tech
- Email
- Twitter
- LinkedIn
- Github
- YouTube
- Google Scholar

## NeRF Explosion 2020

21 minute read

**Published:** December 16, 2020

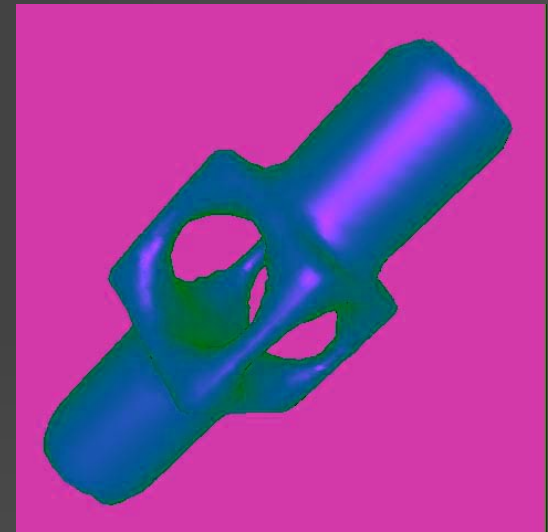
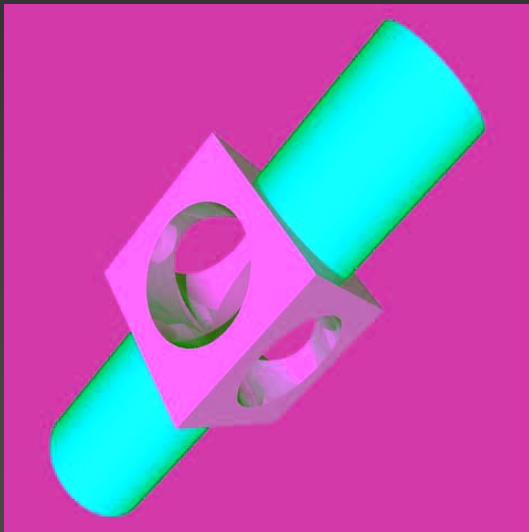


*The result that got me hooked on wanting to know everything about NeRF :-).*

<https://dellaert.github.io/NeRF/>

# This week: from a “graphics origin”

Given a set of **unorganized 3D points**  $X = \{\mathbf{x}_1, \dots, \mathbf{x}_n\}$  sampled from an unknown surface  $M$ , construct a surface  $M'$  that approximates  $M$ .



Surface reconstruction from **unorganized point cloud** data



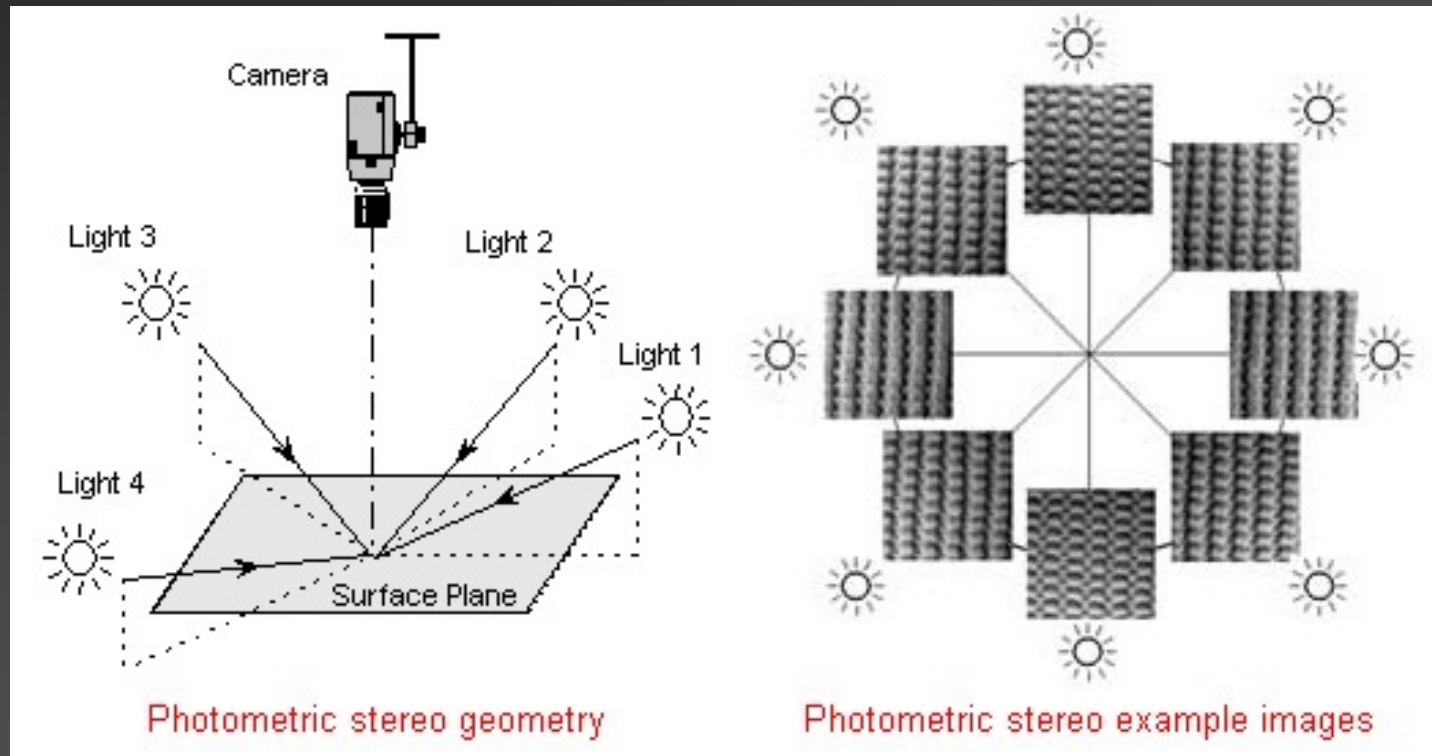
# Background

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- Input: point cloud obtained via laser scanning with **no normal information**
- Output: a triangle mesh
- Surface  $M'$  can either **interpolate** or **approximate**  $X$
- Solve a general problem: no structure or organization of points assumed ...
  - Here **structural information** refers to specific knowledge about the arrangement of the point samples, e.g., contours on **parallel slices in MRI**
  - Some info about the device specs can be known, e.g., **scanning accuracy**
  - Normal information may be available via **photometric stereo** [Woodham 80]

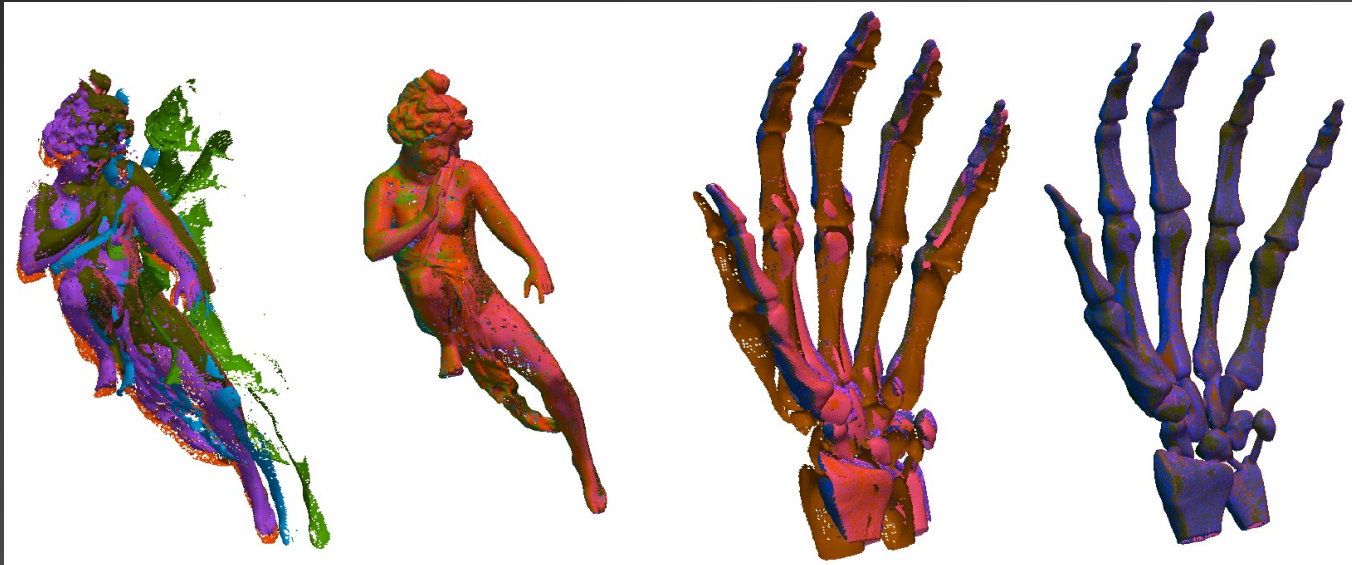
# Photometric stereo

- Estimate surface orientation from different images



# Many related problems

- (Static) surface registration: bring several partial scans to **alignment**



- Key: point or region **correspondence** - a topic we cover later

# Many related problems

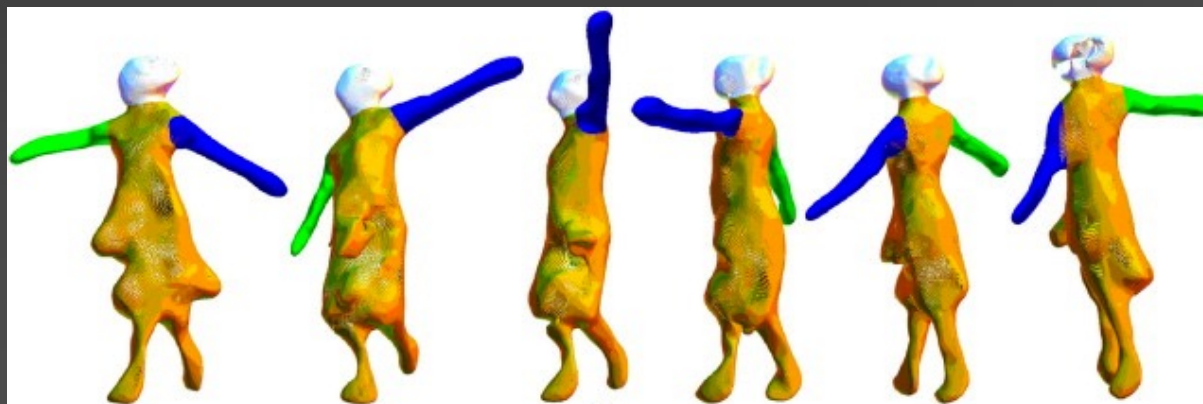
- **Multi-view** geometry reconstruction, e.g., Microsoft photosynth



- Sub-problems: **shape-from-shading**, e.g., photo to point clouds, and (multi-view) point cloud **registration**

