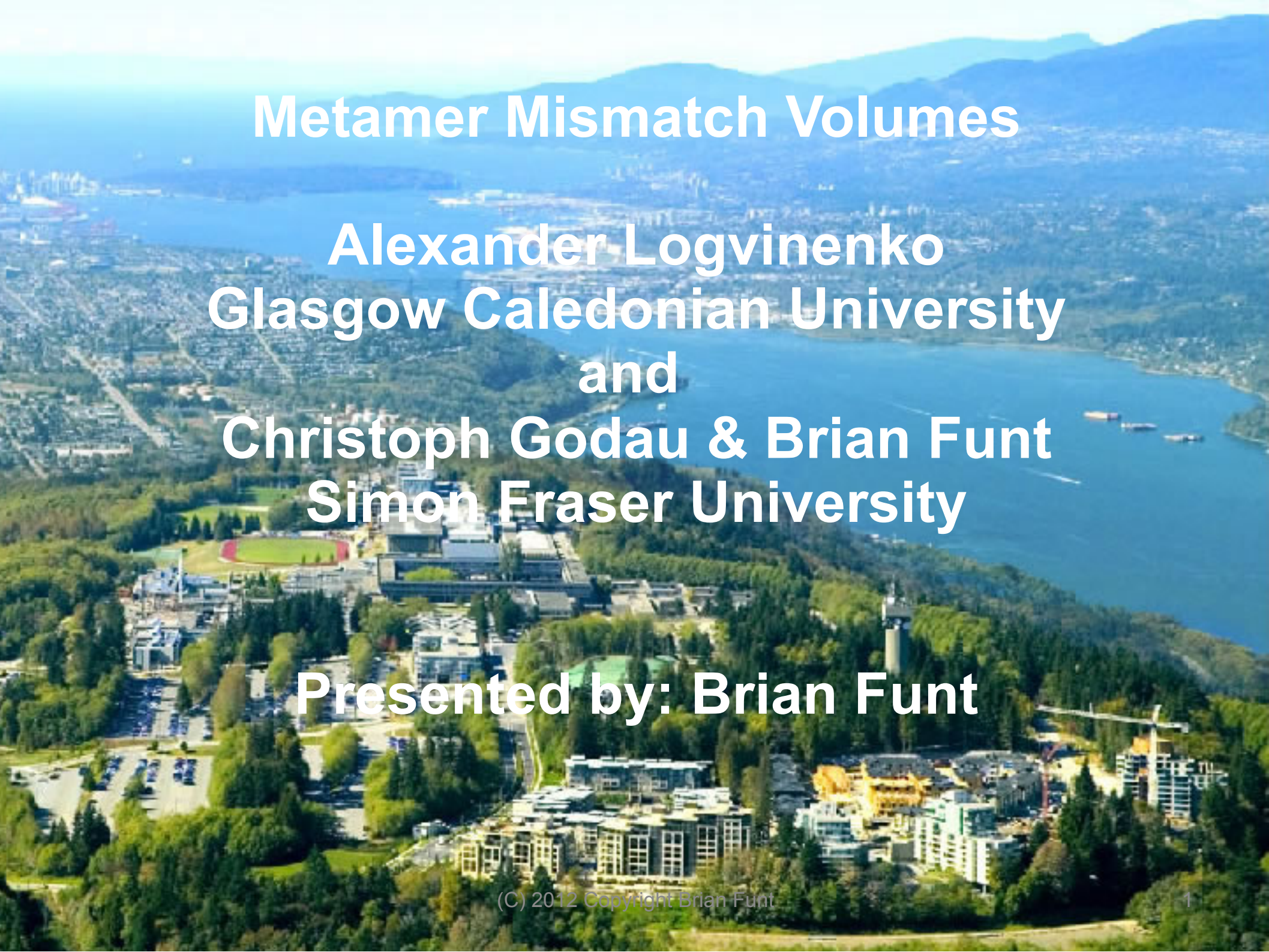

# Metamer Mismatch Volumes


Alexander Logvinenko
Glasgow Caledonian University
and
Christoph Godau & Brian Funt
Simon Fraser University


Presented by: Brian Funt

Alexander Logvinenko

# Talk Goal is Intuition

Formal mathematical details are in the proceedings.

I'm going to talk using words like red, green, blue.

In the paper you can find $\varphi, \phi, \Psi, \Phi$ and so on.

Given an object $x_0 \in \mathcal{X}$, the $\Phi-$pre-image $\Phi^{-1}(\Phi(x_0))$ (i.e., $\Phi^{-1}(\Phi(x_0)) = \{x \in \mathcal{X} | \Phi(x) = \Phi(x_0)\}$) of its colour signal $\Phi(x_0)$ is the set of all the objects metameric to $x_0$ (with respect to $\Phi$), and is referred to as its metamer set. Generally, when this set of metameric objects $\Phi^{-1}(\Phi(x_0))$ is mapped by $\Psi$ into the $\Psi$-colour solid, it will be spread into a non-singleton set. The resulting set is usually referred to as the metamer mismatch volume. Formally, the $\Psi-$image of the set of the $\Phi$-metamers $\Psi(\Phi^{-1}(\Phi(x_0)))$ will be called *the metamer mismatch volume* induced by $x_0$.

Given two colour maps, $\Phi = (\varphi_1, ..., \varphi_n)$ and $\Psi = (\psi_1, ..., \psi_n)$, let us consider a map $\Upsilon : \mathcal{X} \to \mathbf{R}^{2n}$ such that $\Upsilon(x) = (\mathbf{z}; \mathbf{z}')$, where $\mathbf{z} = (\varphi_1(x), ..., \varphi_n(x))$ and $\mathbf{z}' = (\psi_1(x), ..., \psi_n(x))$. The corresponding object-colour solid $\Upsilon(\mathcal{X})$ is a convex subset in $\mathbf{R}^{2n}$. The $\Phi$-object-colour solid, $\Phi(\mathcal{X})$, is the $z-$projection of $\Upsilon(\mathcal{X})$:

$$\Phi(\mathcal{X}) = \{\mathbf{z} \in \mathbf{R}^n : (\mathbf{z}; \mathbf{z}') \in \Upsilon(\mathcal{X}), \quad \mathbf{z}' \in \mathbf{R}^n\}.$$

# We’re all used to using metamers

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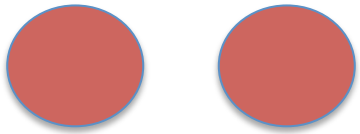
Situation is similar, but a little different with object surfaces…

# Illuminant-Induced Metamer Mismatching



Match

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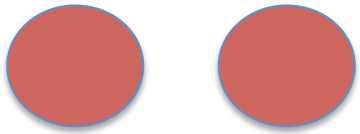
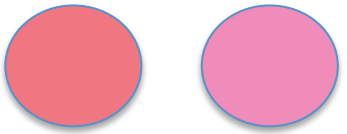
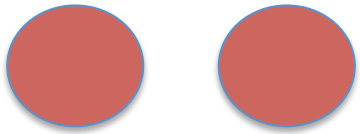




Match

Mismatch!

# Observer-Induced Metamer Mismatching

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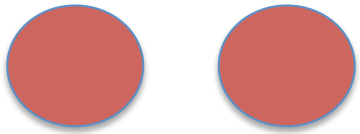

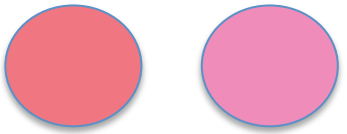

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$$\int_{visible} reflectance(\lambda)illuminant(\lambda)sensor(\lambda)d\lambda$$

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Whether changing one or the other or both,

it's a change of *colour mechanism*.

I'll say 'mechanism' for brevity.

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- The set of possible R'G'B' for second mechanism is the *metamer mismatch volume* for RGB
- Also often referred to as the 'metamer set'.

# MMV Methods with Restrictions

- Wyszecki “Evaluation of metameric colors” JOSA 1958
- Wyszecki & Stiles, “Color Science” 1967
- Finlayson & Morovic “Metamer sets” JOSA 2005.
- Urban & Grigat, “The Metamer Boundary Descriptor Method for Color Correction” JIST 2005

