A Tensor Field Mumford-Shah Segmentation of Neural Pathways in DW-MRI

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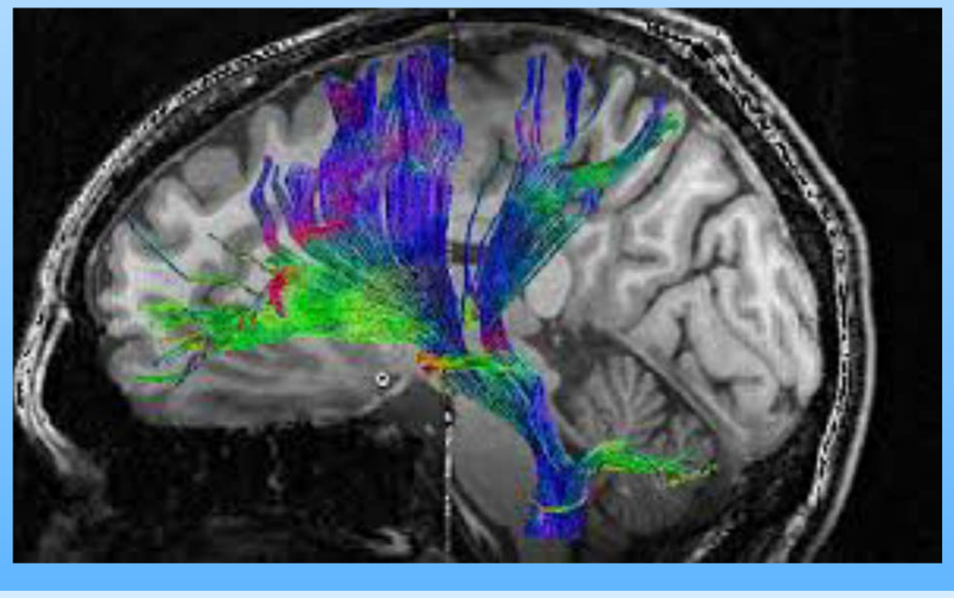
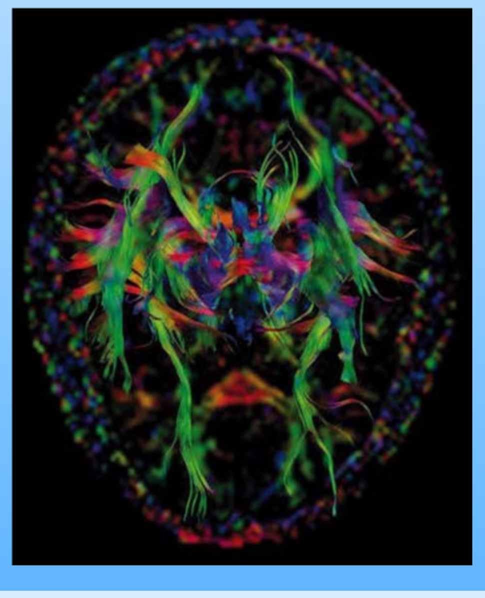
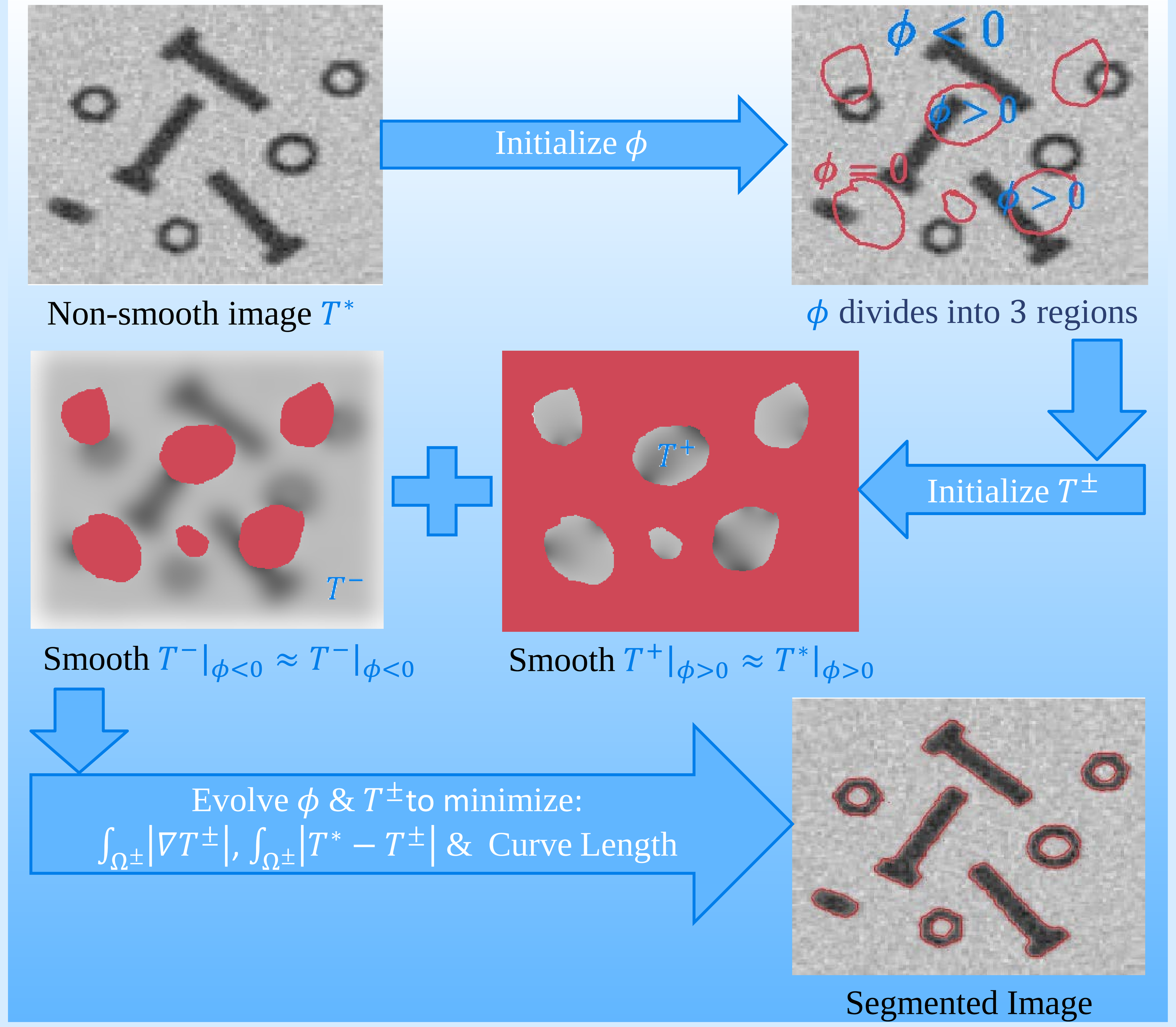
BRAIN IMAGING