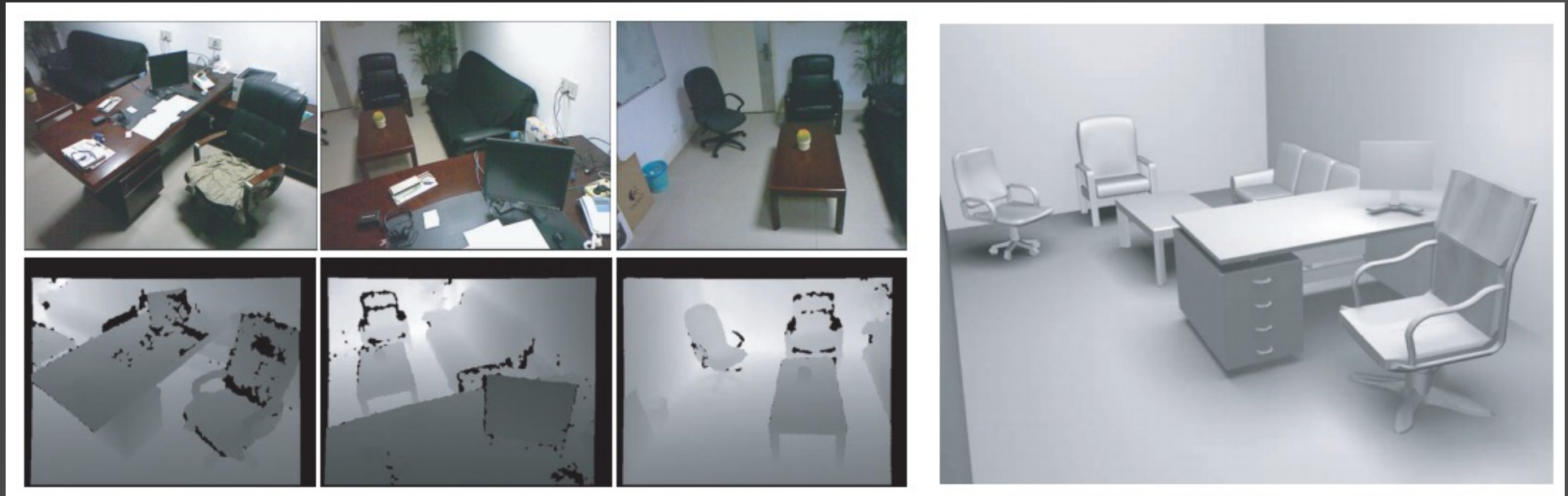
# Many related problems

- **Time-varying** surface **tracking**, e.g., for deformation or animation



# Problem scales

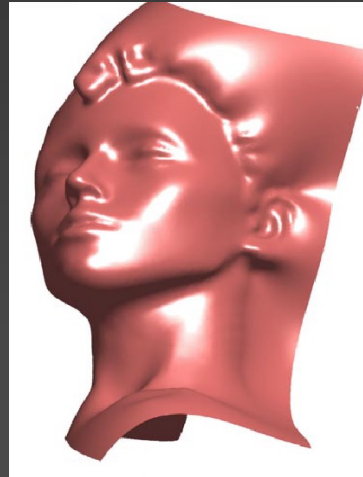
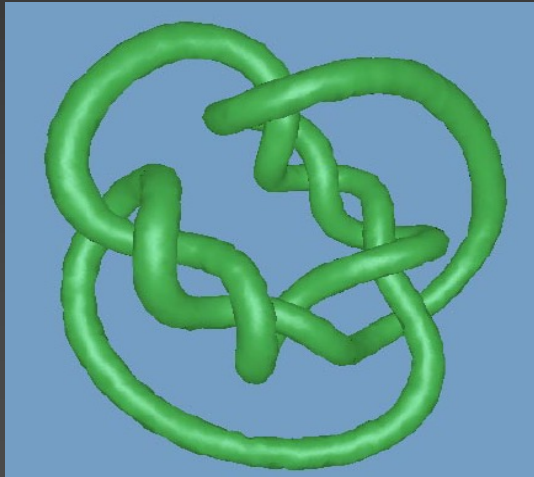
- From single objects (our focus) to scenes to buildings and cities!



Scaling up from **objects to scenes** [Shao et al. *Siggraph Asia* 2012]

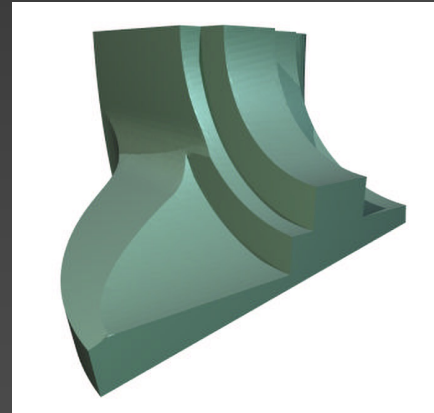
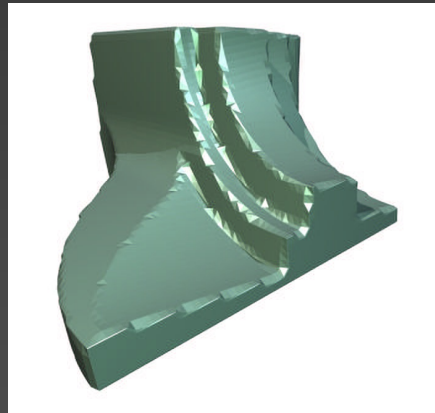
# Our problem: challenges

- Reconstruction should cover a range of shapes
  - Arbitrary topology, even if manifold, and arbitrary details
  - Shapes with **boundaries, holes, missing data**, etc.



# Challenges

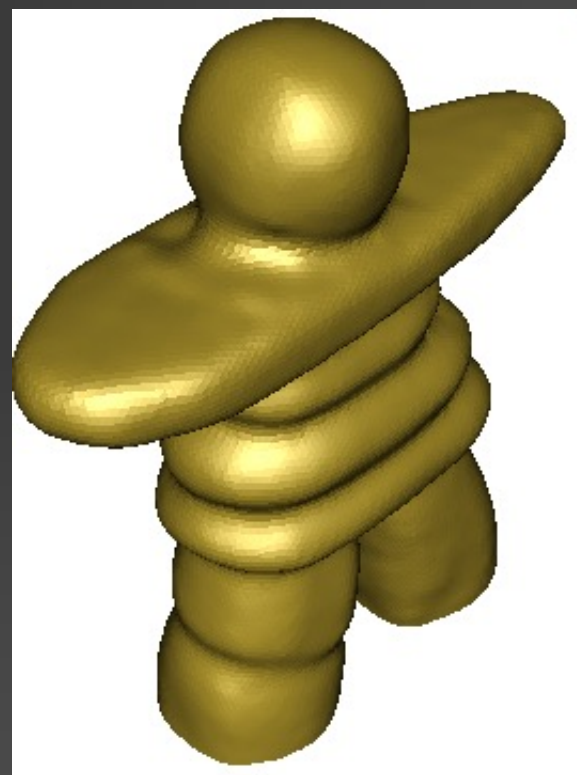
- Ensure consistent surface orientation
- Deal with **noise** in the data
- Recover **sharp features**: not easy if points are not on edges



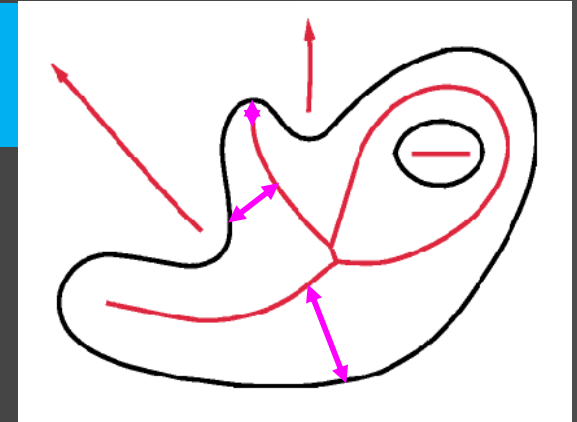
Feature-sensitive reconstruction [Kobbelt et al. 2001]



# Missing data and noise

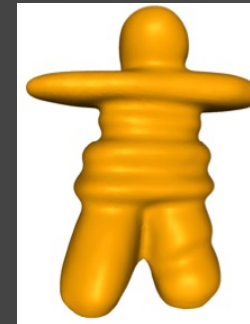
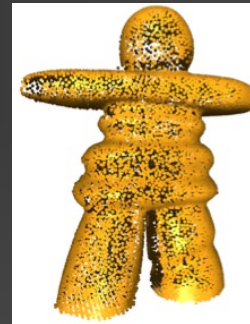


# Theoretical challenge (aside)



- Ensure “correctness” of reconstruction, meaning

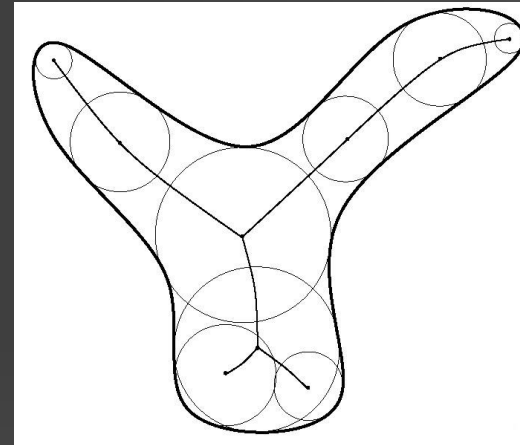
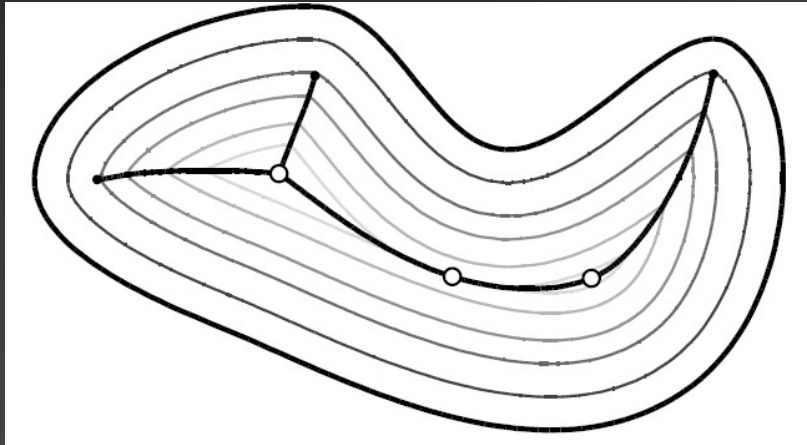
- **Topology** correctness
- **Geometry** precision: as sampling density increases, reconstruction approaches the original surface