# Some Background Concepts

- Object colour solid
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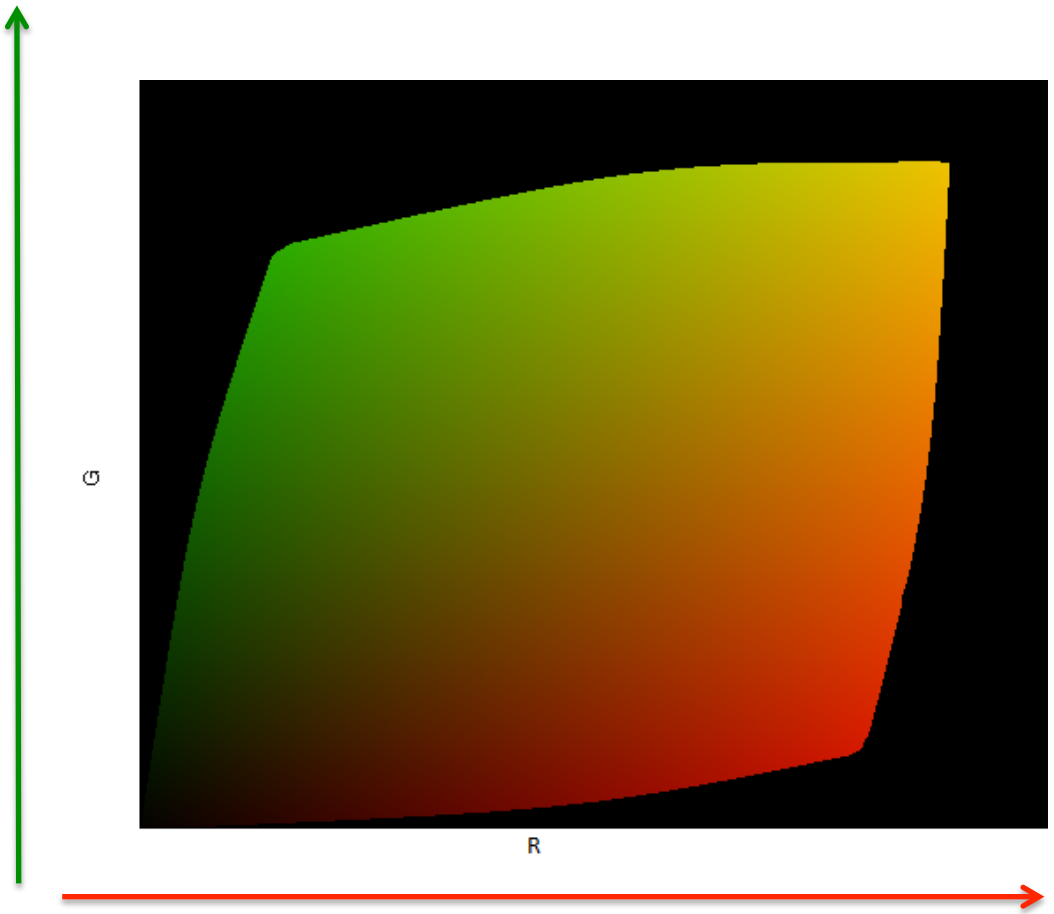
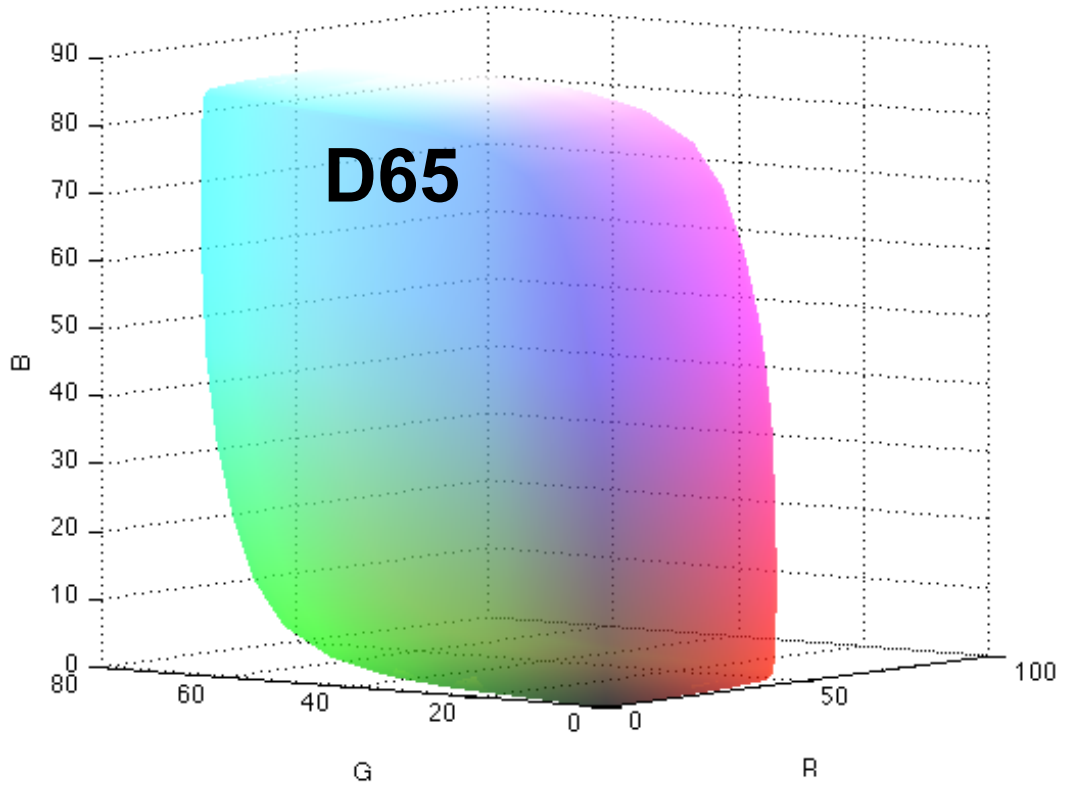
- Object colour solid
  - The set of all possible RGB that can occur
- Optimal reflectance spectra
  - Reflectances producing RGBs on colour-solid boundary

# Two-Channel Red-Green Colour Solid




Set of RGs from all possible surface reflectances under some illuminant.

# RGB Object Colour Solids




Set of RGBs from all possible surface reflectances under D65

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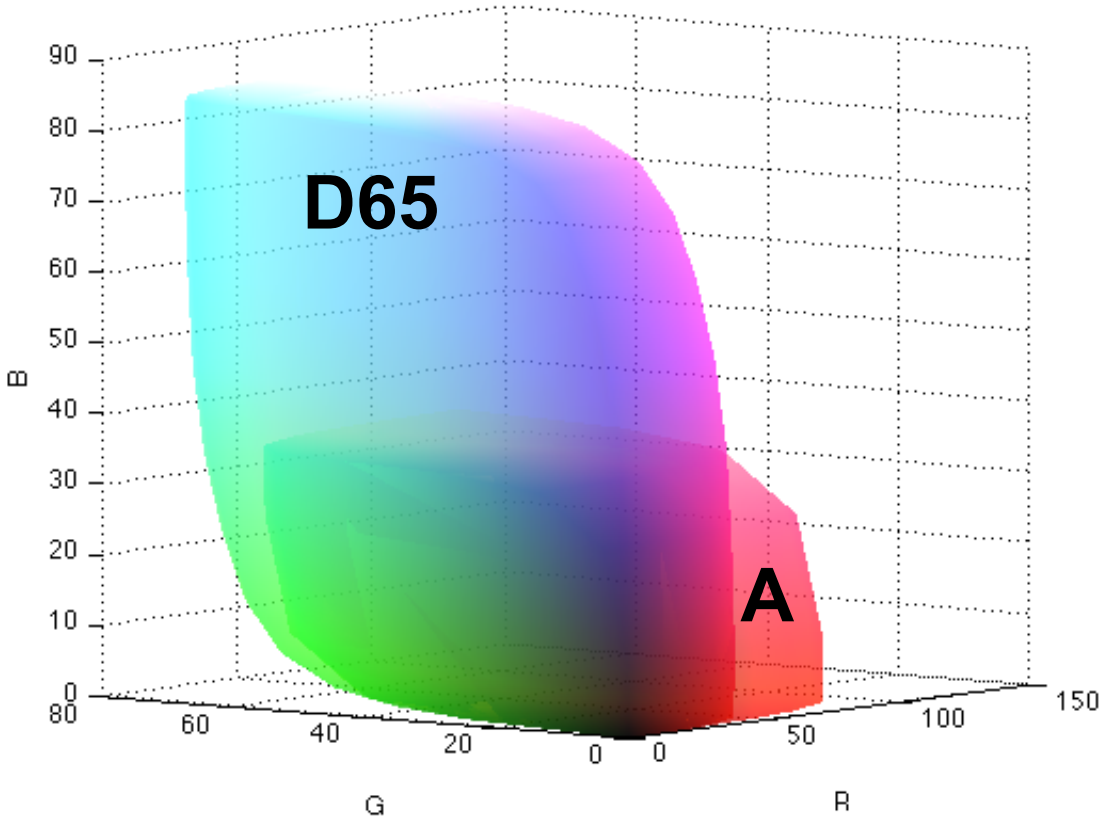
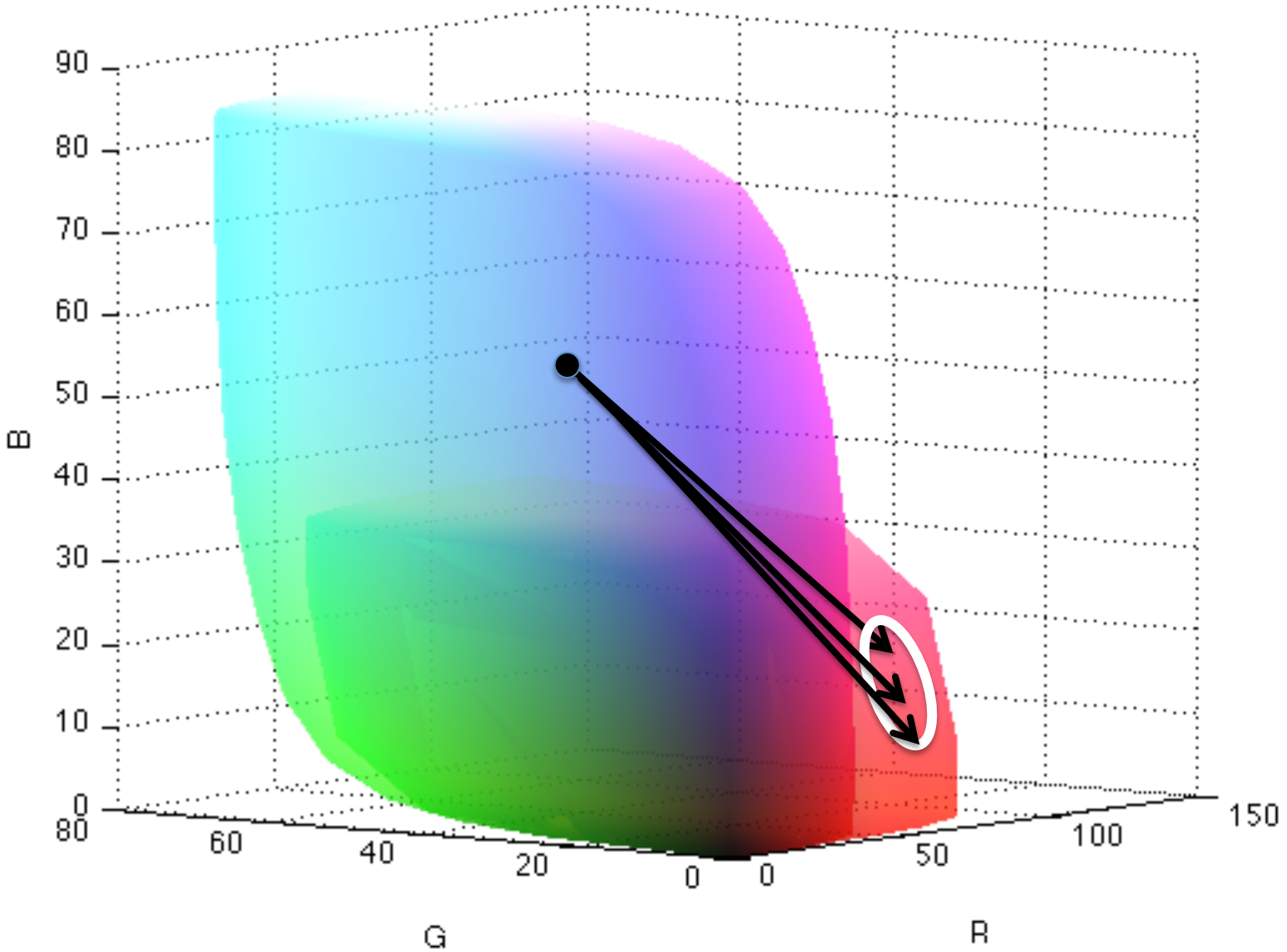



