Subsampling and image pyramids
Overview of today’s lecture

• Image downsampling.
• Aliasing.
• Gaussian image pyramid.
• Laplacian image pyramid.
Image downsampling
This image is too big to fit on the screen. How would you reduce it to half its size?
Naïve image downsampling

Throw away half the rows and columns

delete even rows
delete even columns

What is the problem with this approach?
Naïve image downsampling

What is the 1/8 image so pixelated (and do you know what this effect is called)?
Aliasing
Reminder

Images are a discrete, or sampled, representation of a continuous world.
Sampling

Very simple example: a sine wave

How would you discretize this signal?
Sampling

Very simple example: a sine wave
Sampling

Very simple example: a sine wave

How many samples should I take? Can I take as many samples as I want?
Sampling

Very simple example: a sine wave

How many samples should I take?
Can I take as few samples as I want?
Undersampling

Very simple example: a sine wave

Unsurprising effect: information is lost.
Undersampling

Very simple example: a sine wave

Unsurprising effect: information is lost.
Surprising effect: can confuse the signal with one of lower frequency.
Undersampling

Very simple example: a sine wave

Unsurprising effect: information is lost.
Surprising effect: can confuse the signal with one of lower frequency.
Note: we could always confuse the signal with one of higher frequency.
Aliasing

Fancy term for: *Undersampling can disguise a signal as one of a lower frequency*

Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of lower frequency.

Note: we could always confuse the signal with one of higher frequency.
Aliasing in textures
Aliasing in photographs

This is also known as “moire”
Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise). Mark wheel with dot so we can see what’s happening.

If camera shutter is only open for a fraction of a frame time (frame time = 1/30 sec. for video, 1/24 sec. for film):

Without dot, wheel appears to be rotating slowly backwards! (counterclockwise)
Wagon wheel effect
Anti-aliasing

How would you deal with aliasing?
Anti-aliasing

How would you deal with aliasing?

Approach 1: Oversample the signal
Anti-aliasing in textures

aliasing artifacts

anti-aliasing by oversampling
Anti-aliasing

How would you deal with aliasing?

Approach 1: Oversample the signal

Approach 2: Smooth the signal
  • Remove some of the detail effects that cause aliasing.
  • Lose information, but high frequency doesn’t matter in halved images.

How would you smooth a signal?
Better image downsampling

Apply a smoothing filter first, then throw away half the rows and columns

Gaussian filter
delete even rows
delete even columns

Gaussian filter
delete even rows
delete even columns

1/2

1/4

1/8
Better image downsampling

1/2
1/4 (2x zoom)
1/8 (4x zoom)
Naïve image downsampling

1/2

1/4 (2x zoom)

1/8 (4x zoom)
Anti-aliasing

Question 1: How much smoothing do I need to do to avoid aliasing?

Question 2: How many samples do I need to take to avoid aliasing?

Answer to both: Enough to reach the Nyquist limit.

We’ll see what this means soon.
Gaussian image pyramid
Gaussian image pyramid

The name of this sequence of subsampled images
Constructing a Gaussian pyramid

Algorithm

repeat:
  filter
  subsample
until min resolution reached

Question: How much bigger than the original image is the whole pyramid?
Constructing a Gaussian pyramid

Algorithm
repeat:
    filter
    subsample
until min resolution reached

Question: How much bigger than the original image is the whole pyramid?
Answer: Just 4/3 times the size of the original image! (How did I come up with this number?)
Some properties of the Gaussian pyramid

What happens to the details of the image?
Some properties of the Gaussian pyramid

What happens to the details of the image?
• They get smoothed out as we move to higher levels.

What is preserved at the higher levels?
Some properties of the Gaussian pyramid

What happens to the details of the image?
• They get smoothed out as we move to higher levels.

What is preserved at the higher levels?
• Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?
Some properties of the Gaussian pyramid

What happens to the details of the image?
• They get smoothed out as we move to higher levels.

What is preserved at the higher levels?
• Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?
• That’s not possible.
Blurring is lossy

What does the residual look like?

level 0

level 1 (before downsampling)

residual
Blurring is lossy

level 0

- level 1 (before downsampling)

residual

Can we make a pyramid that is lossless?
Laplacian image pyramid
Constructing a Laplacian pyramid

It’s a Gaussian pyramid.
At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?
At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?  
• Yes we can!

What do we need to store to be able to reconstruct the original image?

Laplacian image pyramid
What do we need to construct the original image?

(1) residuals

(2) smallest image

$f_2$

$h_1$

$h_0$

$f_0$
Reconstructing the original image

Algorithm

repeat:
  upsample
  sum with residual
until orig resolution reached
Constructing a Laplacian pyramid

It’s a Gaussian pyramid.
Gaussian vs Laplacian Pyramid

Shown in opposite order for space.

Which one takes more space to store?
Why is it called a Laplacian pyramid?
Reminder: Laplacian of Gaussian (LoG) filter

As with derivative, we can combine Laplace filtering with Gaussian filtering

input

Laplacian of Gaussian

output

“zero crossings” at edges
Why is it called a Laplacian pyramid?

\[ \text{unit} \times I (= I) \quad \text{Gauss} \times I \quad (\text{unit} - \text{Gauss}) \times I \]

\[ \text{unit} \quad \text{Gaussian} \quad \text{Laplacian} \]

Difference of Gaussians approximates the Laplacian
Why Reagan?
Why Reagan?

Ronald Reagan was President when the Laplacian pyramid was invented.

The Laplacian Pyramid as a Compact Image Code (1983)
Peter J. Burt, Edward H. Adelson
Still used extensively

foreground details enhanced, background details reduced user-provided mask
Other types of pyramids

Steerable pyramid: At each level keep multiple versions, one for each direction.

Wavelets: Huge area in image processing
What are image pyramids used for?

- Image compression
- Multi-scale
- Texture mapping
- Focal stack compositing
- Denoising
- Multi-scale detection
- Multi-scale registration
References

Basic reading:
• Szeliski textbook, Sections 3.5

Additional reading:
• Burt and Adelson, “The Laplacian Pyramid as a Compact Image Code,” IEEE ToC 1983. the original Laplacian pyramid paper