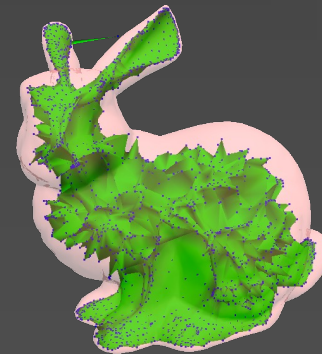
- Correctness guarantees possible if sampling is sufficiently “good”
  - not easy to achieve or define “goodness” [Amenta et al. 98]
  - Related to **local feature size: distance to medial axis**

# Medial axis (aside)

- **Singularities or meeting fronts** of a “grass-fire flow”



- Set of all points that have **at least two closest points** to the boundary
- Medial axes for 3D shapes have **sheets** rather than curves



# Classical main approaches

---

- Reconstruct **zero-set of a 3D scalar field**, e.g., via **marching cubes**
    - Use of tangent plane estimators – [Hoppe et al. 92]
    - Use of radial basis functions – [Carr et al. 01]
  - Utilizing **Voronoi diagrams or Delaunay Tetrahedralizations** – [Amenta et al. 01, Boissonnat 84, Dey & Goswami 03, Kolluri et al. 04]
    - Power crust algorithm – [Amenta et al. 01]
  - **Deform-to-fit** with energy minimization
    - e.g., inflating a balloon from inside the object – [Terzopoulos, Witkin, and Kass 88 & 91, Miller 91]
-

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# Our coverage



Hugues Hoppe

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TITLE	CITED BY	YEAR
<b>Progressive meshes</b> H Hoppe Proceedings of the 23rd annual conference on Computer graphics and ...	4785	1996
<b>Surface reconstruction from unorganized points</b> H Hoppe, T DeRose, T Duchamp, J McDonald, W Stuetzle Proceedings of the 19th annual conference on Computer graphics and ...	4035	1992

- H. Hoppe et al., “Surface Reconstruction from Unorganized Points.” *SIGGRAPH 92*
- W. Lorensen and H. Cline, “Marching Cubes: A High Resolution 3D Surface Construction Algorithm,” *SIGGRAPH 87*
  - A sub-algorithm of the surface reconstruction algorithm
  - One of most fundamental surface reconstruction algorithms itself
  - Input is volumeric data or scalar field of signed distances to surfacee
  - Algorithm constructs approximation of the **zero-set** of the scalar field

# Our coverage



Hugues Hoppe

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*SIGGRAPH 92*
- W. Lorensen and H. Cline, “Marching Cubes: A High Resolution 3D



Bill Lorensen

FOLLOW

GE Global Research (retired)

Verified email at nycap.rr.com - [Homepage](#)

TITLE	CITED BY	YEAR
<a href="#">Marching cubes: A high resolution 3D surface construction algorithm</a> WE Lorensen, HE Cline ACM siggraph computer graphics 21 (4), 163-169	16729	1987

<https://www.computer.org/csdl/magazine/cg/2020/02/09020249/1hS2S5b2V6E>

# Assumptions

## ■ ... on **data noise (measurement error)**

- The samples  $X = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n\}$  are  **$\delta$ -noisy**, i.e., each sample is no farther than  $\delta$  away from its true position
- Features of size less  $< \delta$  cannot be recovered reliably

## ■ ... on **sampling density**

- **$\rho$ -dense**: within each sphere centered at a point on surface  $M$  having radius  $\rho$ , at least one sample is drawn
- This assumption is necessary in order to distinguish between holes in surface (boundary) and holes in the sampling
- If there is an empty sphere with radius  $(\delta + \rho)$  embedded in the sampling, then it is a hole in the model



# Overview of Hoppe's approach

- Input: set  $X$  of unorganized 3D points ( $\delta$ -noisy;  $\rho$ -dense) sampled near surface  $M$
- Algorithm in two stages
  1. Obtain an **implicit function**  $f: D \rightarrow \mathbf{R}$ , where  $D \subseteq \mathbf{R}^3$ , is a **region near true surface**  $M$ , and  $f(\mathbf{p})$  estimates the **signed distance** from  $\mathbf{p}$  to  $M$
  2. The **zero-set**  $Z(f)$  of  $f$  is an estimate of  $M$ . A **contouring** or **marching-cube** algorithm approximates  $Z(f)$  by a triangle mesh
- Output: a connected, consistently oriented 2-manifold triangle mesh
- A general paradigm: implicit function  $f$  can be obtained in various ways, Hoppe paper uses a set of approximate **tangent planes**

# Learning of implicit/signed distance functions

- Generates surfaces with best visual quality so far

## Learning Implicit Fields for Generative Shape Modeling

Zhiqin Chen, Hao Zhang

*(Submitted on 6 Dec 2018 (v1), last revised 5 Apr 2019 (this version, v3))*

## Occupancy Networks: Learning 3D Reconstruction in Function Space

Lars Mescheder, Michael Oechsle, Michael Niemeyer, Sebastian Nowozin, Andreas Geiger

*(Submitted on 10 Dec 2018)*

## DeepSDF: Learning Continuous Signed Distance Functions for Shape Representation

Jeong Joon Park, Peter Florence, Julian Straub, Richard Newcombe, Steven Lovegrove

*(Submitted on 16 Jan 2019)*

## Deep Level Sets: Implicit Surface Representations for 3D Shape Inference

Mateusz Michalkiewicz, Jhony K. Pontes, Dominic Jack, Mahsa Baktashmotlagh, Anders Eriksson

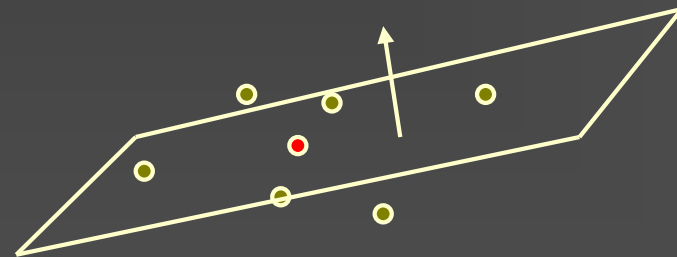
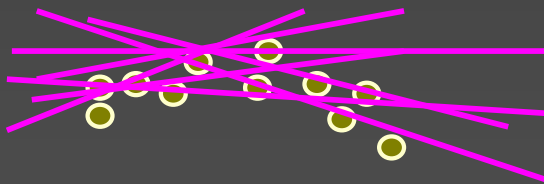
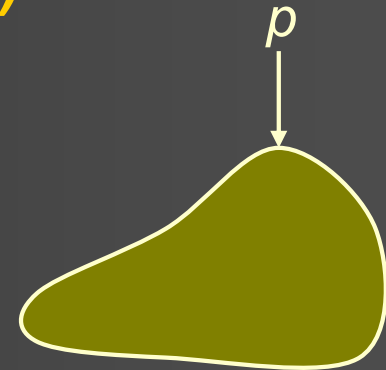
*(Submitted on 21 Jan 2019)*

Awesome implicit neural representations:

<https://github.com/vsitzmann/awesome-implicit-representations>

# Signed distance function (SDF)

- Distance from a point  $p$  to a surface  $M$  is the distance from  $p$  to a **closest point** on  $M$
- Sign depends on which side of  $M$  point  $p$  lies
- Since  $M$  is unknown, it is approximated by a set of **oriented tangent planes – one per data point**
- Tangent plane for  $x_i$  is defined by a center  $o_i$  and a unit normal  $n_i$

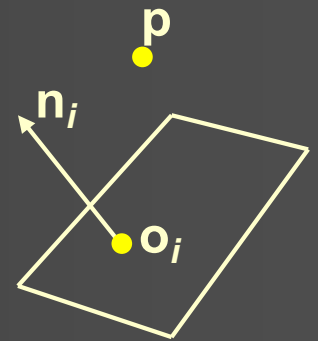


# Computing SDF, given tangent planes

- Determine **region  $D$  close to surface  $M$**
- If  $\mathbf{p} \in D$ , the signed distance from  $\mathbf{p}$  to  $M$  is a projection

$$f(\mathbf{p}) = (\mathbf{p} - \mathbf{o}_i) \cdot \mathbf{n}_i$$

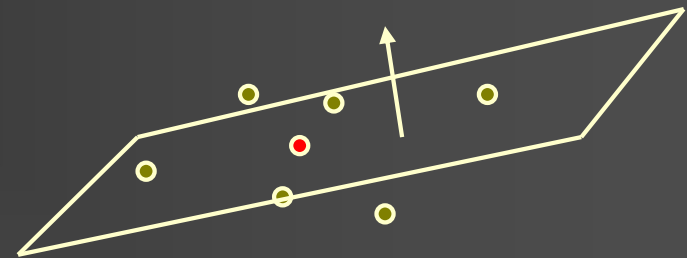
where  **$\mathbf{o}_i$  is the tangent plane center that is closest to  $\mathbf{p}$**



- If the shortest distance from a point  $\mathbf{p}$  to the point set  $X$  is  $> (\delta + \rho)$ , then  $\mathbf{p}$  cannot be on the surface  $M$ 
  - Otherwise, the sphere centered at  $\mathbf{p}$  with radius  $\delta + \rho$  must contain a point from  $X$ , since the samples are  $\delta$ -noisy and  $\rho$ -dense
  - $\mathbf{p}$  is possibly near a hole on the surface  $\rightarrow$   **$f(\mathbf{p})$  is undefined**
  - The remaining set of  $\mathbf{p}$  define  $D$

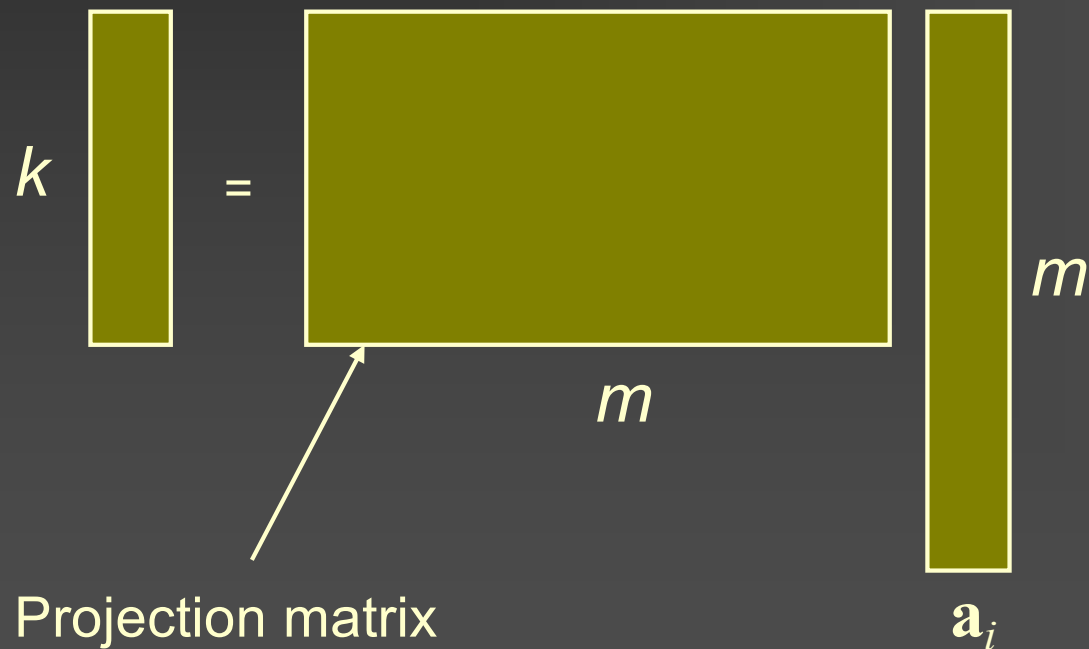
# Tangent plane estimation – key!