Set of RGBs from all possible surface reflectances under D65 versus A

# Metamer Mismatch Volume Example



# Multi-Spectral 6-Sensor Colour Solid

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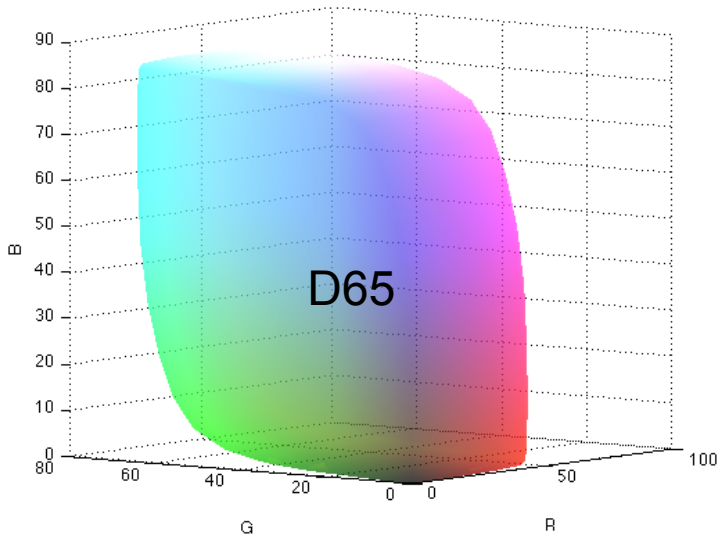
# Optimal Reflectances

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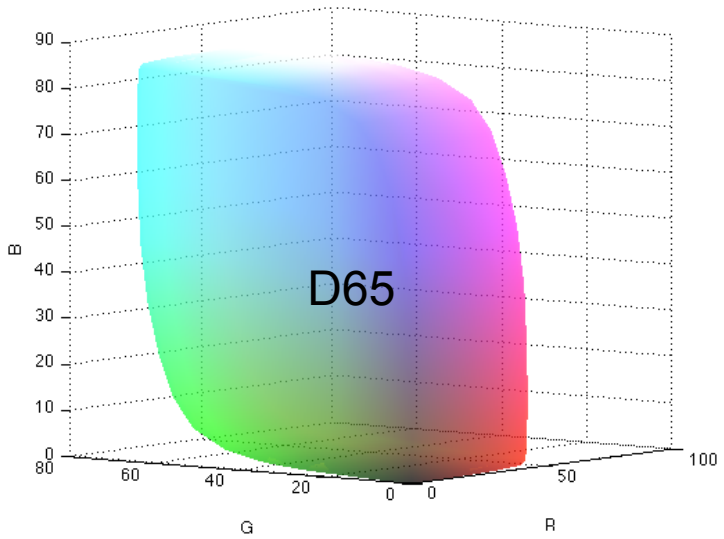


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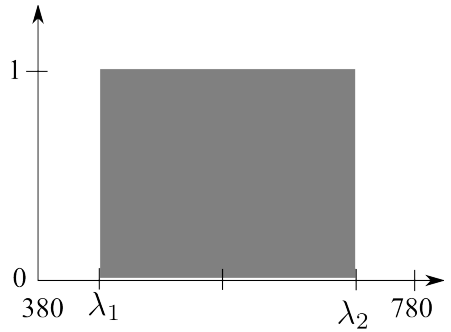
**Schrödinger's answer:** Only those that take values 0 or 1 with at most 2 transitions.

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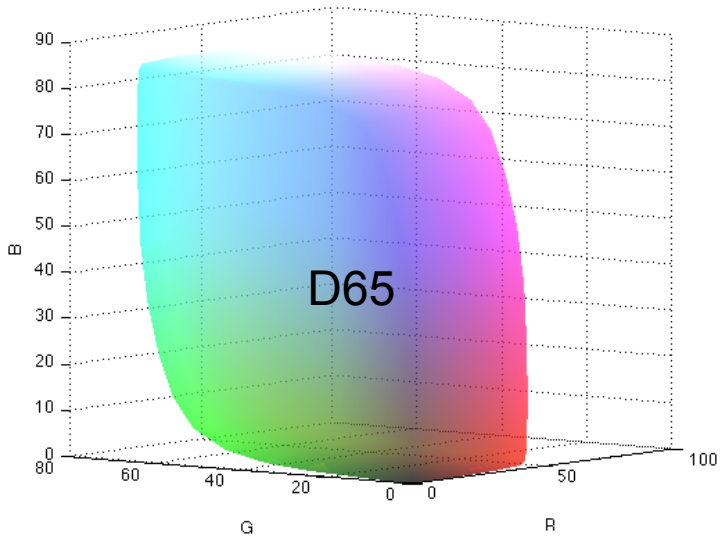


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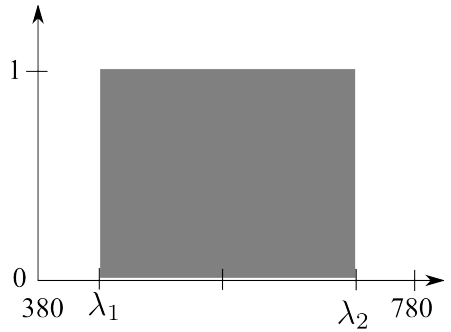


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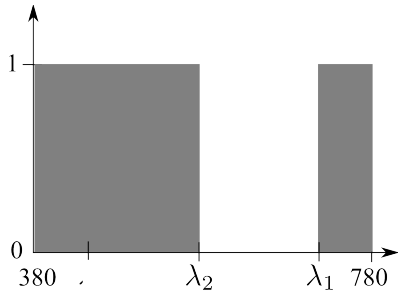


