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Color Discrimination Ellipses Explained by Metamer Mismatching

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ABSTRACT

Many psychophysical experiments have shown that color discrimination varies substantially with the region of color space in which the colors reside. Many models of the experimental data have been proposed, and many uniform color spaces have been developed that attempt to represent color in a coordinate system such that equally discriminable colors are equal distances apart, but all of them are based on fits to the experimental data. Many provide good fits to the data, but they remain data models and do not explain why color discrimination varies in the way it does. In contrast, this paper outlines a theory of color discrimination based on the uncertainties reflected in the extent of metamer mismatching. The greater its extent, the more finely a color needs to be discriminated.

Keywords: *Color discrimination, metamer mismatching*

INTRODUCTION

The threshold for discriminating one color from a very similar one is known to vary as a function of the color involved. This paper addresses the question of why color discrimination varies as it does.

It has become common to represent color discrimination in terms of ellipsoids in color space or ellipses in chromaticity space. Chromaticity discrimination ellipses were initially measured by MacAdam [1], and his results first revealed the non-uniformity of the CIE 1931 color space. Many other color discrimination experiments have followed and have been used in the development and testing of several new uniform color spaces and color difference formulas.

Luo et al. [2] tested the performance of the CIECAM02 color appearance model and introduced a modified version of it to fit what are known as the SCD (small color difference) and LCD (large color difference) data sets. Wen [3] proposed a method for calculating color differences and showed that it outperforms CIEDE2000 in predicting threshold color differences. Huang et al. conducted an experiment using 466 pairs of printed samples surrounding 17 color centers to evaluate 10 color difference formulas [4]. In another study, Luo and Rigg combined the data from different sources to produce a consistent set of ellipses [5]. Berns et al. [6] generated a color-difference tolerance dataset of 19 color centers for fitting and testing of color-difference metrics. Sharma et al. [7] provided a data set for additional test on CIEDE2000 formula. Pridmore and Melgosa [8] analyzed four different data sets to model the ellipse area and dimension.

All of the models and uniform color spaces derived from these experiments are based on fits to the experimental data. Many provide good fits to the data, but they remain data models and do not explain why color discrimination varies in the way it does. What is the underlying reason that color discrimination varies as it does? The hypothesis investigated here is that it is due to metamer mismatching.

HYPOTHESIS

Metamer mismatching refers to the extent to which two physically distinct reflectances that match (i.e., lead to identical LMS cone response color signals) under one light fail to match under a second light. Metamer mismatching arises from the fact that normal trichromatic color vision is based on only 3 weighted-sum measurements of the reflected light's spectrum impinging at any given point on the retina, whereas that spectrum—the product of the illuminating light's spectrum and the surface's underlying spectral reflectance function—is much more complex. The study by Zhang et al. [9] of metamer mismatching showed that it is most severe for grey and least severe for highly saturated colors. See Figure 1. Our hypothesis is that in order to be able to reliably discriminate physically distinct surfaces from one another observers must be more sensitive to the differences between colors for which metamer mismatching creates significant uncertainty (i.e., when the metamer mismatch bodies are large), and least sensitive for colors for which metamer mismatching creates little uncertainty.

The volume of the metamer mismatch body (MMB) (i.e., the set of all possible color signals that can arise under the second light given the color signal under the first light) for a given color signal is a measure of the possible variability in the nature of the underlying physical reflectance. The larger the MMB, the larger and more varied is the set of reflectances that are all metameric (i.e., create the same LMS cone response) under a given light. Hence, for colors with large MMBs there is more uncertainty as to the exact nature of the underlying surface reflectance function. Intuitively, it is clear that there are likely more reflectance functions that lead to a mid-grey where the entire range of the visible spectrum is likely to be involved than there are to a saturated red, for example, where mainly the long-wave portion of the spectrum is likely to be involved. For an observer wishing to identify a given physical surface by its color, it is therefore more important to distinguish the exact shade of a gray surface as precisely as possible and less important to distinguish the exact tint of a red one. Similarly, there are very few reflectances leading to pure white, with the limit being the ideal white created by a uniform 100% reflectance. In fact, for any color signal on the boundary of the object color solid, there is only one possible reflectance creating it, so the volume of the MMB drops to zero for such color signals. This is illustrated by the plot in Figure 1.