- Neuronal bundles form structural connections between various brain regions
- Brain areas ‘communicate’ through these connections
- Connections’ abnormalities are indicative of brain diseases
- Water diffuses faster along the neuronal tracts and slower across them
- Diffusion weighted MRI measures diffusion rates along various directions
- Assuming Gaussian diffusion, we obtain a field of 3 × 3 positive definite matrices
- Main eigenvectors give main diffusion directions, thus neuronal tracts directions





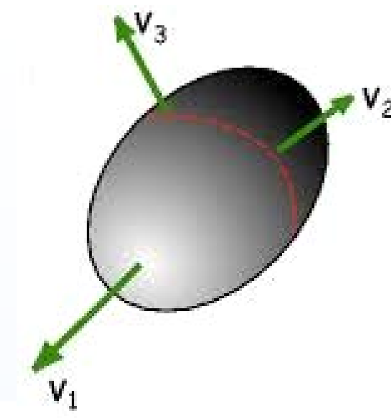
MUMFORD-SHAH for 2D SCALAR FIELD



MUMFORD-SHAH for 3D TENSOR FIELD

Given a *positive definite* 3 × 3 tensor (matrix) field T^* on a 3D bounded *domain* Ω, with

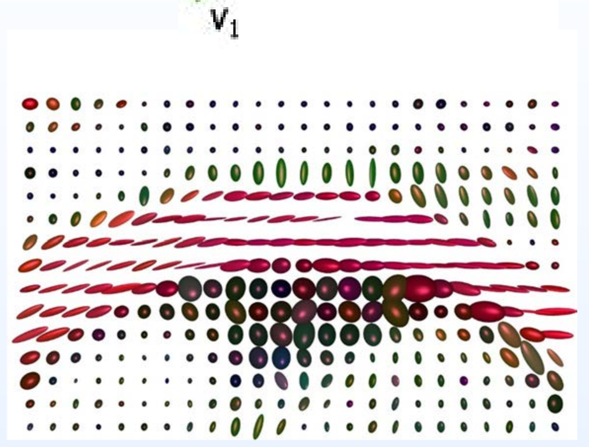
- Spectral decomposition $T^* = V\Lambda^* V^T$
- Eigenvectors $V = (V_1|V_2|V_3)$
- Eigenvalues $\Lambda^* = diag(\alpha_1^*, \alpha_2^*, \alpha_3^*)$



Let ϕ be a smooth function with $\phi = 0$ level surface dividing $\Omega = \Omega^+ \cup \{\phi = 0\} \cup \Omega^-$

Let $T^{\pm}$ be smooth approximations of T^* on the respective regions $\Omega^{\pm}$, with

- Spectral decompositions $T^{\pm} = U^{\pm}\Lambda^{\pm}(U^{\pm})^T$
- Eigenvectors $U^{\pm} = (U_1^{\pm}|U_2^{\pm}|U_3^{\pm})$
- Eigenvalues $\Lambda^{\pm} = diag(\alpha_1^{\pm}, \alpha_2^{\pm}, \alpha_3^{\pm})$ (with $\alpha_k^{\pm} = \int_{\Omega^{\pm}} \alpha_k^* \, dx$)
- *Alignment measure* $A_k^{\pm} = \alpha_k^{\pm}\left(\alpha_k^{\pm} U_k^{\pm} - \sum_{m=1}^{3} \alpha_m^*(U_k^{\pm} \cdot V_m)V_m\right)$

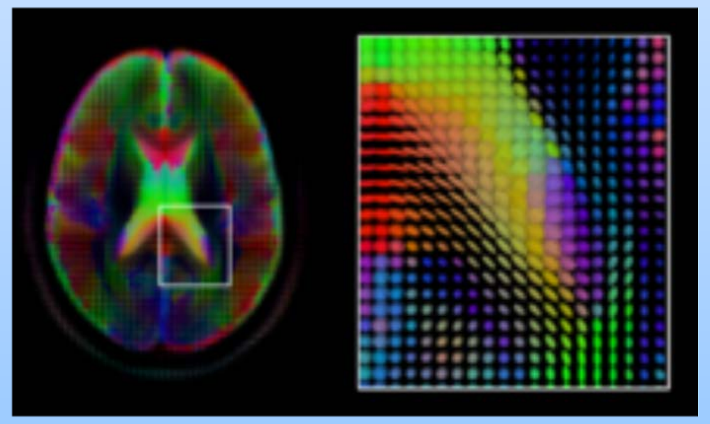


Let $G_k^{\pm} = \sum_{i=1}^{3} \nabla U_{ik}^{\pm} \left(\nabla U_{ik}^{\pm}\right)^T$ be the *structural tensor* of vector $U_k^{\pm}$ (note $|dU_k^{\pm}| = dx^T G_k^{\pm} dx$), with