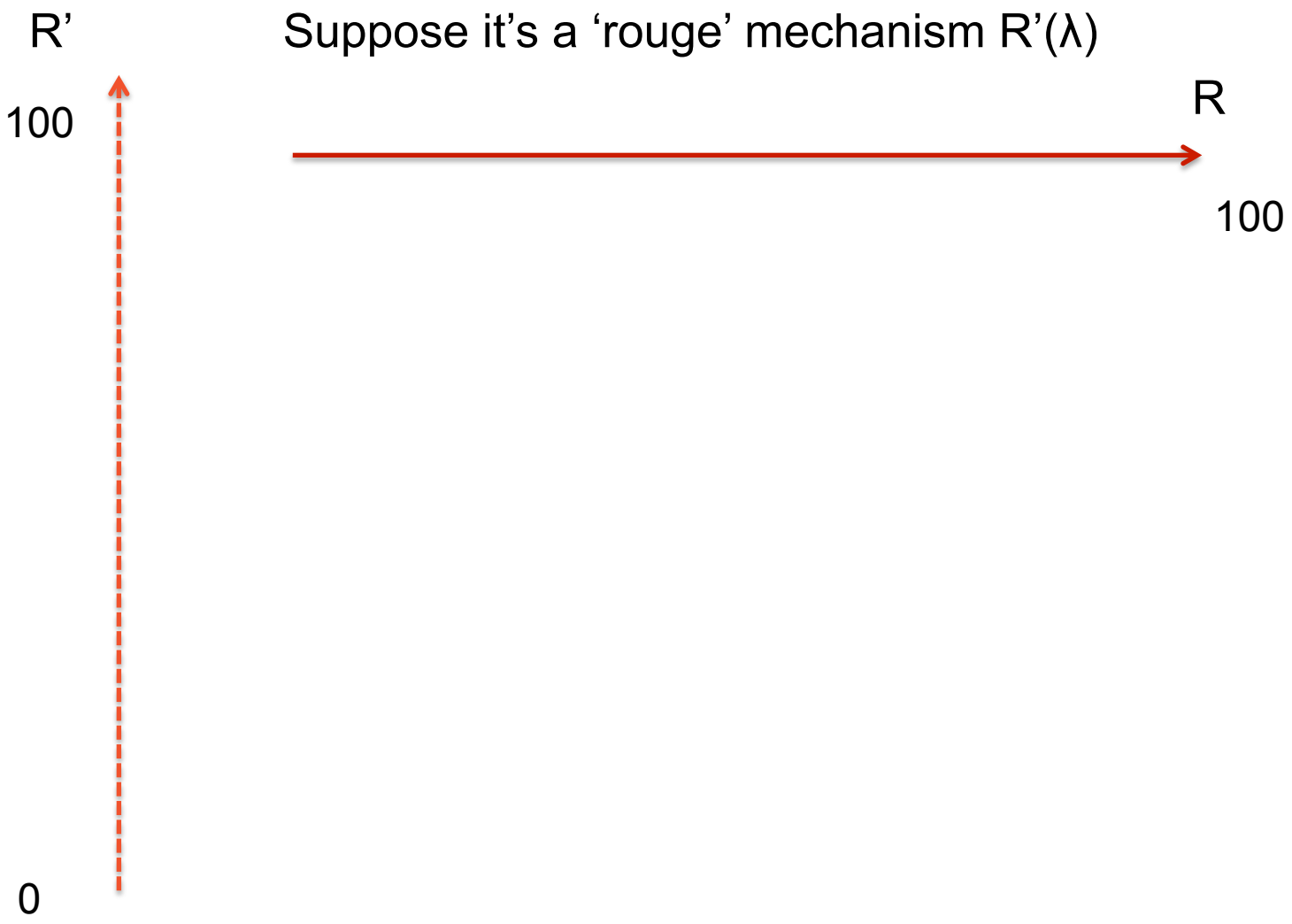

Type II Optimal Reflectance

# Metamer Mismatch Volume Calculation

# Example: Monochromatic Case

# Consider a Single-Channel ‘red’ Colour Mechanism

# Consider Also a Second Single-Channel Mechanism



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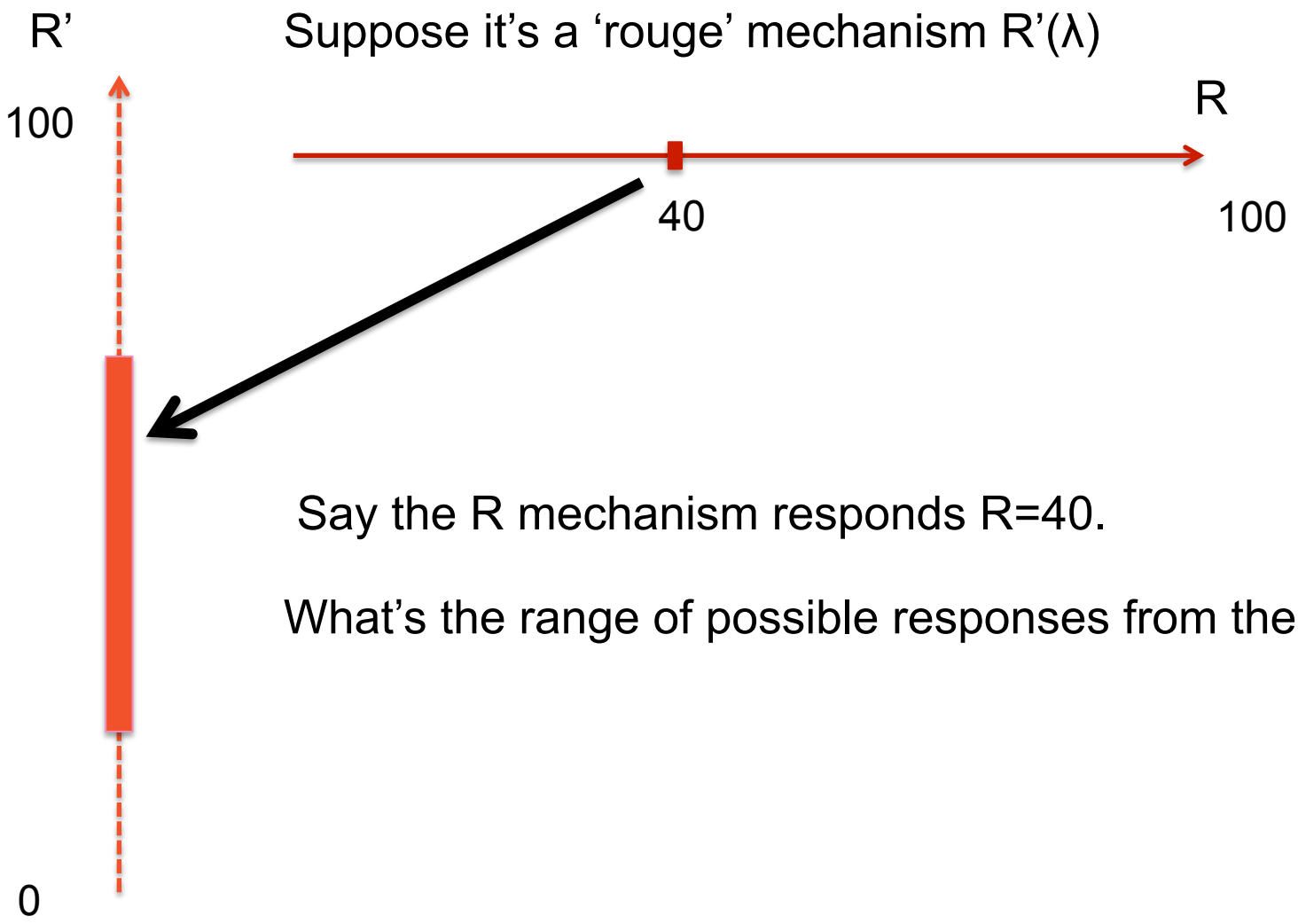


Say the R mechanism responds R=40.

What's the range of possible responses from the R' mechanism?

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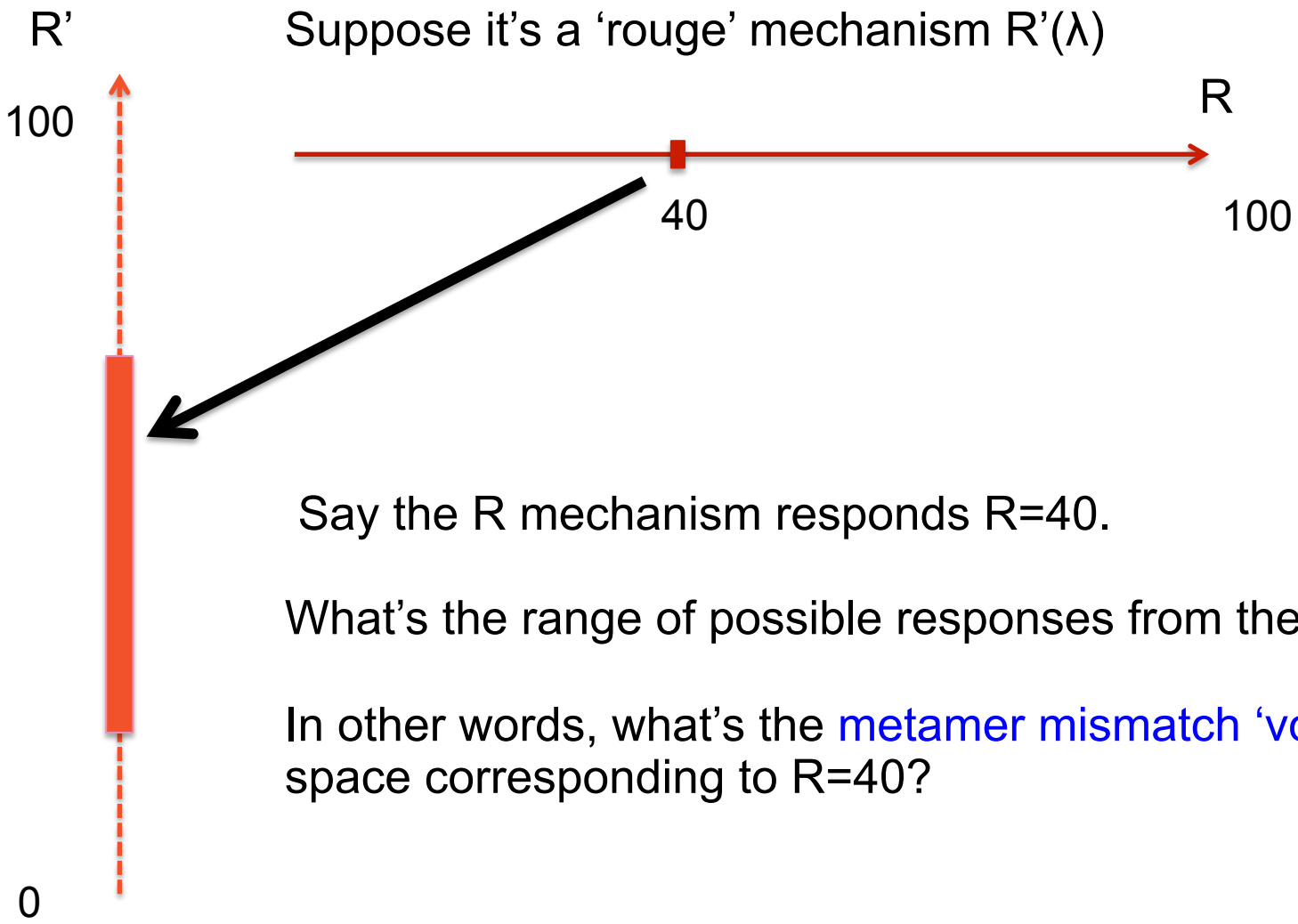


Say the R mechanism responds R=40.

What's the range of possible responses from the R' mechanism?

In other words, what's the metamer mismatch 'volume' in R' space corresponding to R=40?

# Calculating the MMV

Consider all possible pairs of responses (R, R') that can ever arise

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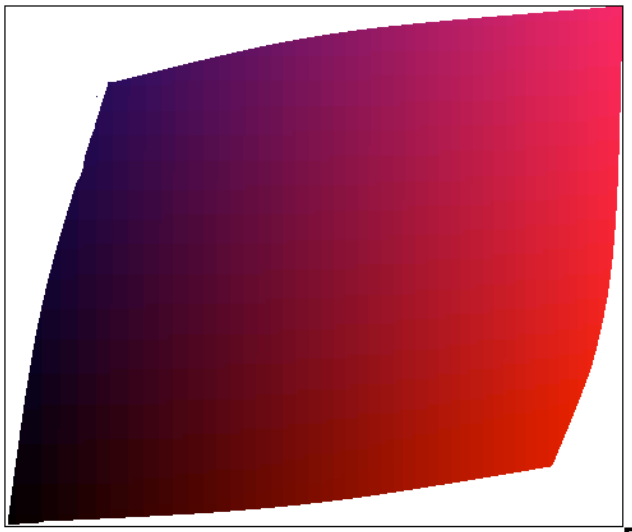
It's the object colour solid for colour mechanisms R and R'.