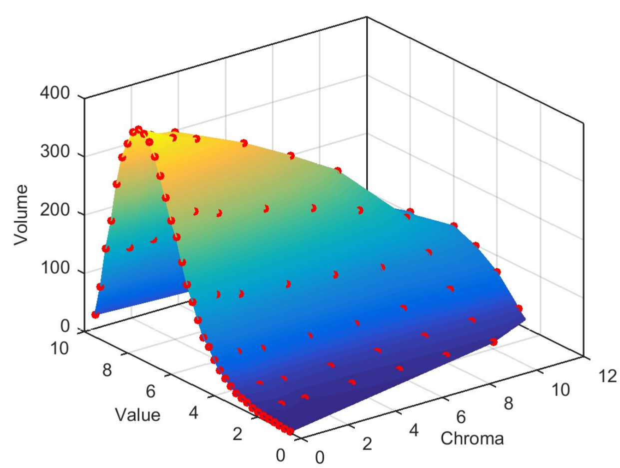

Figure 1: Plot of MMB volume averaged over all hues showing how the MMB volume decreases with distance in Munsell value and/or chroma from grey (value 6, chroma 0).

This trend of the MMB volumes is also clear in the xy-chromaticity plot of Figure 2, which shows how the size of the MMBs decreases towards the spectral locus (i.e., as the saturation increases).

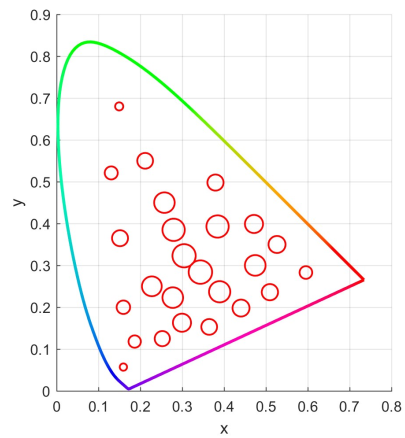

Figure 2: A plot in xy-chromaticity space of metamer mismatch body volumes as a function of color center, with the area of each red circle being proportional to the corresponding MMB volume.

RESULTS

Huang et al. [4] measured color discrimination ellipsoids for 17 color centers and Berns et al. [6] measured 19. While the experimental methods differed, both sets of experiments are based on colored surface samples, not lights. These datasets are therefore useful comparison to predictions based on the MMB volumes.

The results of both studies are reported in CIELAB color space. For comparison to the MMBs, boundary points of the ellipsoid in CIELAB are converted to CIE XYZ and then to LMS cone space via the Hunt-Pointer-Estevez matrix. The volume, *E*, is then computed as the volume of the convex hull of those boundary points in LMS space. For each color center, the volume, *M*, of the corresponding MMB for a change in illuminant from CIE D65 to CIE A is computed directly in LMS space using the algorithm of Logvinenko et al. [10].

In order to make comparisons in 'linear' space the cube root of both *E* and *M* are used as being representative of their 'radii'. The cube root of *E* is then normalized by the Euclidean distance, *C,* from the ellipsoid's center to the LMS origin. This normalization eliminates effects due to the intensity/luminance and is similar to converting to chromaticity space. The hypothesis that metamer mismatching is at least partly responsible for the variation in color discrimination as a function of color center then is evaluated by comparing $\boldsymbol{E^{1/3}/C}$ to $\boldsymbol{1/M^{1/3}}$.

This metamer mismatching hypothesis is then evaluated against the null hypothesis that there is no linear relationship between $\boldsymbol{E^{1/3}/C}$ and $\boldsymbol{1/M^{1/3}}$ for both the Huang [4] and Berns [6] datasets. The null hypothesis is rejected at the 5% significance level, with p-values in all cases being less than 0.006. The R-squared results along with plots of the fits are shown in Figure 3 both in linear and log-log plots.