- How to define a tangent plane associated with a sample  $\mathbf{x}_i$ ?
- Define:  $Nbr(\mathbf{x}_i, k)$  = the set of  **$k$  nearest neighbors** ( $k$ NN) of a data point  $\mathbf{x}_i$ , where  $k$  is a user input value
- Center  $\mathbf{o}_i$  is the **centroid** of  $Nbr(\mathbf{x}_i, k)$
- Normal  $\mathbf{n}_i$  is determined by **principal component analysis (PCA)**
- The oriented plane passing through  $\mathbf{o}_i$  having normal  $\pm \mathbf{n}_i$  provides the **least squares best fit** to points in  $Nbr(\mathbf{x}_i, k)$



# PCA: Linear dimensionality reduction

- **Linearly map** a set of  $m$ -dimensional vectors  $\{\mathbf{a}_1, \dots, \mathbf{a}_n\}$ , to an  $k$ -dimensional subspace,  $k < m$ , so as to minimize the approximation error in the least square sense



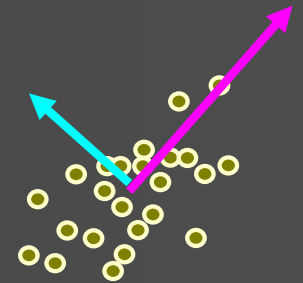
# Principal component analysis (PCA)

- Project data points  $\mathbf{a}_i$  onto the leading  $k$  eigenvectors (for the  $k$  **largest eigenvalues**) of the **covariance matrix**  $\Sigma$  for the original data set  $\mathbf{a}$

$$\Sigma = (\mathbf{a} - \bar{\mathbf{a}}\mathbf{1}^T)(\mathbf{a} - \bar{\mathbf{a}}\mathbf{1}^T)^T = \sum_{j=1..n}(\mathbf{a}_j - \bar{\mathbf{a}}) \cdot (\mathbf{a}_j - \bar{\mathbf{a}})^T \in \mathbf{R}^{m \times m}$$

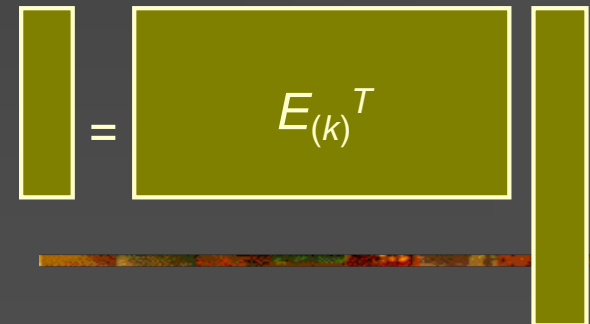
where  $\bar{\mathbf{a}}$  is the (uniform) mean of data points in  $\mathbf{a}$ .

- Eigenvectors: orthogonal and **major modes of variations**
- A  **$k$ -dimensional embedding** is obtained by



$$\hat{\mathbf{a}}_{(k)} = E_{(k)}^T \mathbf{a},$$

where  $E_{(k)} \in \mathbf{R}^{m \times k}$  has  $k$  columns of leading eigenvectors of  $\Sigma$ .



# PCA

- Project data points  $\mathbf{a}_i$  onto the leading  $k$  eigenvectors (for the  $k$  **largest eigenvalues**) of the **covariance matrix**  $\Sigma$  for the original data set  $\mathbf{a}$

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where  $\bar{\mathbf{a}}$  is the (uniform) mean of data points in  $\mathbf{a}$ .



# Normal of the tangent plane

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- Covariance matrix  $\Sigma$  of 3D points in  $Nbr(\mathbf{x}_i, k)$  is a symmetric (positive semi-definite)  $3 \times 3$  matrix
- The normal chosen for  $Nbr(\mathbf{x}_i, k)$  is  $+/-$  of the eigenvector of  $\Sigma$  corresponding to the **smallest eigenvalue** of  $\Sigma$
- The 2-dimensional subspace, i.e., the plane, is spanned by the other two eigenvectors
- The exact **sign** of the normal is chosen so that nearby tangent planes are **consistently oriented**

# Derivation of PCA (aside)

- Given a set of 3D points  $x_1, \dots, x_k$ , find a **best fitting plane**  $(o, n)$  in the **least squares sense**, where  $o$  is a point on the plane and  $n$  is the **unit** plane normal

- The minimization problem:

$$\min \sum_{i=1}^k [(x_i - o)^T \cdot n]^2 \text{ subject to } n^T n = 1$$

- Use **Lagrange Multiplier**:

$$\min F(o, n, \lambda) = \sum_{i=1}^k [(x_i - o)^T \cdot n]^2 - \lambda(n^T n - 1)$$

- We assume that  $n = (n_x, n_y, n_z)^T \neq 0$ .

- Differentiate  $F$  with respect to  $o$ , we have

$$\left[ \sum_{i=1}^k (x_i - o)^T \right] \cdot n = 0$$

- Differentiate  $F$  with respect to  $n_x, n_y, n_z$  and then combine into matrix form, we have

$$\left[ \sum_{i=1}^k (x_i - o)(x_i - o)^T \right] \cdot n = \lambda n$$

# Derivation of PCA (aside)

$$\left[ \sum_{i=1}^k (x_i - o)(x_i - o)^T \right] \cdot n = \lambda n$$

- So the normal  $n$  is an eigenvector of the covariance matrix and there are three local minima corresponding to three eigenvectors
- Alternatively, the minimization problem is really

$$\min \sum_{i=1}^k [(x_i - o)^T \cdot n]^2 = n^T \Sigma n, \text{ subject to } n^T n = 1$$

- By Courant-Fischer Theorem, this is just an eigenvalue problem

# Consistent normal orientation

- A harder part of the algorithm – it tells **topological information**
- One can model it as a global graph optimization problem
  - One node  $N_i$  per tangent plane
  - Two nodes connected if the corresponding centers are sufficiently close (where consistent orientation is enforced)
  - Cost of edge  $(N_i, N_j)$  is  $\mathbf{n}_i \cdot \mathbf{n}_j$  (maximum if coplanar)
  - Problem: **Find orientation to maximize the total cost in graph**
- But this optimization problem is NP-hard (i.e., its decision version is NP-complete)

# Approximate solution

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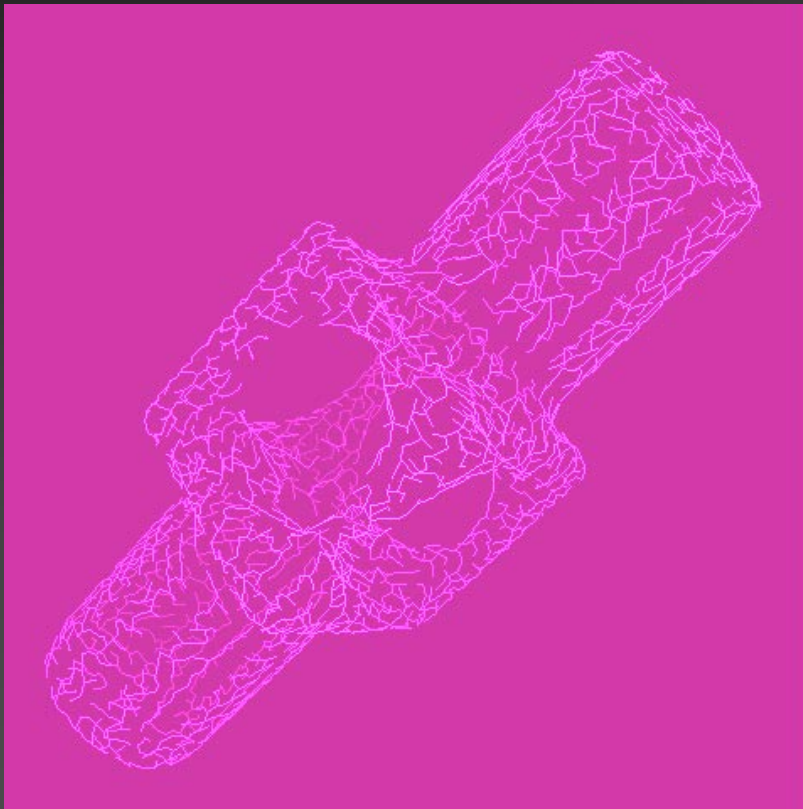
- First, build a **Riemannian graph** on tangent plane centers
  - Riemannian graph: encodes the geometric proximity of the tangent plane centers
  - Riemannian graph is built upon the **Euclidean minimum spanning tree** (EMST) – connected, tends to connect near neighbors, but there are not enough edges
  - Add an edge  $(N_i, N_j)$  to EMST if  $\mathbf{o}_i$  is one of the  $k$  closest neighbors of  $\mathbf{o}_j$  or vice versa

# Recall: EMST

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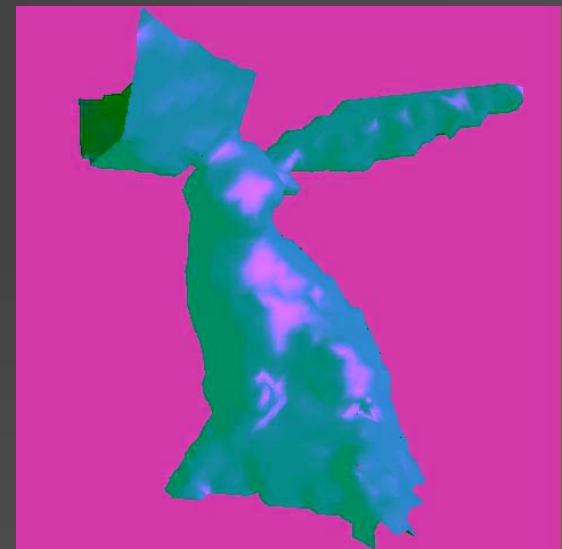
- Given a set of points  $L$ , an EMST is a spanning tree of  $L$  with the **minimum total cost** (edge cost measured by Euclidean distance)
- Can be obtained via **Kruskal's minimum spanning tree algorithm**
  - Conceptually consider complete graph on  $L$  with Euclidean distances as edge weights
  - Greedily add shortest edges that do not form a cycle
  - Stop when no edges can be added any more