R'



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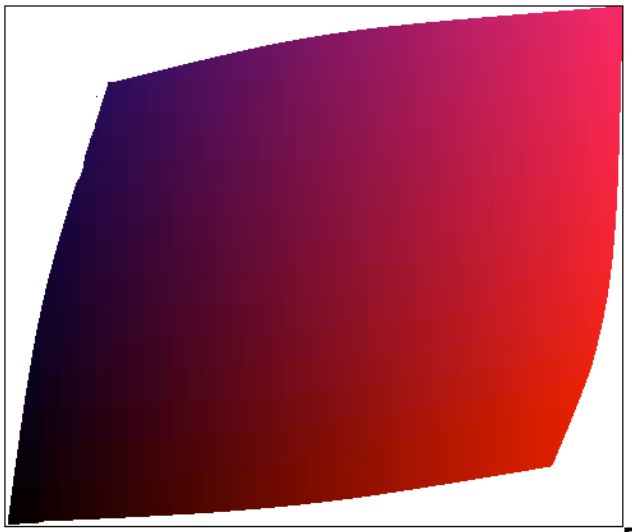
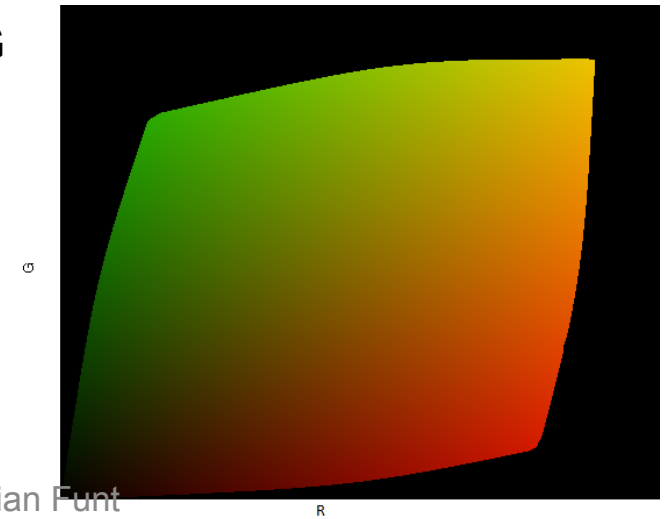
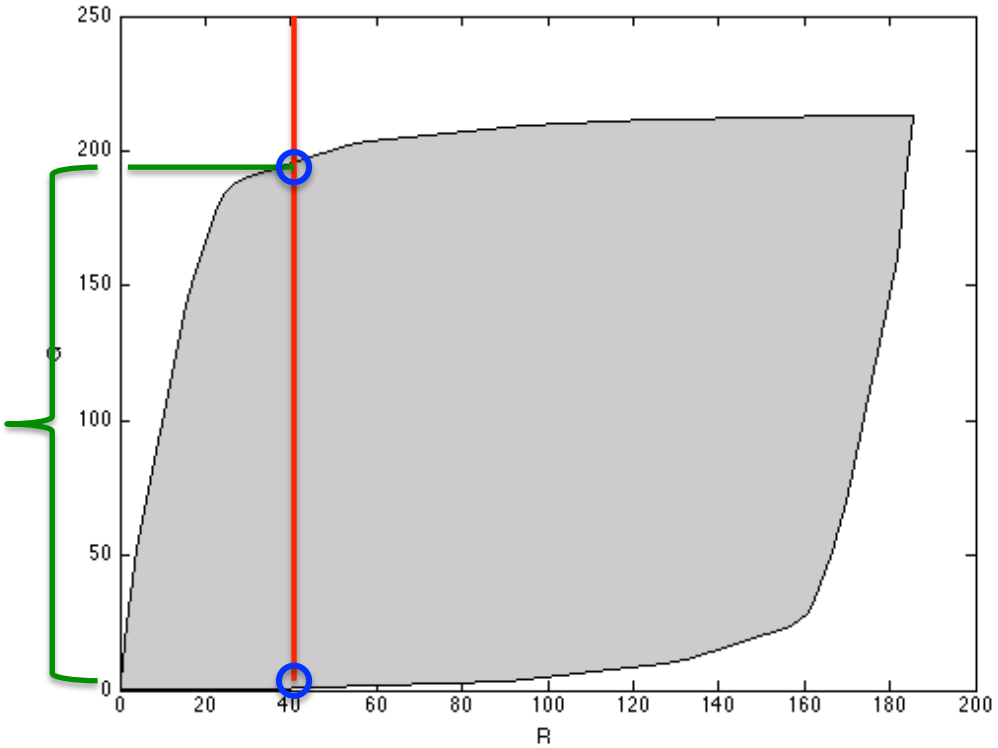
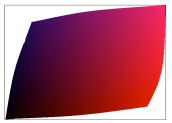


# Cross-Section of Object Colour Solid



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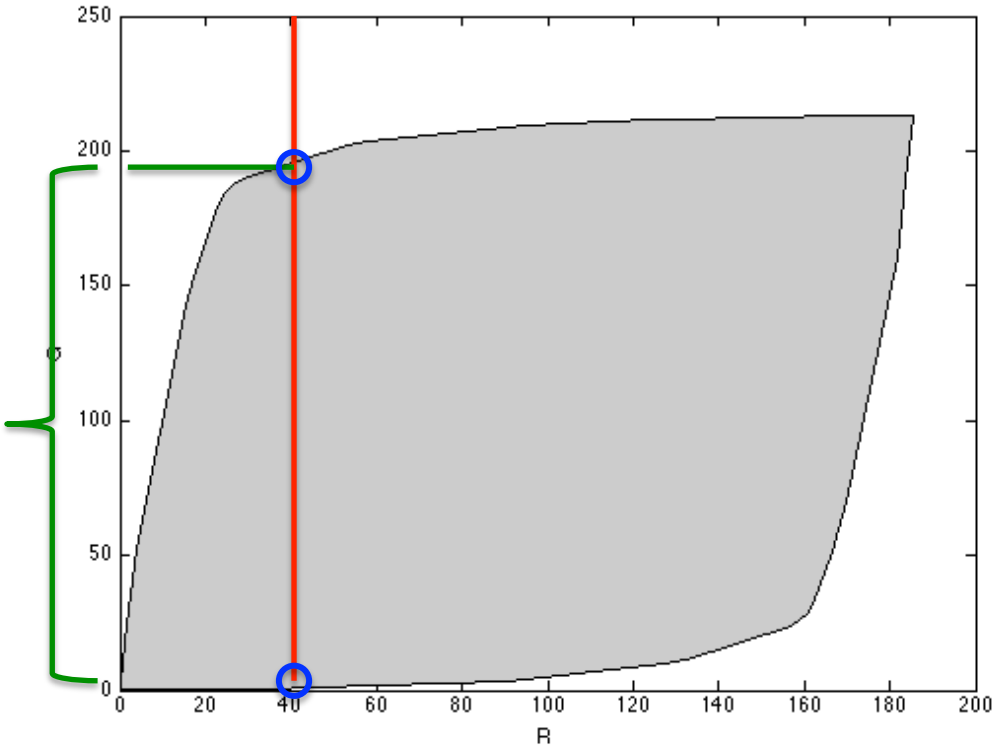


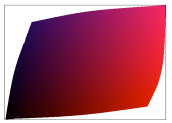

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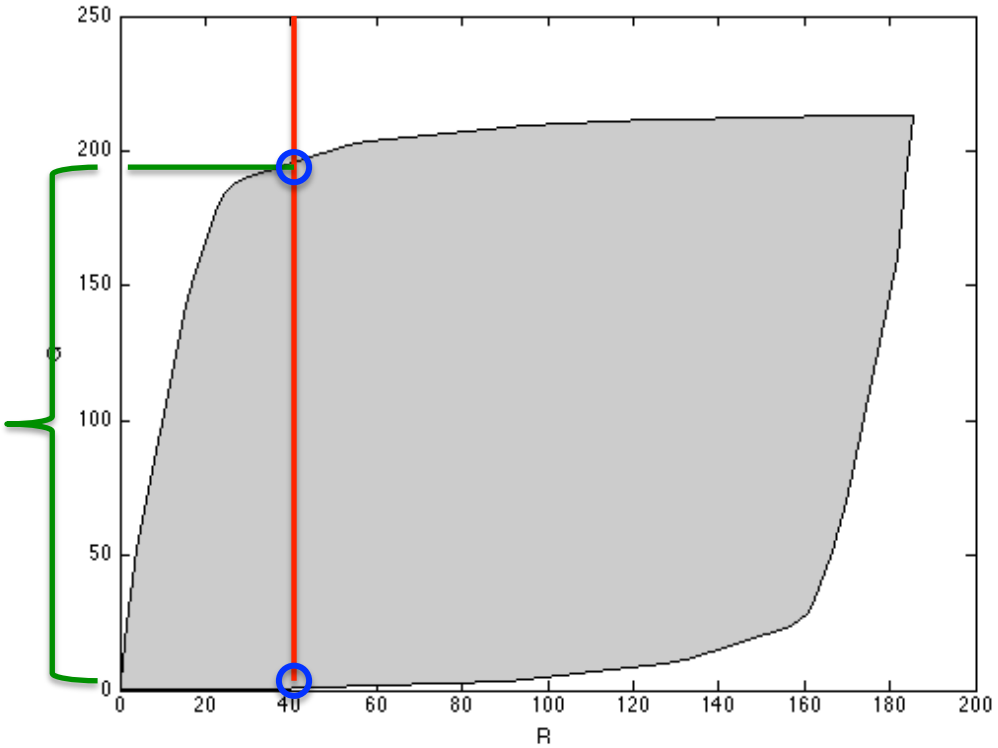


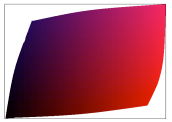

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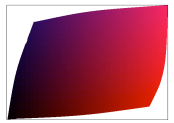