- Largest eigenvalue $\lambda_k^{\pm}$
- Corresponding eigenvector $\theta_k^{\pm}$
- *Diffusion* tensor $D_k^{\pm} = H(\phi)\theta_k^{\pm}\left(\theta_k^{\pm}\right)^T$ (H Heaviside function)

Define the following *energy* functionals

- *Fidelity* $E_1 = \int_{\Omega^{\pm}} Tr(B^{\pm}(B^{\pm})^T)dx$ (where $B^{\pm} = T^* - T^{\pm}$)
- Level *surface area* $E_2 = \int_{\Omega} |\nabla H(\phi)| dx$
- Eigenvectors *smoothness* $E_3 = \sum_{k=1}^{3} \int_{\Omega} \lambda_k^{\pm} H(\phi) dx$
- *Ortho-normality* constrain $E_4 = \int_{\Omega} H(\phi) \sum_{p,q=1}^{3} \eta_{pq}^{\pm}\left(U_p^{\pm} \cdot U_q^{\pm} - \delta_{pq}\right) dx$



Resulting *Euler-Lagrange* evolution of $U_k^{\pm}$ is

$$\frac{\partial U_k^{\pm}}{\partial t} = -L_k^{\pm} + \sum_{q=1}^{3}\left(L_q^{\pm} \cdot U_k^{\pm}\right)U_q^{\pm}$$

Here $L_{ik}^{\pm} = -\mu\nabla \cdot \left(D_k^{\pm}\nabla U_{ik}^{\pm}\right) + 2\lambda A_{ik}^{\pm} H(\phi)$. The first term is a *diffusion* term along the main eigenvector $\theta_k^{\pm}$ of $G_k^{\pm}$, and the second *fidelity* term tries to align the main eigenvectors of T^* and $T^{\pm}$

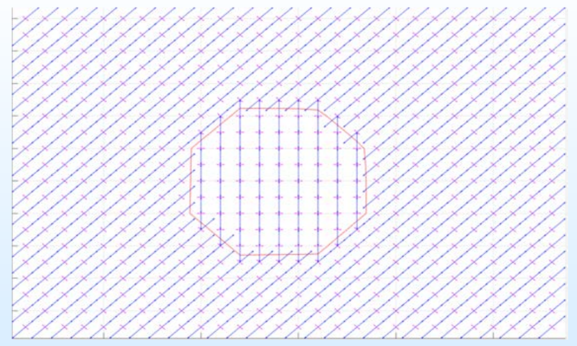
Resulting *Euler-Lagrange* evolution of ϕ is

$$\frac{\partial \phi}{\partial t} = \delta^{\epsilon}(\phi)\left(\nu\nabla \cdot \left(\frac{\nabla\phi}{|\nabla\phi|}\right) + \lambda\left(d^2(T^*, T^+) - d^2(T^*, T^-)\right) + \mu\left(\sum_{k=1}^{3}\lambda_k^+ - \sum_{k=1}^{3}\lambda_k^-\right)\right)$$

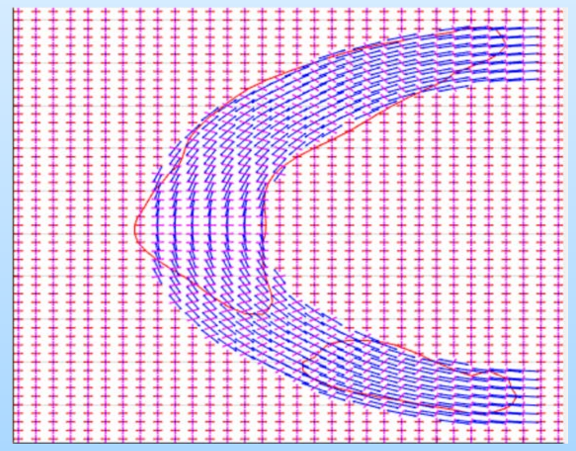
$$+\delta^{\epsilon}(\phi)\left(\sum_{p,q=1}^{3}\lambda_{pq}^+\left(U_p^+ \cdot U_q^+ - \delta_{pq}\right) - \sum_{p,q=1}^{3}\lambda_{pq}^-\left(U_p^- \cdot U_q^- - \delta_{pq}\right)\right)$$