# EMST and Riemannian graph



# Orientation propagation

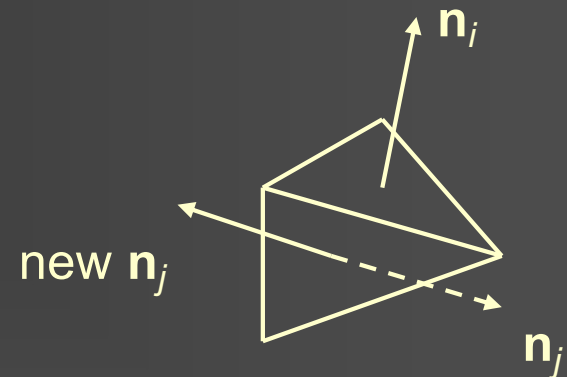
- To start propagation, choose orientation for an initial plane
- Propagate this orientation to its nearby planes by traversing the Riemannian graph
- **Traversal order is important**
- A heuristic: propagate along **low curvature** directions –
  - favor propagation from plane  $i$  to  $j$  if they are almost parallel
  - less likely to be a mistake



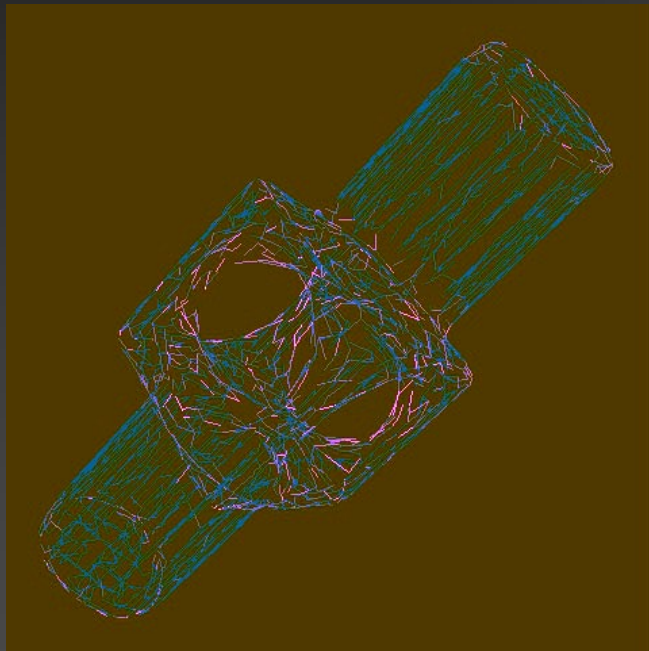


# Algorithm

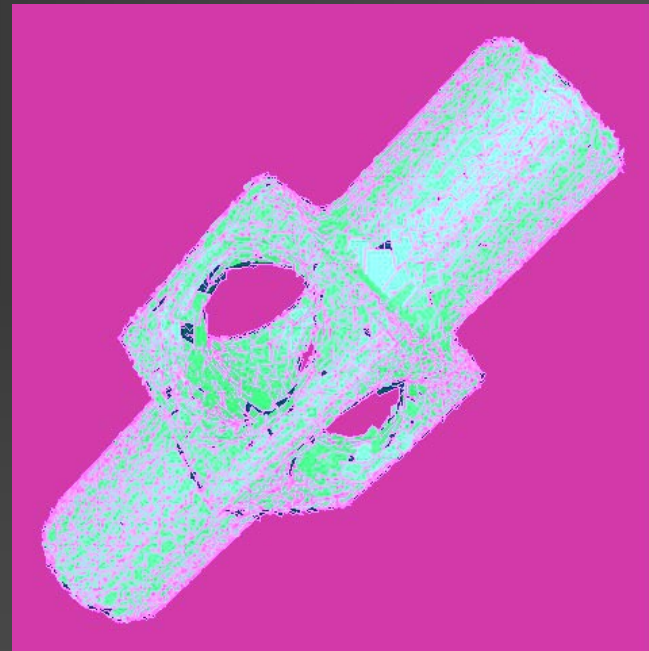
- Assign weight  $(1 - |\mathbf{n}_i \cdot \mathbf{n}_j|)$  to edge  $(N_i, N_j)$
- Propagate along edges of **minimum spanning tree** of the resulting graph (**depth-first search**)
- How to propagate from  $\mathbf{n}_i$  to next plane  $j$ ?
  - If  $\mathbf{n}_i \cdot \mathbf{n}_j < 0$ ,  $\mathbf{n}_j = -\mathbf{n}_j$
- How to choose an initial orientation?
  - Normal of plane whose center has largest  $z$  value is forced to point to  $+z$  direction



# Result



MST of normal variation graph with  
edge costs colored



Oriented tangent planes as shaded  
triangles

# Recall SDF

- $f(\mathbf{p})$  is signed distance from  $\mathbf{p}$  to “closest tangent plane”
- Since sampling is  $\delta$ -noisy and  $\rho$ -dense, if  $f(\mathbf{p}) > \delta + \rho$ , then  $\mathbf{p}$  cannot be on the surface  $M$ 
  - $f(\mathbf{p})$  is undefined in this case
- Otherwise, the signed distance from  $\mathbf{p}$  to  $M$  is a projection

$$f(\mathbf{p}) = (\mathbf{p} - \mathbf{o}_i) \cdot \mathbf{n}_i$$

where  $\mathbf{o}_i$  is the tangent plane center that is closest to  $\mathbf{p}$

Why is  $f(\mathbf{p})$  not the closest distance from  $\mathbf{p}$  to any tangent plane?

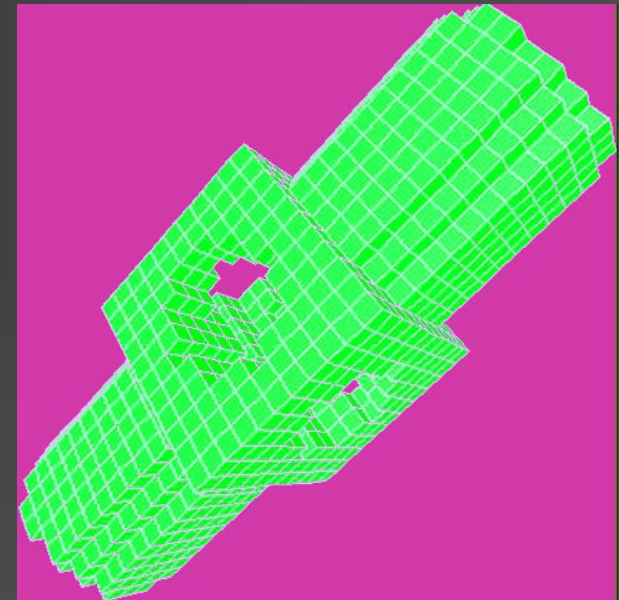
# Contour tracing

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- Given the set of **oriented** tangent planes, SDF from points to these planes can be computed
- Next, need to extract the iso-surface corresponding to the **zero-set** of the signed distance function
- This can be done using a **Marching Cubes (contour tracing)** algorithm or one of its variants

# Preparation for cubes marching

- Divide 3D space into cubical grids
- **Sample signed distance values at cube vertices**
- Only choose cubes that intersect the zero iso-surface for efficiency
- Size of cube  $d \approx \delta + \rho$ , why?
  - if  $d \gg \delta + \rho$ , may fill holes or join boundaries
  - if  $d$  too small, complexity too high
- No intersection between zero-surface and cube if a vertex has undefined  $f(\mathbf{p})$



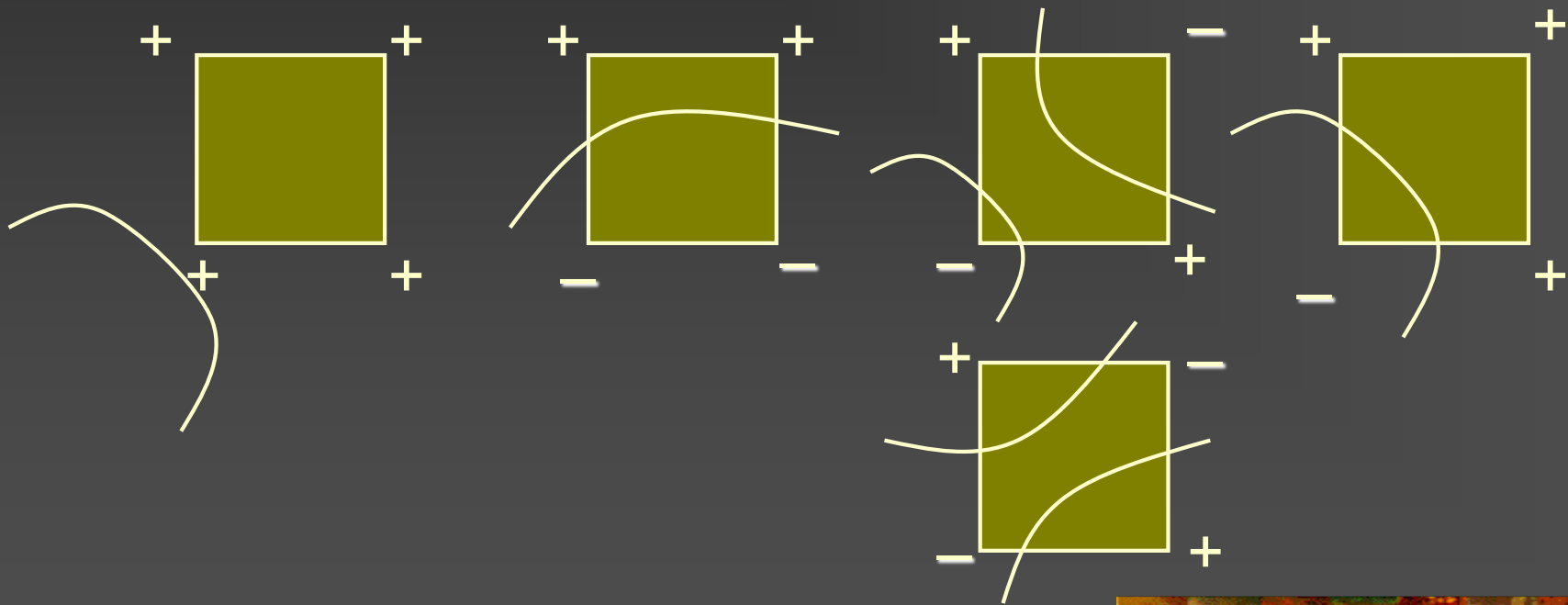
# Marching cubes algorithm

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- Input: a scalar field sampled over the vertices of a cubical grid
- Output: a set of triangles approximating the zero iso-surface of the scalar field
- Basic idea:
  - Process (march) cubes one at a time
  - Look at scalar values at vertices to decide how the iso-surface intersects the cube
  - Generate triangles reflecting these intersections