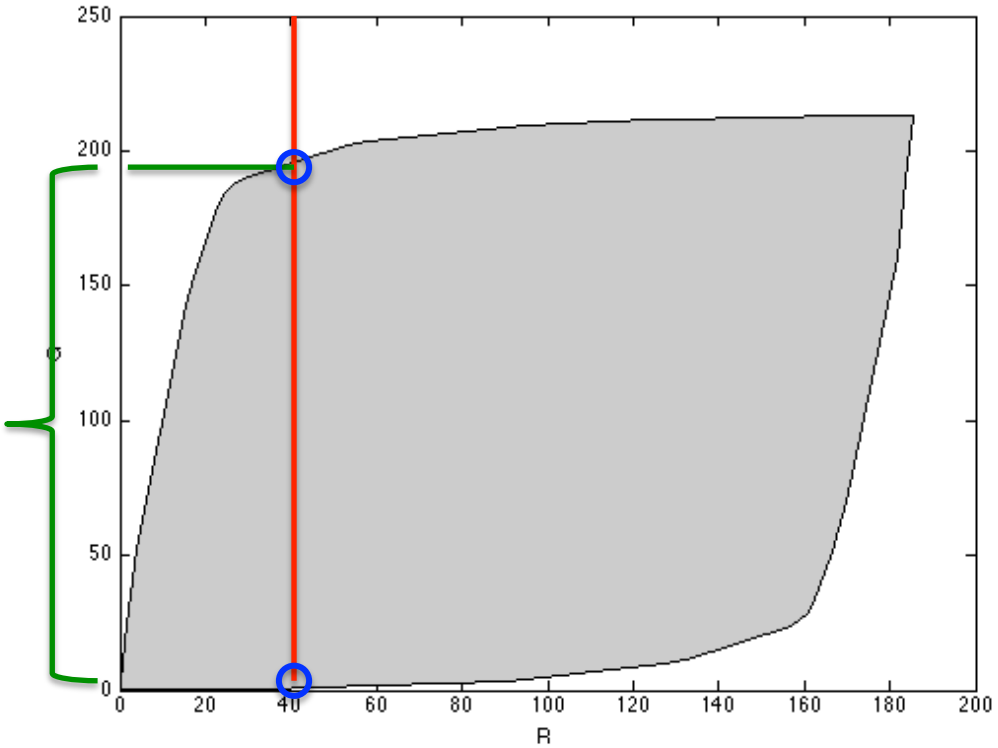


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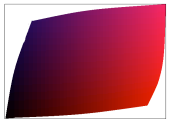

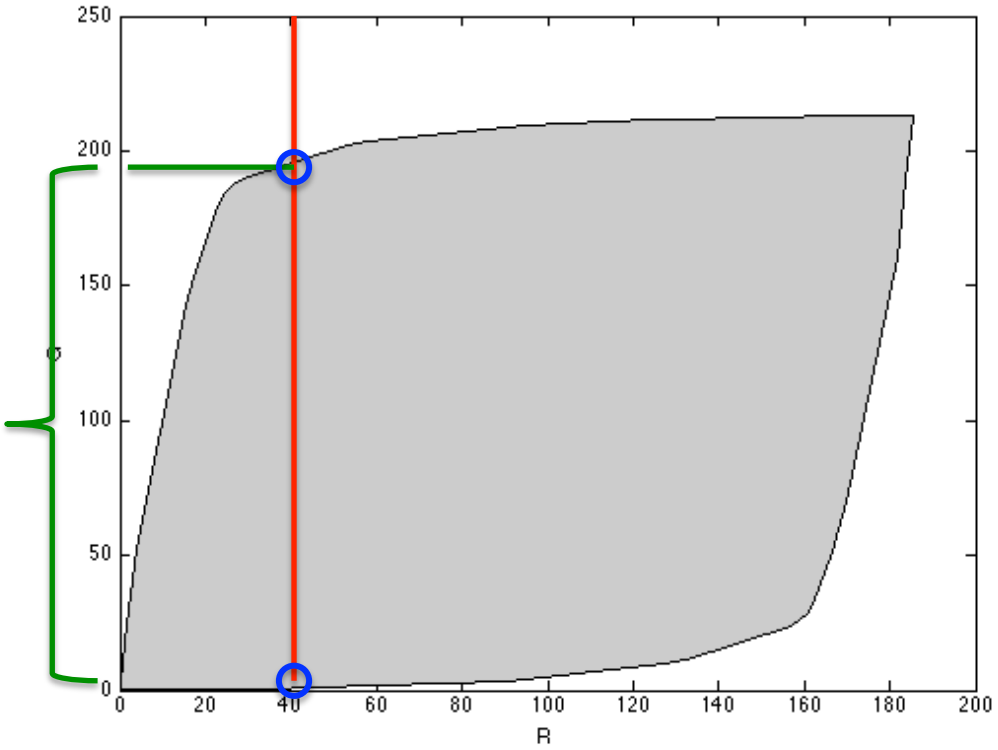


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MMV boundary:
intersection of Object Colour Solid Boundary with the R=40 cross-section

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- Intersect boundary of Colour Solid with its R=40 cross-section
- That intersection is 2 points in this 1D case.
  They define the boundary of the MMV (i.e., range of R')

# Dichromatic Metamer Mismatch Volumes

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  - Each is now two-channel

    [R(λ) and G(λ)]  and [R’(λ) and G’(λ)]

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  - All 4-tuples (R, G, R', G') arising in response to all reflectances
- Metamer mismatch volume boundary
  - Intersection of cross-section of 4D solid and solid's boundary

# Dichromatic: 2*2D = 4D case

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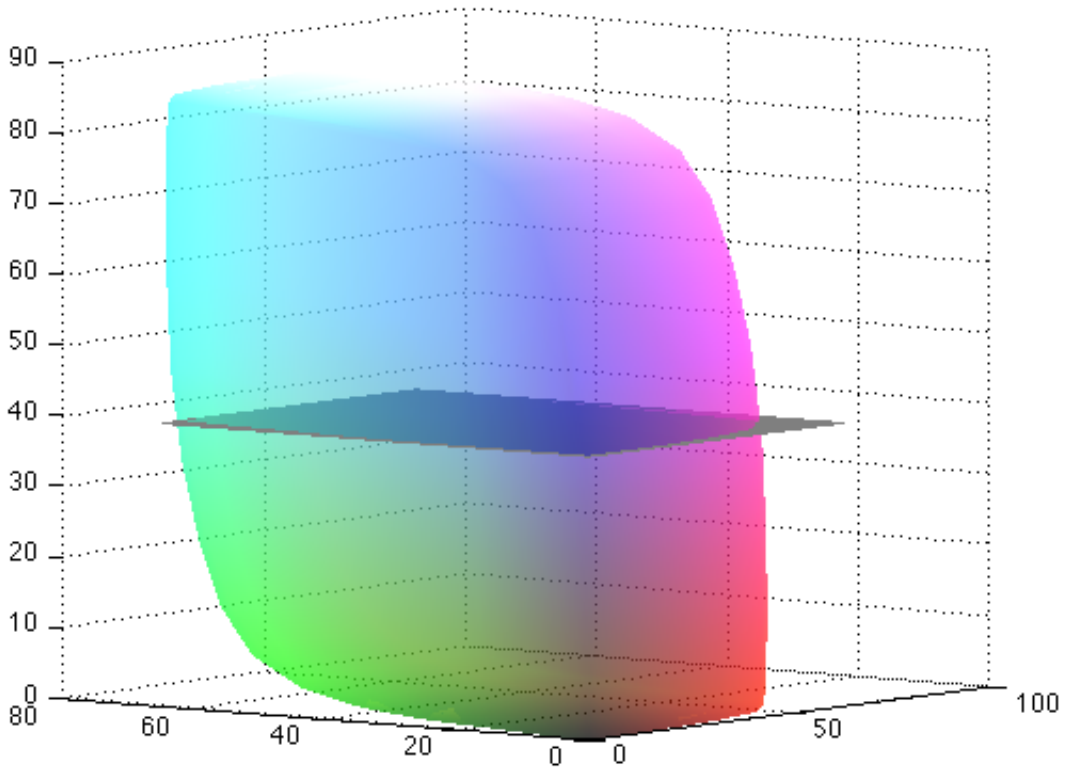
- Colour Solid Volume is 4D
  - Suppose we want MMV for R=20, G=50
- Cross-section is all 4-tuples (20, 50, R', G' )
  - It's a 2D (i.e., planar cross-section)
  - Result is a 2D "volume"

# 3D Cross-Section Example (Can't display 4D volume)




Analogous to metamer volume problem

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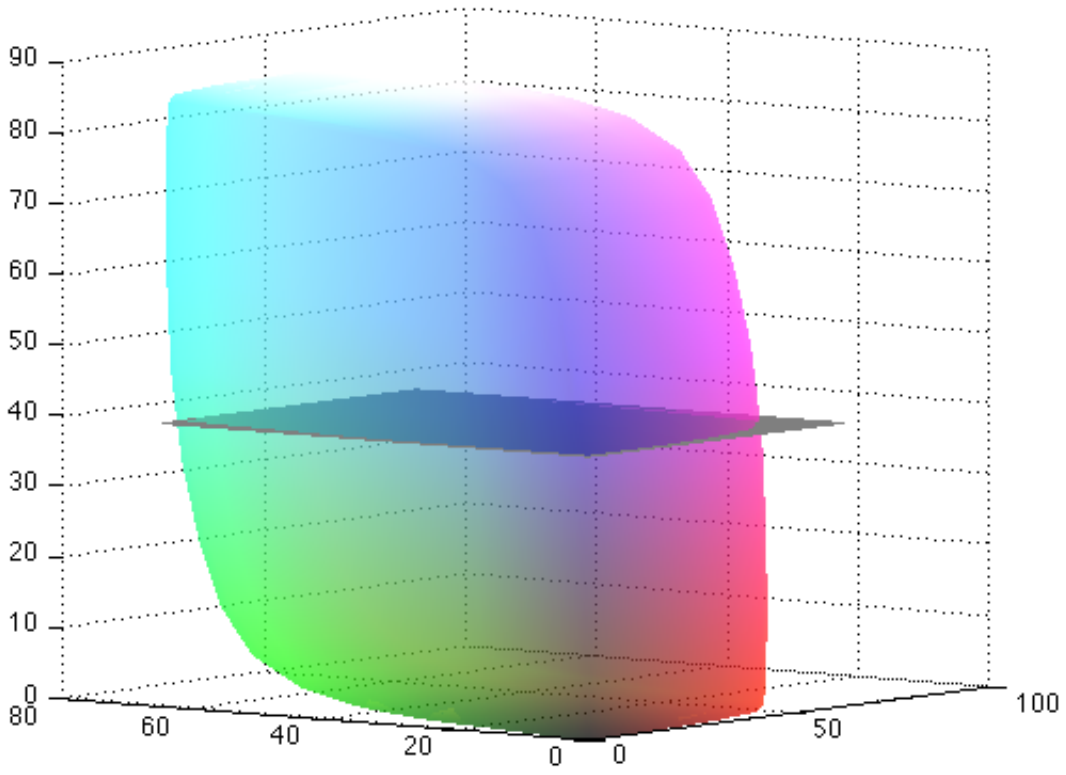