where $\delta^{\epsilon}(\phi)$ is the Dirac delta function. Here $\phi = 0$ is evolved to reduce the differences between T^* and $T^{\pm}$ in each respective region $\Omega^{\pm}$, while maintaining a small gradient of the eigenvectors $U_k^{\pm}$ away from the level surface. This is done while trying to reduce the irregularity and surface area of $\phi = 0$

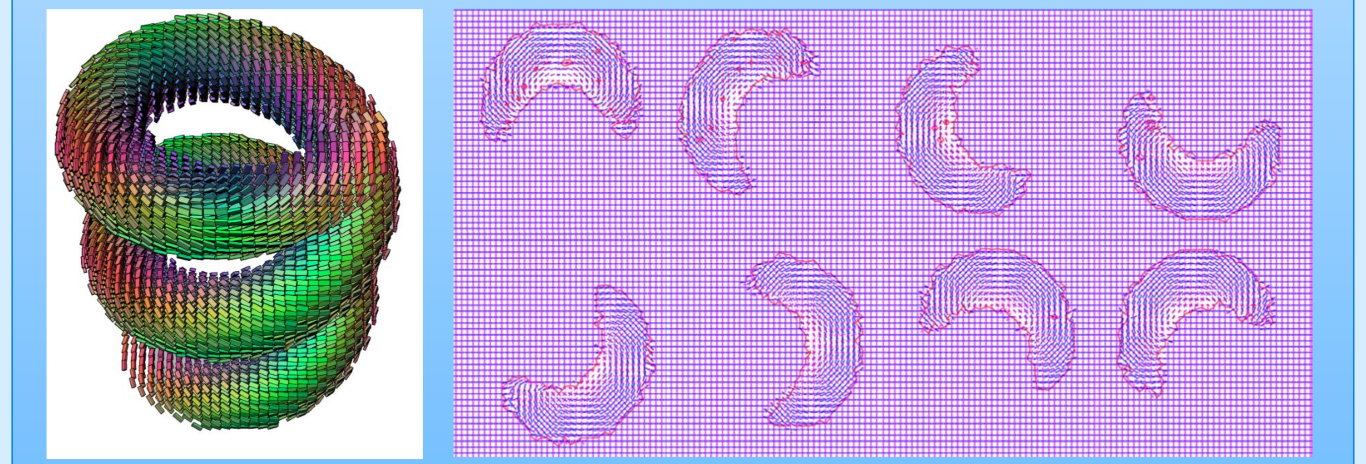
SIMULATIONS for SYNTHETIC DATA



Fiber perpendicular to page
Main eigenvector pointing vertically. Background is anisotropic with main eigenvector pointing diagonally. Horizontal cross-section shown



Semi-annulus
main eigenvector following the annulus direction. Background is anisotropic with main eigenvector perpendicular to page. Horizontal cross-section shown



Helix perpendicular to page
Main eigenvector following the helix direction. Background is isotropic. Horizontal cross-sections shown

CONCLUSIONS and FUTURE WORK

- Managed to segment simple cases of synthetic data
- Aim to use better optimization algorithms than gradient descent to address the non-convexity issue
- Aim to then test the algorithm on brain images

REFERENCES

- Vese, L.A. and Chan, T.F., 2002. A multiphase level set framework for image segmentation using the Mumford and Shah model. *International journal of computer vision*, *50*(3), pp.271-293
- Tschumperlé, D. and Deriche, R., 2001. Diffusion tensor regularization with constraints preservation. In *Computer Vision and Pattern Recognition, 2001. CVPR 2001. Proceedings of the 2001 IEEE Computer Society Conference on*, vol. 1, pp. I-948 - I-953

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