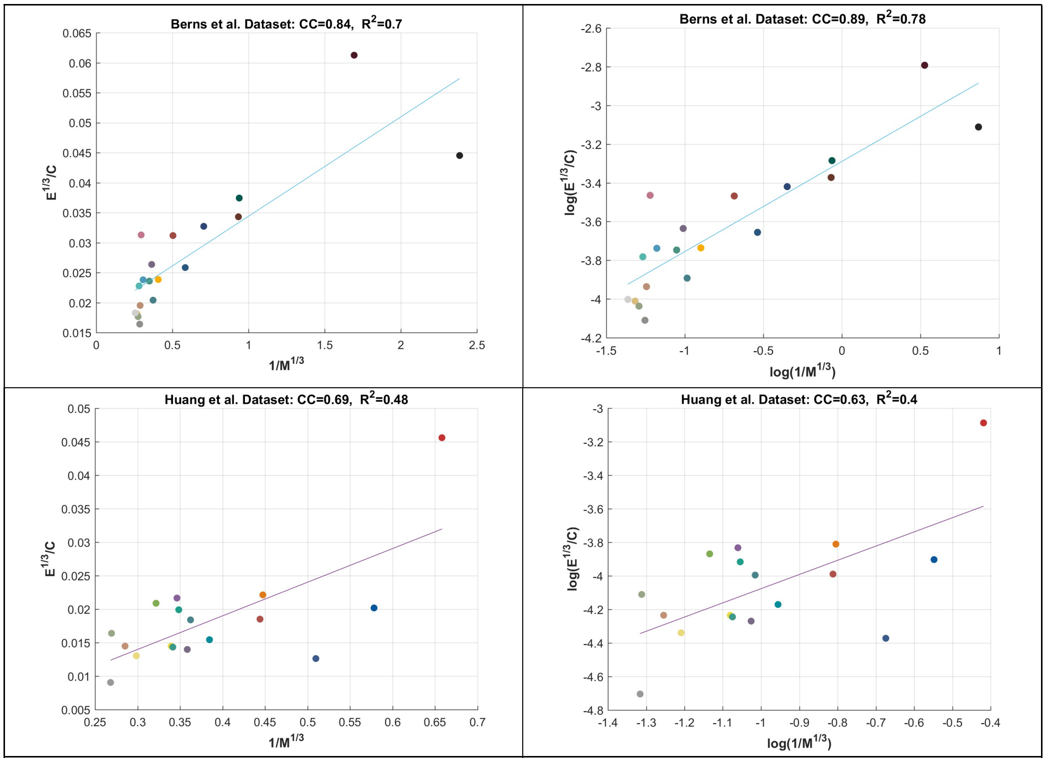

Figure 3: Plots of the normalized 'radii' (cube root of volumes) of the color discrimination ellipsoids in LMS space as a function of the inverse of the radii of the corresponding metamer mismatch bodies for both datasets. The plot titles specify the correlation coefficient (CC) and R-squared (R^2) for the linear fits.

DISCUSSION and CONCLUSION

It is common to represent color discrimination in terms of ellipsoids in color space and ellipses in chromaticity space. In this paper, two sets of experimental data on color discrimination are used for testing. The results reported above indicate a correlation between color discrimination and metamer mismatching. In particular, as the extent of metamer mismatching increases, color discrimination thresholds decrease. The fits shown in Figure 3 do indicate the hypothesized relationship, but they are far from perfect. Of course, the experimental data contains noise, but other unaccounted for factors need to be considered and investigated. The fact that the Berns data is modelled better than the Huang data can be accounted for by the fact that the Huang data focused on measuring discrimination ellipses rather than ellipsoids. Huang et al. [5] specifically state that the ellipsoid volumes they report are likely to be less reliable than the ellipse areas.

This paper has explored the hypothesis that the need to overcome the uncertainty due to metamer mismatching is the reason for more precise discrimination between colors in some regions of color space. Since Zhang et al. [9] showed that metamer mismatching is greatest for grey, high for colors of low saturation, and decreases with increasing saturation, the hypothesis correctly predicts that color discrimination is finest near grey and coarsest for the saturated colors near the object color solid boundary. In other words, metamer mismatching provides a possible explanation for why color discrimination varies in the way it does.

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