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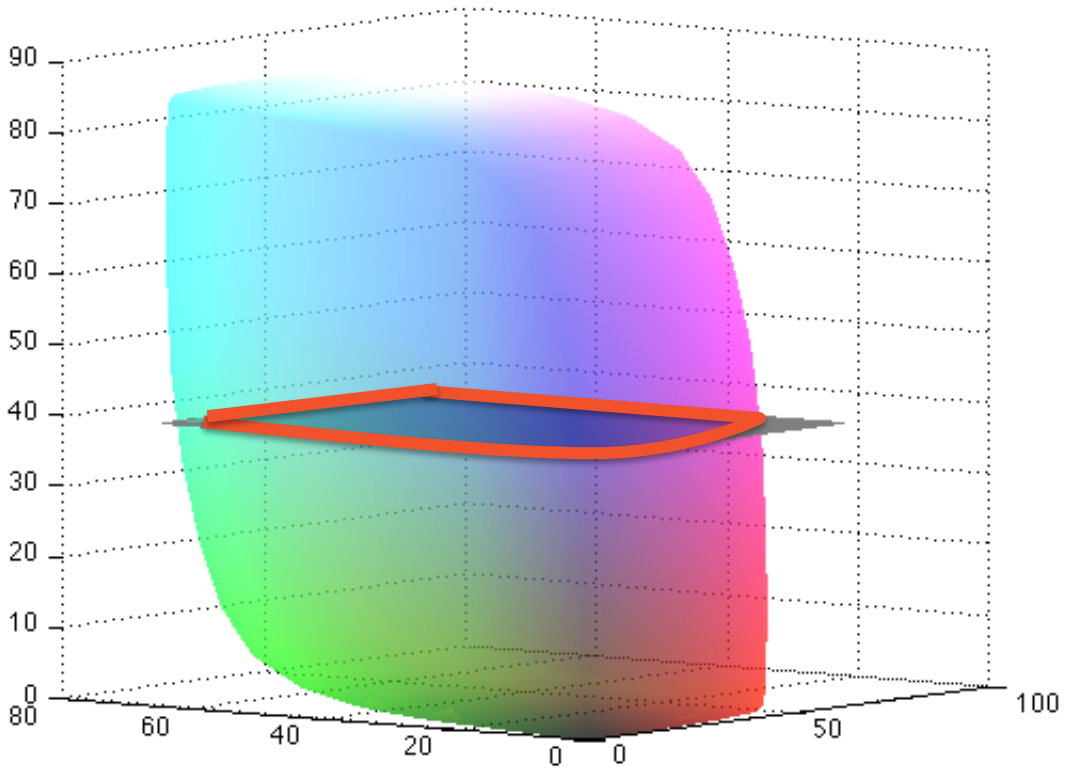
It's all (R,G) on the B=40 plane.

"Volume" here is a plane.

# 3D Cross-Section Example (Can't display 4D volume)




Analogous to metamer volume problem

What's the range of (R, G, B)

such that B = 40?

It's all (R,G) on the B=40 plane.

"Volume" here is a plane.

Boundary is a curve in the plane.

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- Metamer mismatch volume boundary
  - Defined by intersection of cross-section with 6D solid

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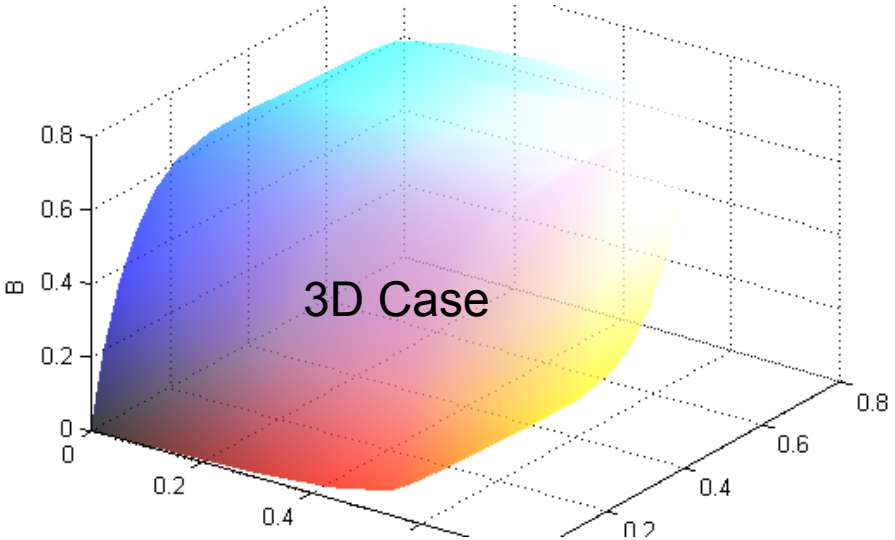
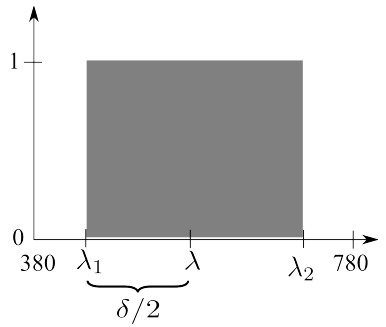
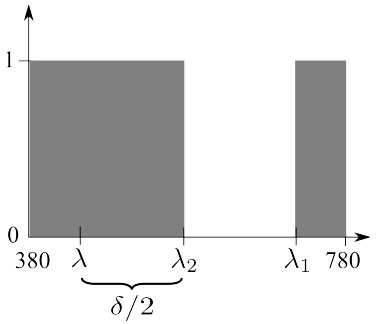
  - Boundary is a surface in 3D

# So what are the reflectances on the boundary of 6D Object Colour Solid?

# Boundary of 6D Object Colour Solid?



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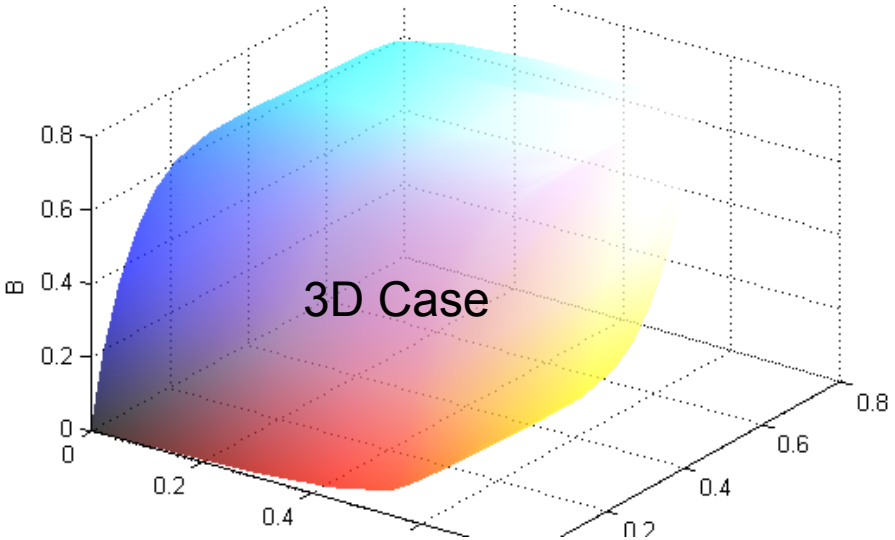


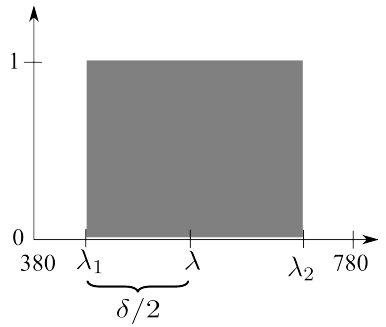


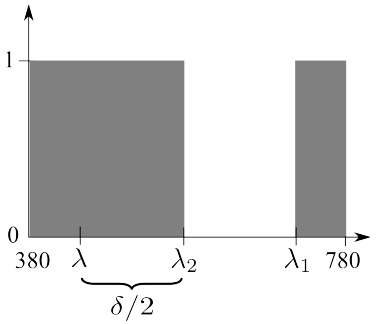


**6D Colour Solid Boundary is analogous to 3D color solid boundary**

What are the optimal reflectances?