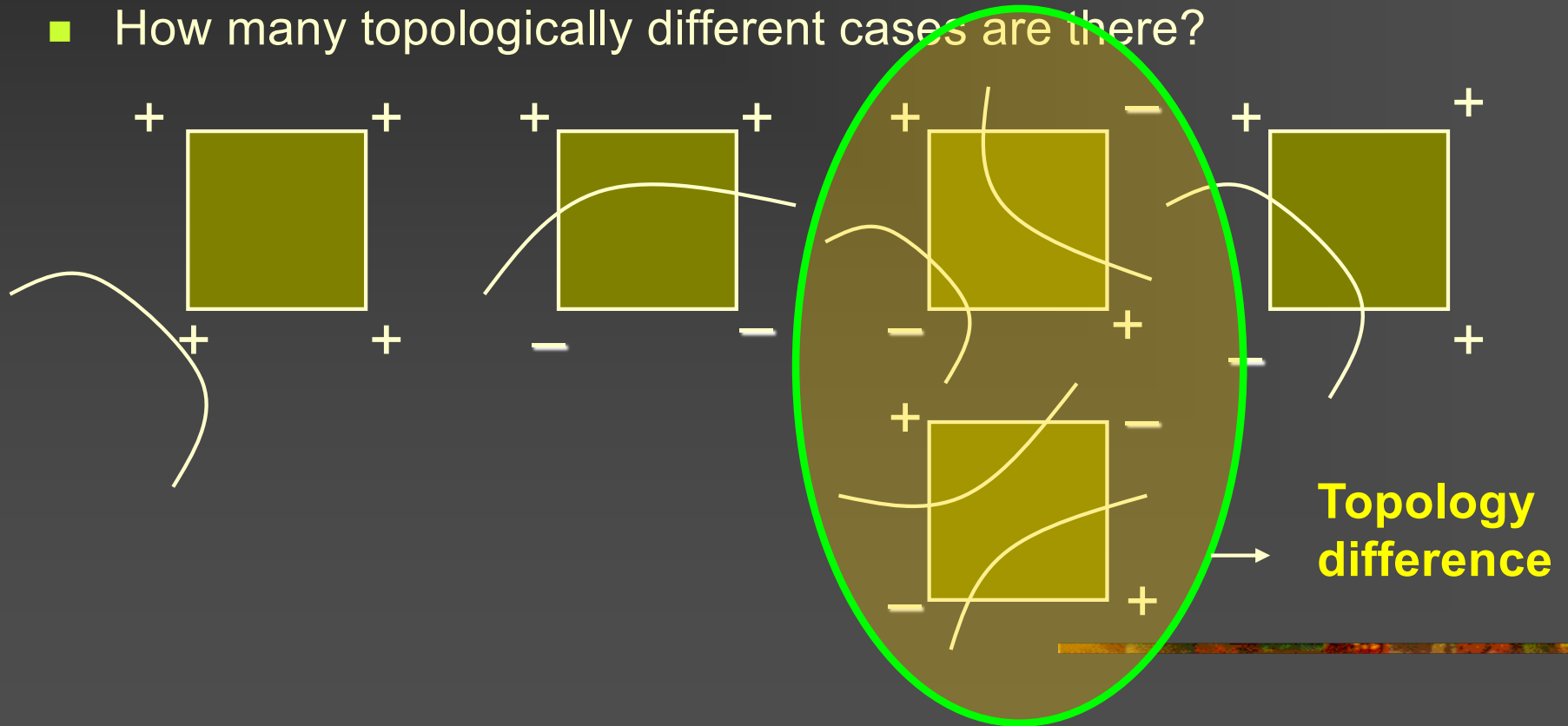
# 2D case: iso-contouring

- Inside iso-curve  $\equiv <$  and iso-value  $\equiv -$
- Outside iso-curve  $\equiv >$  and iso-value  $\equiv +$
- How many topologically different cases are there?



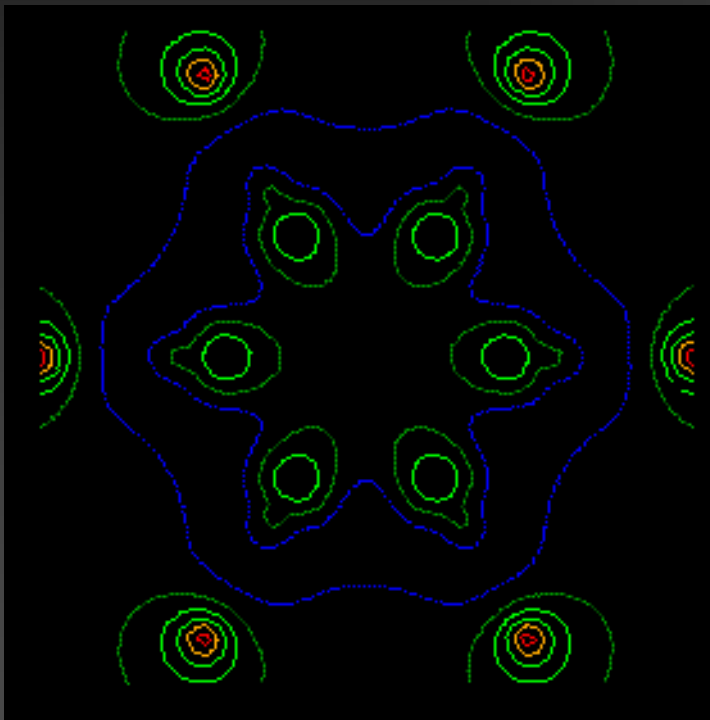
# 2D case: iso-contouring

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# Iso-contouring algorithm sketch

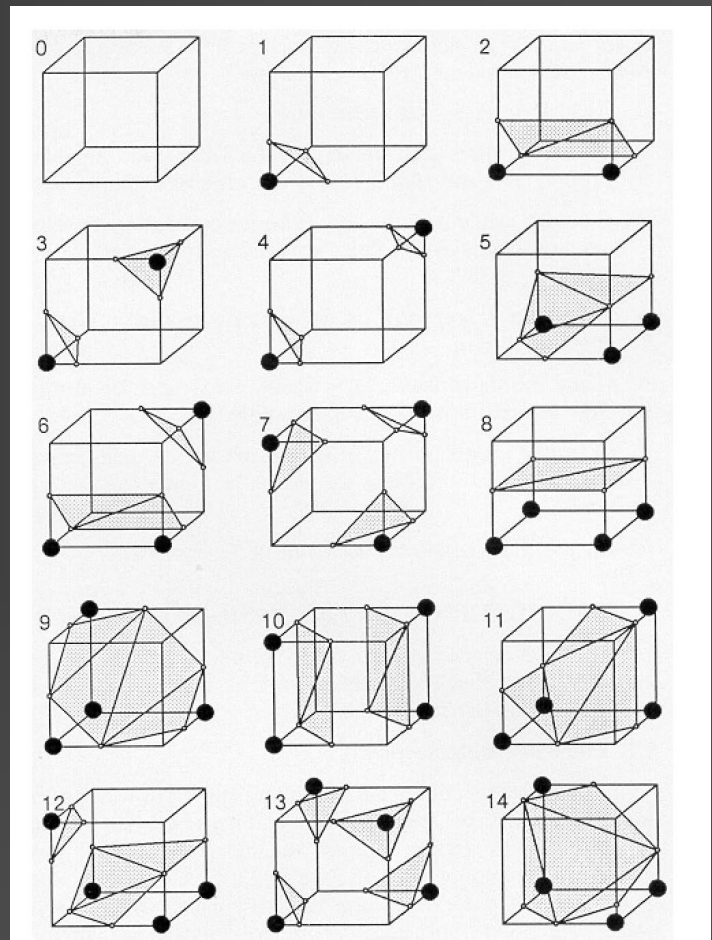


## Divide-and-conquer

1. Look at (march) one cell at a time
2. Compare the values at 4 corners with iso-value
3. **Linear interpolate** along edges for intersection points
4. Connect interpolated points together

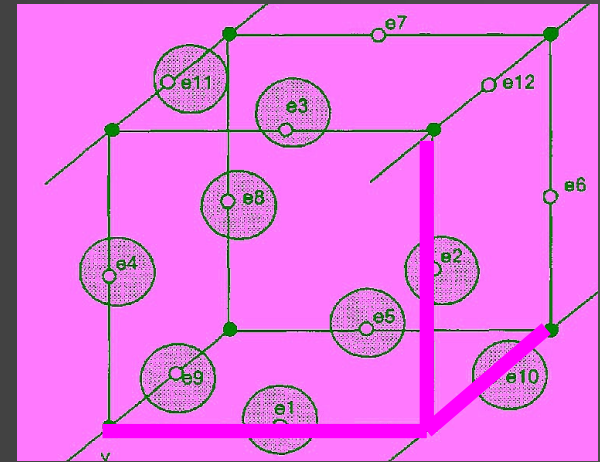
# Marching cubes

- Generalize iso-contour algorithm to 3D
- March cubes one at a time
- Linear interpolation again
- There are more cases:
  - Total of  $2^8 = 256$  cases
  - Reduce to 15 topological cases relying on value and rotational symmetry



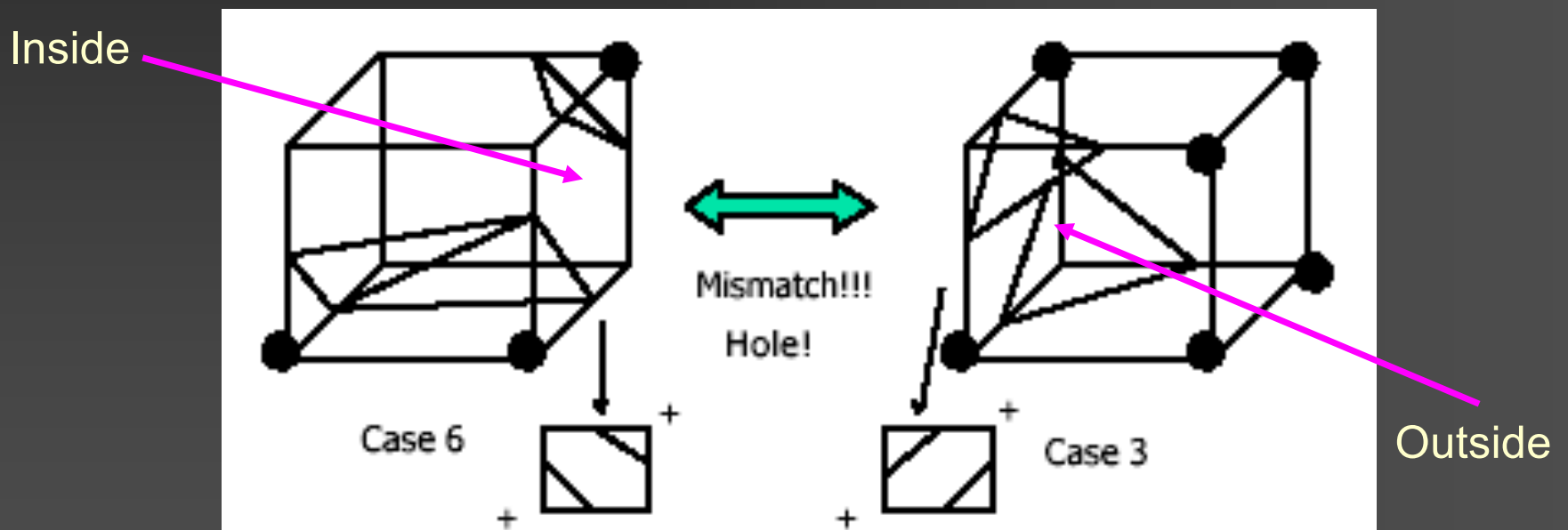
# Improvements

- Exploit spatial coherence
  - e.g., for an interior cube, only three new linear interpolations are needed, if cubes are visited in scan-line order
- Need to find efficient ways for cube traversal
  - Typically, roughly  $n^2$  cubes intersect an iso-surface in  $n^3$  cube grid
  - e.g., can use an octree to skip empty regions – a great deal of research along this line



