Color-Difference Ellipsoids Follow from Metamer Mismatching

Emitis Roshan and Brian Funt; School of Computing Science, Simon Fraser University, Vancouver, Canada

Many psychophysical experiments have shown that color discrimination thresholds vary as a function of the color center under consideration, MacAdam ellipses being a prime example. Color discrimination thresholds are usually modelled either as ellipses in chromaticity space or ellipsoids in 3D color space. Various color difference models, such as those of Luo et al. [1] [2], have been developed based on fits to the data obtained from psychophysical experiments (e.g., Li et al. [3] [4]). However, such models do not explain why the color difference thresholds vary across color space as they do. Funt et al. [5] show that the uncertainty created by metamer mismatching provides a very interesting explanation. Extending that theory as described below enables it to predict, not only the ellipsoid volumes, but the shapes and orientations of the ellipsoids as well, thereby providing further evidence of its validity.

Metamer mismatching results from the fact that the relationship between cone triples and surface reflectances is one-to-many. Under some fixed illuminant, there are many different surface reflectances leading to the same cone response triple. Such reflectances are generally referred to as metamers. Under a second illuminant these different reflectances will usually lead to quite distinct cone response triples. The set of all such cone triples is known as the metamer mismatch body [6]. This phenomenon is referred to as metamer mismatching. Because of the one-to-many relationship between a cone response triple and surface reflectances, the eye cannot uniquely identify a surface's reflectance and so might, for example, misidentify a poisonous plant as an edible one. The likelihood of this misidentification depends on the extent of metamer mismatching, which varies across color space. The more serious the metamer mismatching, the more precisely colors need to be discriminated from one another to avoid mistakes; and hence, the need for an inverse relationship between the extent of metamer mismatching and color discrimination thresholds.

Color difference thresholds are usually represented by ellipses in a 2D chromaticity space or ellipsoids in 3D color space. Funt et al. [5] show that the volume, M, of the Metamer Mismatch Body (MMB) normalized by the cube of the distance, C, between a color center and the origin of the coordinate system is inversely proportional to the volume of the color discrimination ellipsoid (E_{vol}). In other words, $C^3/M \propto E_{vol}$. This relationship—based solely on the hypothesis that the uncertainty created by metamer mismatching underlies color difference thresholds—predicts the experimental data on the volume of color discrimination ellipsoids better than CAM16-UCS, even though CAM16-UCS is based on a direct fit to similar experimental data.

Although the relationship $C^3/M \propto E_{vol}$ based on metamer mismatching is effective in predicting the volume of color discrimination ellipsoids, can metamer mismatching predict the shape and orientation of the color discrimination ellipsoids as well?

To model the general shape of an MMB, we use its equivalent ellipsoid as proposed by Roshan et al. [7]. The radii and principal axes of the equivalent ellipsoid represent the MMB's size and orientation respectively. Let the radii of its equivalent ellipsoid be r_1, r_2 and r_3. Based on the inverse relationship between the normalized MMB volume and the discrimination ellipsoid volume (i.e., strong correlation coefficients between C^3/M and E_{vol}) it is reasonable to expect an ellipsoid with radii C/r_1, C/r_2 and C/r_3, to be similar to the discrimination ellipsoid.

The proposed theory predicts the shape and orientation of color discrimination ellipsoids based on metamer mismatching. In order to evaluate the accuracy of the method's predictions, an ellipsoid similarity measure is needed. One such measure is that defined by Merritt [8]. The Merritt similarity measure varies from 0 to 1, with 1 indicating two identical ellipsoids.

The MMBs for the 19 color centers in the Melgosa dataset [9] (derived from the original data in RIT-DuPont dataset [10]), the 17 color centers in Huang dataset [11], the 5 color centers in Cheung dataset [12] and the 4 in Witt dataset [13] are calculated for illuminant change D65 to A using the algorithm of Logvinenko et al. [14]. The equivalent ellipsoids corresponding to these MMBs are then calculated and compared to the experimental data using the Merritt similarity measure (Table 1).

Dataset	Merritt
Melgosa	0.79
Huang	0.78
Cheung	0.80
Witt	0.84

Table 1 Merritt similarity measure comparing the predicted color discrimination ellipsoids to the measured ellipsoids from four datasets.

Color discrimination thresholds are known to vary throughout color space. The degree of metamer mismatching (which reflects the underlying degree of uncertainty as to the actual reflectance producing the color) also varies throughout color space. Hence, color discrimination thresholds and the degree of metamer mismatching are inversely related. This principle can be used to predict the orientation and dimensions of color discrimination ellipsoids quite well.

References

[1] Luo, M. R., Cui, G., and Li, C., "Uniform colour spaces based on CIECAM02 colour appearance model" *Color Research & Application*, 31(4), 320–330 (2006).

[2] Luo, M. R, Cui, G. and Rigg, B., "The development of the CIE 2000 colour difference formula: CIEDE2000" *Color Research & Application*, 26(5), 340–350 (2001).

[3] Li, C., Li, Z., Wang, Z., Xu, Y., Luo, M. R., Cui, G., "A revision of CIECAM02 and its CAT and UCS" *Proc. Color and Imaging Conference,* pp. 208–212 (2016).

[4] Li, C., Li, Z., Wang, Z., Xu, Y., Luo, M. R., Cui, G., "Comprehensive color solutions: CAM16, CAT16, and CAM16-UCS" *Color Research & Application*, 42(6), 703–718 (2017).

[5] Funt, B., and Roshan, E., “Metamer mismatching underlies color difference sensitivity” *Journal of Vision* 21, 12, pp. 1-11 (2021).

[6] Zhang, X., Funt, B., and Mirzaei, H. “Metamer mismatching in practice versus theory” *J. Opt. Soc. Am. A* 33, pp. A238–A247 (2016).

[7] Roshan, E., and Funt, B., “Color Sensor Accuracy Index Utilizing Metamer Mismatch Radii” *Sensors* 20, 15, 4275 pp. 1-10 (2020).

[8] Merritt, E.A., “Comparing anisotropic displacement parameters in protein structures” *Acta Crystallographica Section D: Biological Crystallography*, *55*(12), pp.1997-2004 (1999).

[9] Melgosa, M., Hita, E., Poza, A., Alman, D. H., and Berns, R. S. “Suprathreshold color-difference ellipsoids for surface colors” *Color Research & Application,* 22(3), 148-155 (1997).

[10] Berns, R. S., Alman, D. H., Reniff, L., Snyder, G. D., and Balonon-Rosen, M. R. “Visual determination of suprathreshold color-difference tolerances using probit analysis” *Color Research & Application,* 16(5), 297-316 (1991).

[11] Huang, M., Liu, H., Cui, G., and Luo, M. R. “Testing uniform colour spaces and colour-difference formulae using printed samples” *Color Research & Application,* 37(5), 326–335 (2012).

[12] Cheung, M., and Rigg, B. “Colour-difference ellipsoids for five CIE colour centres” *Color Research & Application*, 11(3), 185–195, (1986).

[13] Witt, K. “Three-dimensional threshold of color-difference perceptibility in painted samples: variability of observers in four CIE color regions” *Color Research & Application,* 12(3), 128–134, (1987).

[14] Logvinenko, A.D., Funt, B., and Godau, C., “Metamer Mismatching” *IEEE Trans. on Image Processing*, 23, 1, pp. 34-43, (2014).