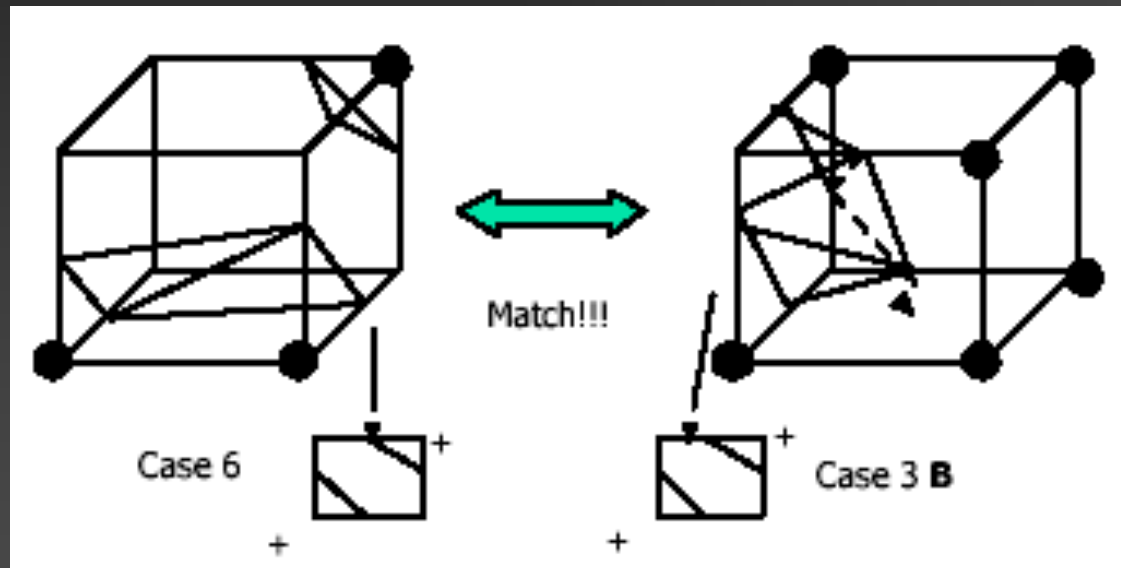
# The ambiguity problem

- Certain marching cube cases have more than one possible triangulations – may create a hole mistakenly



# Fixing the ambiguity problem

- One consistent way to do it

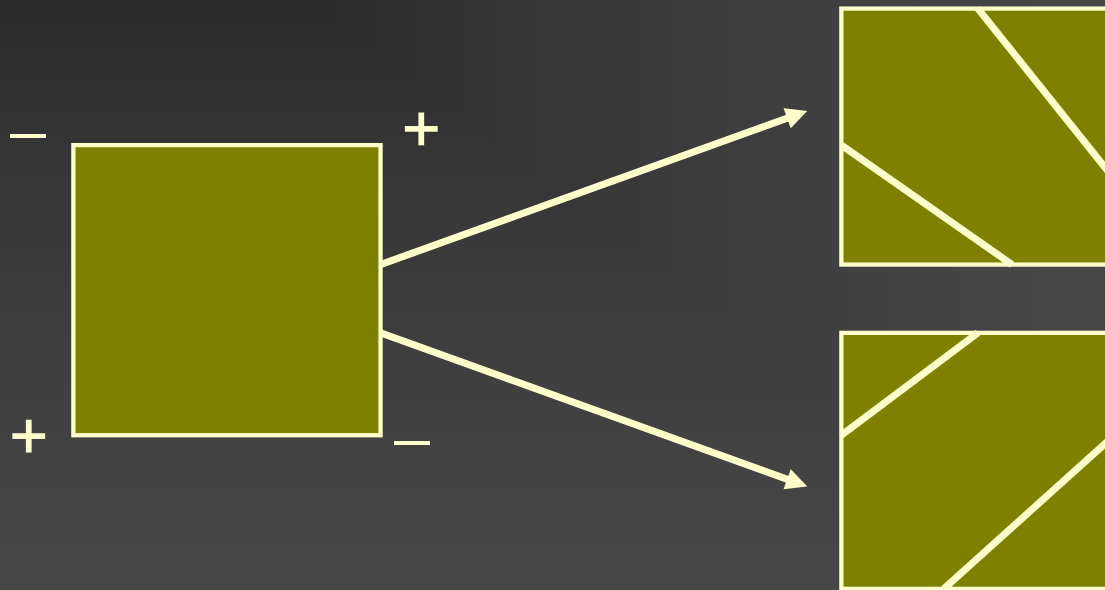


There is another **opposite case:**  
keep case 3 and  
change case 6 to  
6A

- Need to come up with these **consistent triangulations**

# Ambiguous faces

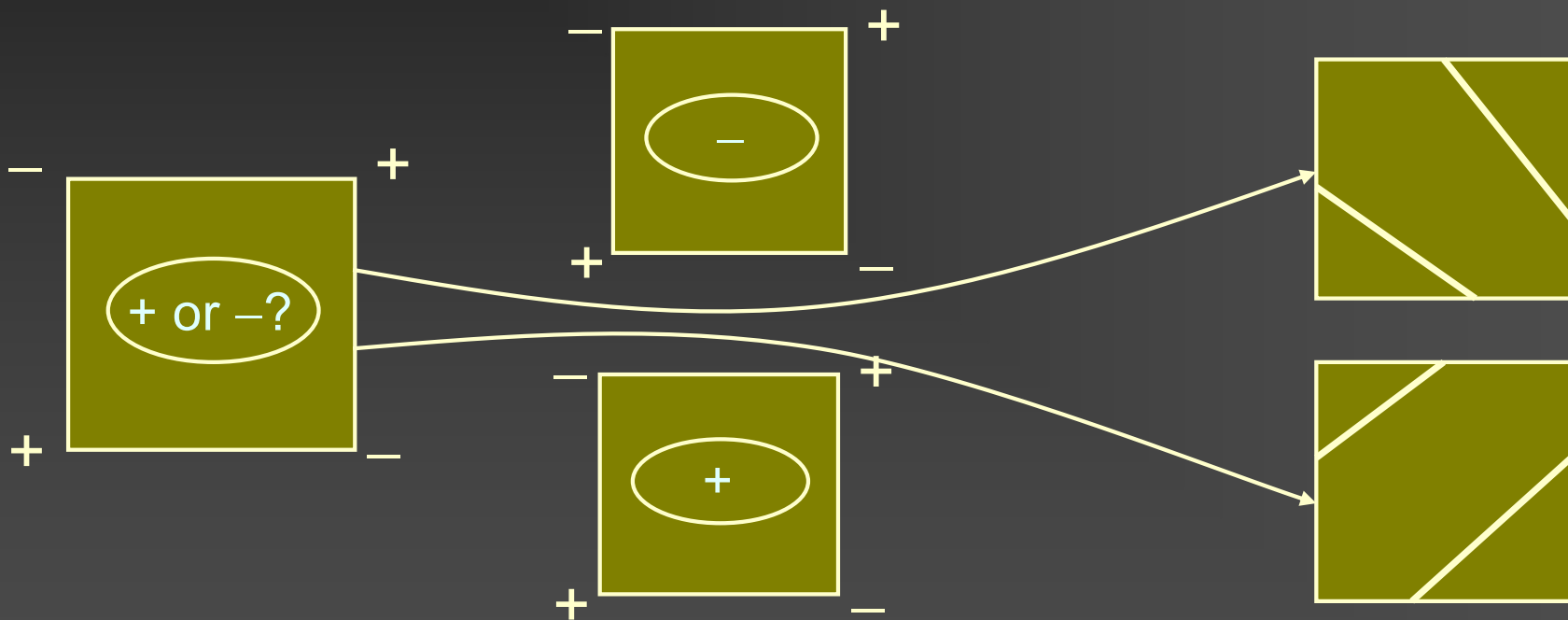
- A face with two opposite vertices having the same sign



- How to resolve this ambiguity? — use the **asymptotic decider** [Nielson & Hamman 91] — somewhat complex and adds cases to original marching cubes

# Asymptotic decider: rough idea

- Need to **examine iso-values inside the face**



- Inside values are unknown, approximate via **bi-linear interpolation**

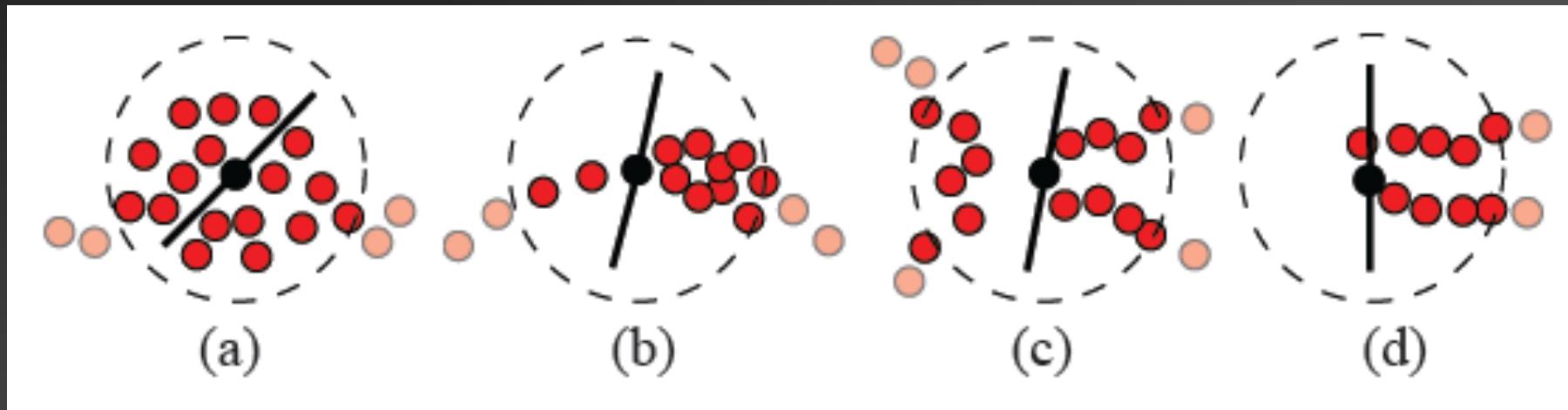
# Summary of Hoppe's approach

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- Surface reconstruction from unorganized points through iso-surface extraction over a signed distance field computed with respect to a set of oriented tangent planes approximating the surface
- Space subdivision helps speed up algorithm (empty cube skipping)
- Constructed surface **approximates** point cloud
- No theoretical guarantee that the surface is correct
- No mechanism for feature preservation

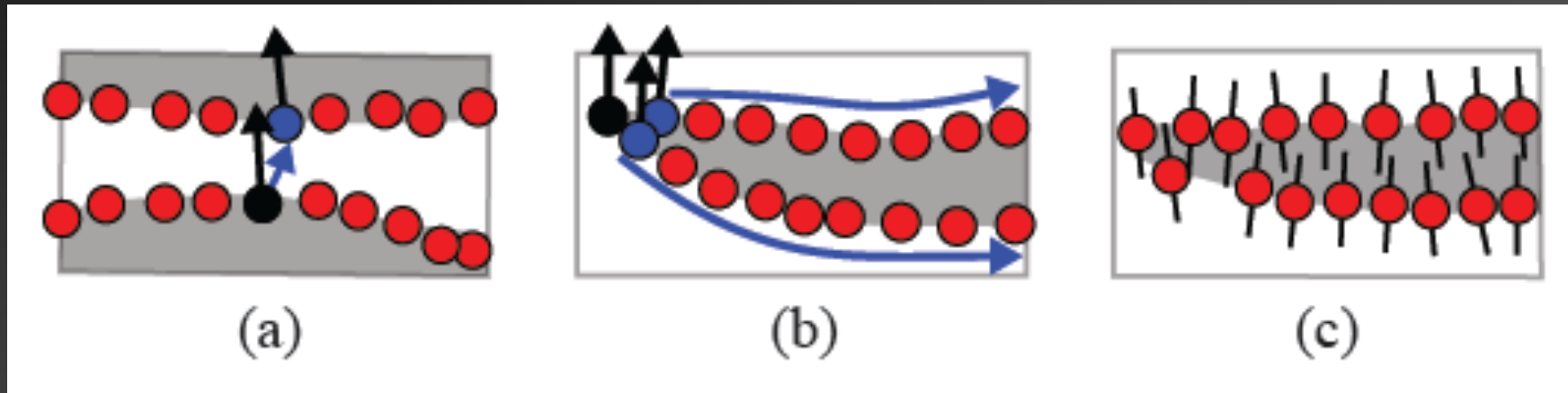


# Unreliability of PCA



- Thick point cloud – need thinning
- Non-uniform point distribution
- Close-by surface sheets

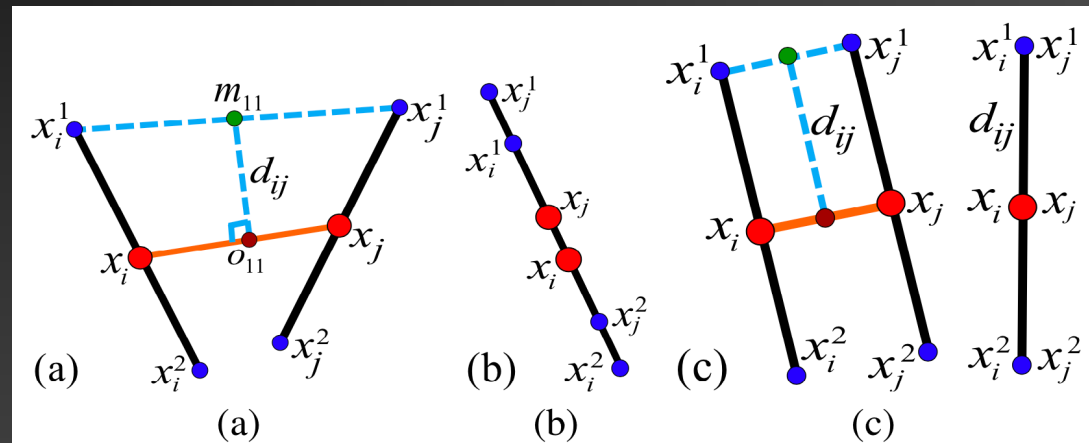
# New propagation cost (aside)



- Again, the close-by surface sheets problem
- Possible solution: also look at the **propagation direction**
- Sharp feature detection: should prevent propagation there

**Hui Huang, Dan Li, Hao Zhang, Uri Ascher, and Daniel Cohen-Or, "Consolidation of Unorganized Point Clouds for Surface Reconstruction," *ACM Trans. on Graphics (Proceeding of SIGGRAPH Asia 2009)*, Article 176.**

# New propagation cost (aside)



$$\mathcal{D}_{ij} = 1 - |\langle \mathbf{v}_i, \mathbf{v}_j \rangle| \cdot \frac{\max_{r,s \in \{1,2\}} \|m_{rs} - o_{rs}\|}{1 + \|x_i - x_j\|}. \quad (4)$$

Note that  $\mathcal{D}_{ij} \in [0, 1]$ ; it combines Euclidean distance (the denominator), angular distance  $|\langle \mathbf{v}_i, \mathbf{v}_j \rangle|$ , and a third term  $d_{ij} = \max_{r,s \in \{1,2\}} \|m_{rs} - o_{rs}\|$ , which is designed to weigh in propagation direction.

**Hui Huang, Dan Li, Hao Zhang, Uri Ascher, and Daniel Cohen-Or, "Consolidation of Unorganized Point Clouds for Surface Reconstruction," *ACM Trans. on Graphics (Proceeding of SIGGRAPH Asia 2009)*, Article 176.**

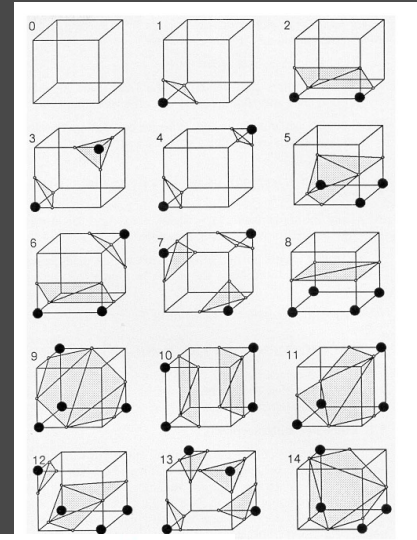
# Other classical approaches

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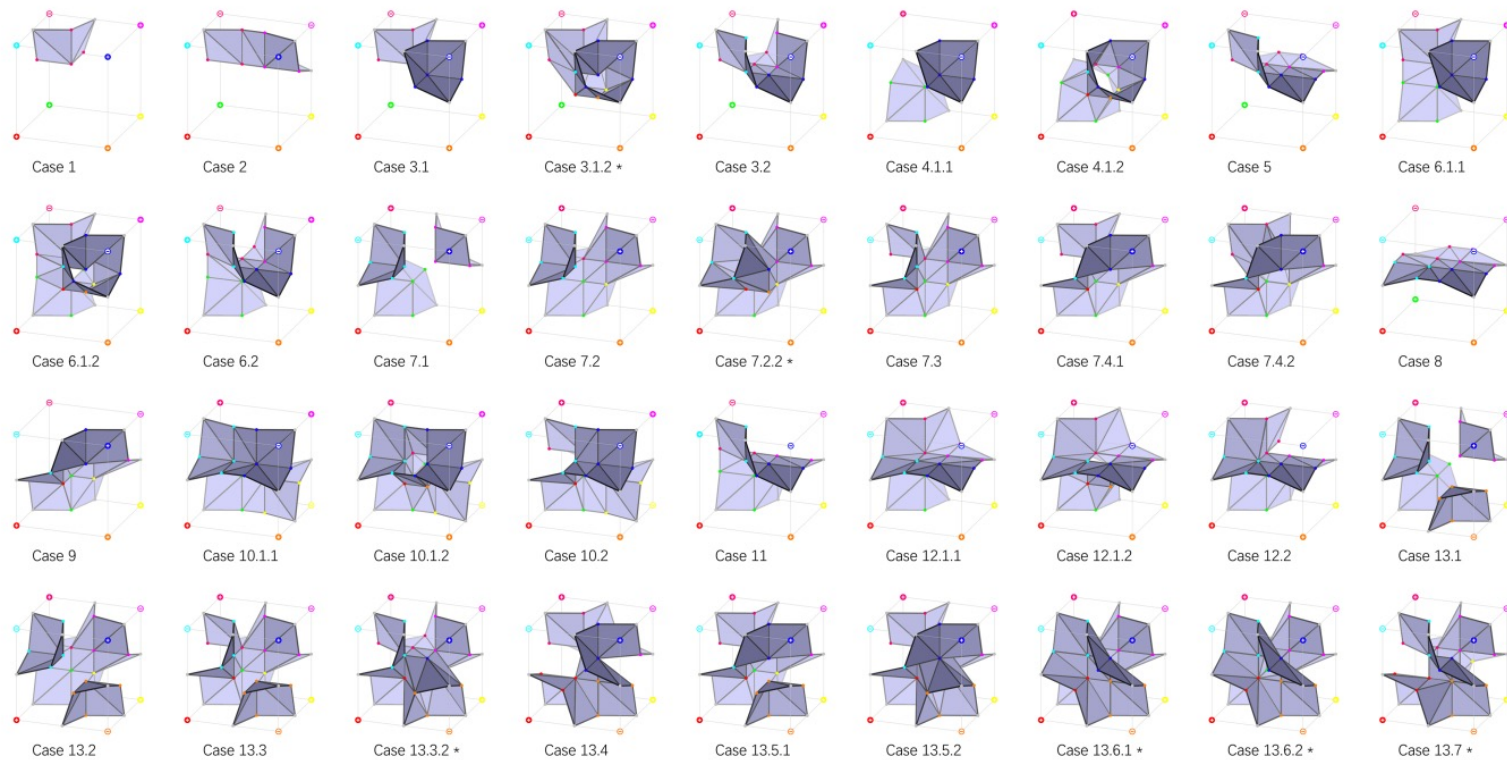
- Voronoi-based with **theoretical guarantee** – by N. Amenta et al., “A New Voronoi-based Surface Reconstruction Algorithm,” *SIGGRAPH 98*
  - $\alpha$ -shape based approaches – [Bajaj, Bernardini 95]
  - **Deform-to-fit** with energy minimization (e.g., inflating a balloon in the object) – [Terzopoulos, Witkin, and Kass 88 & 91, Miller 91]
  - Use of **radial basis functions** – [Carr et al. 01, Iske 02]
  - Use of **Poisson reconstruction** – [Kazhdan et al. 06]
  - Definition of point set surfaces, e.g., **MLS = Moving Least Squares** – [Levin et al. 01, Alexa et al. 02]
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# From MC to NMC

- Use **machine learning** to improve iso-surfacing



(a) Our cube tessellations



# Neural Marching Cubes (NMC) – next week

## Neural Marching Cubes

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