For 3D, they were 2-transition 0-1 functions

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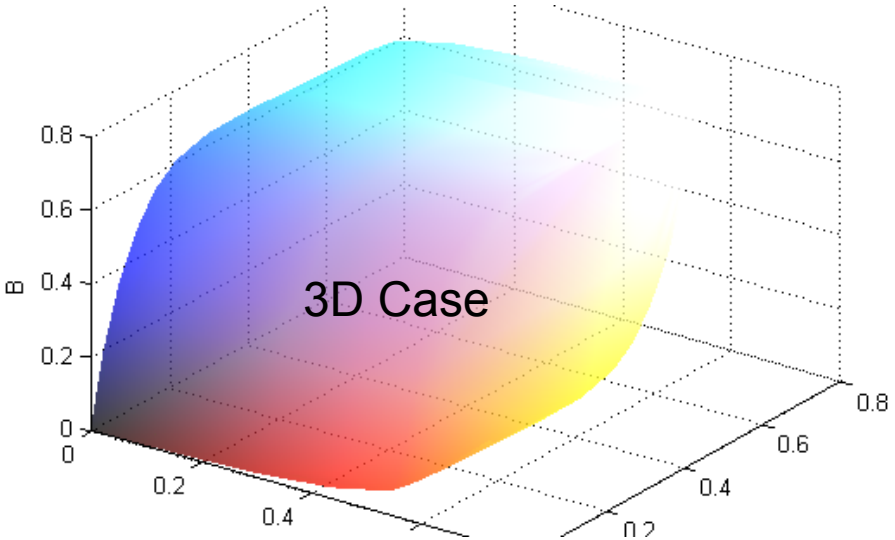


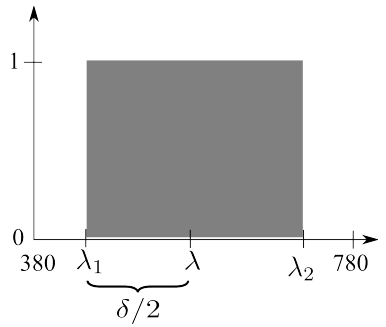


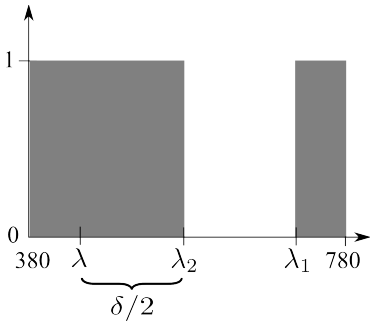


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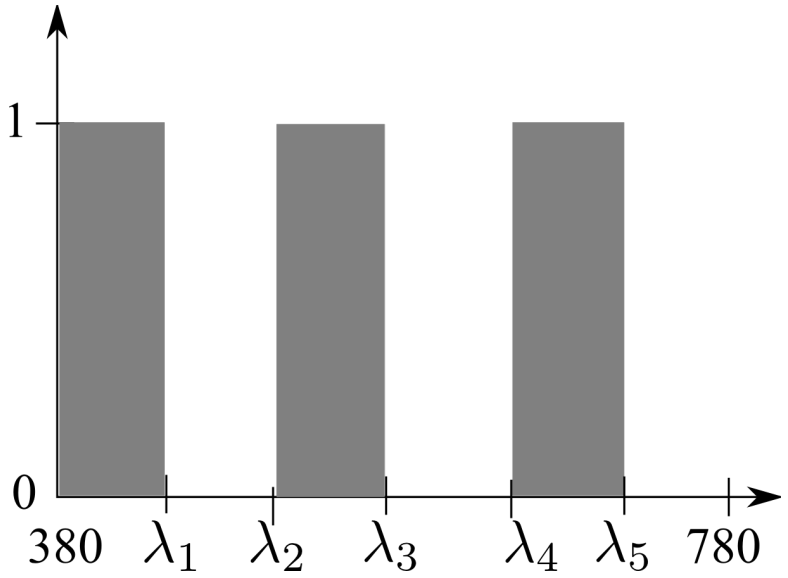
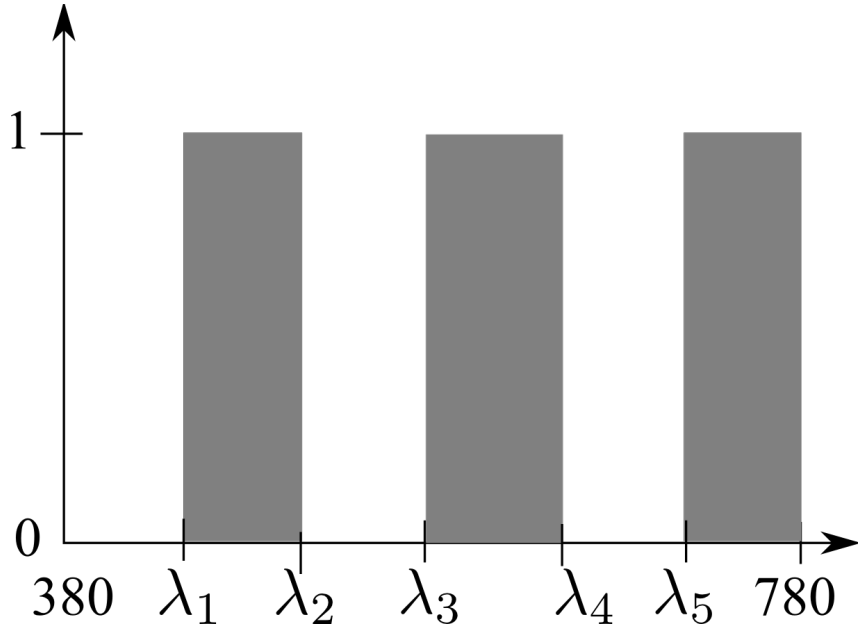
**For 6D, they are 5-transition 0-1 functions**

# 5-transition optimal reflectances




Type I and Type II 5-transition step functions

Define boundary of 6-dimensional object color solid.

# Trichromatic MMV Boundary

Looking for intersection of 6D solid boundary with the cross-section through it defined by (R,G,B) = (20,50,40) (for example).

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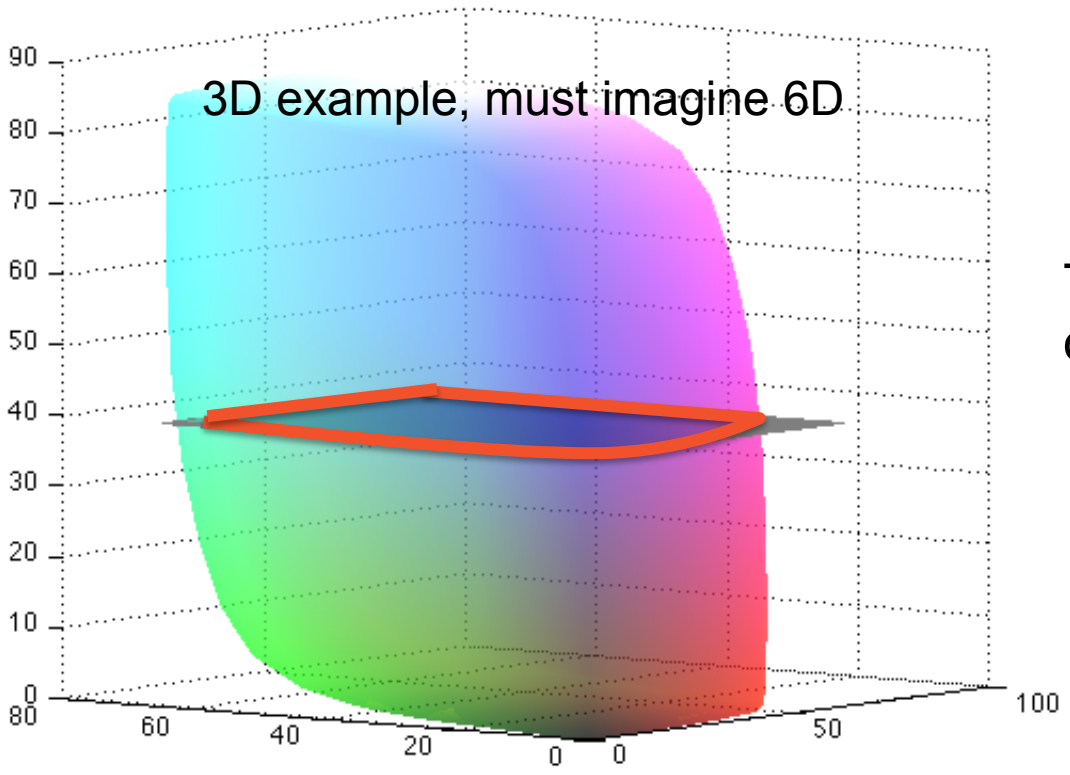


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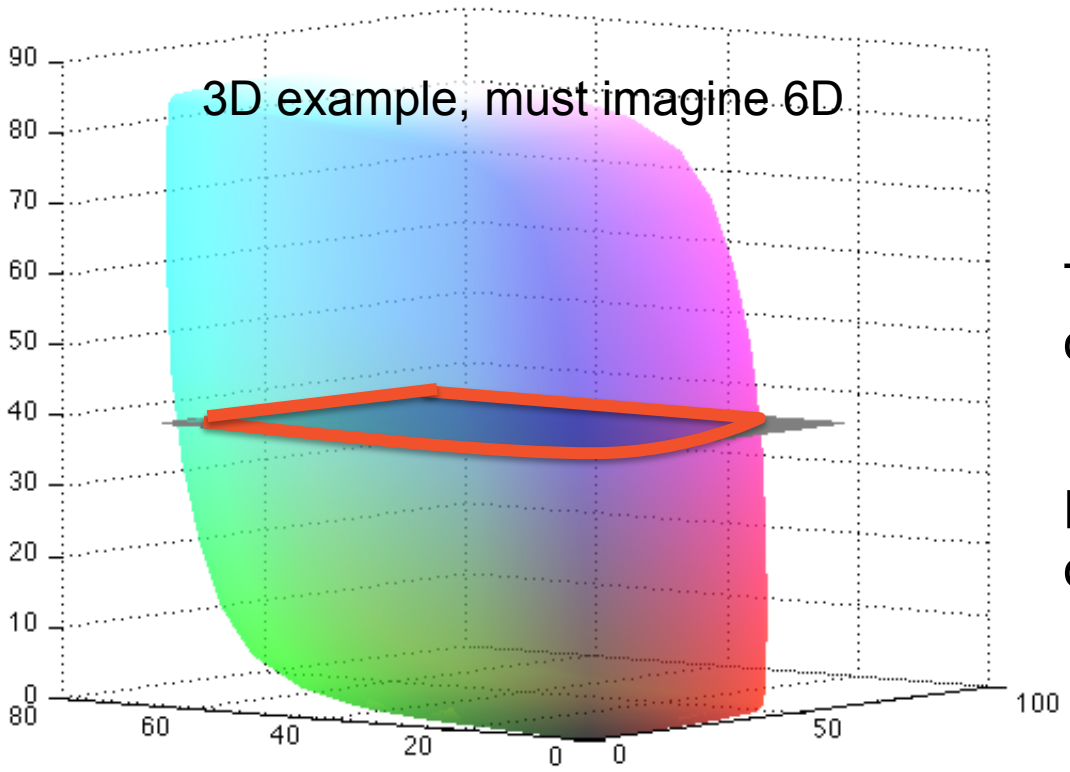


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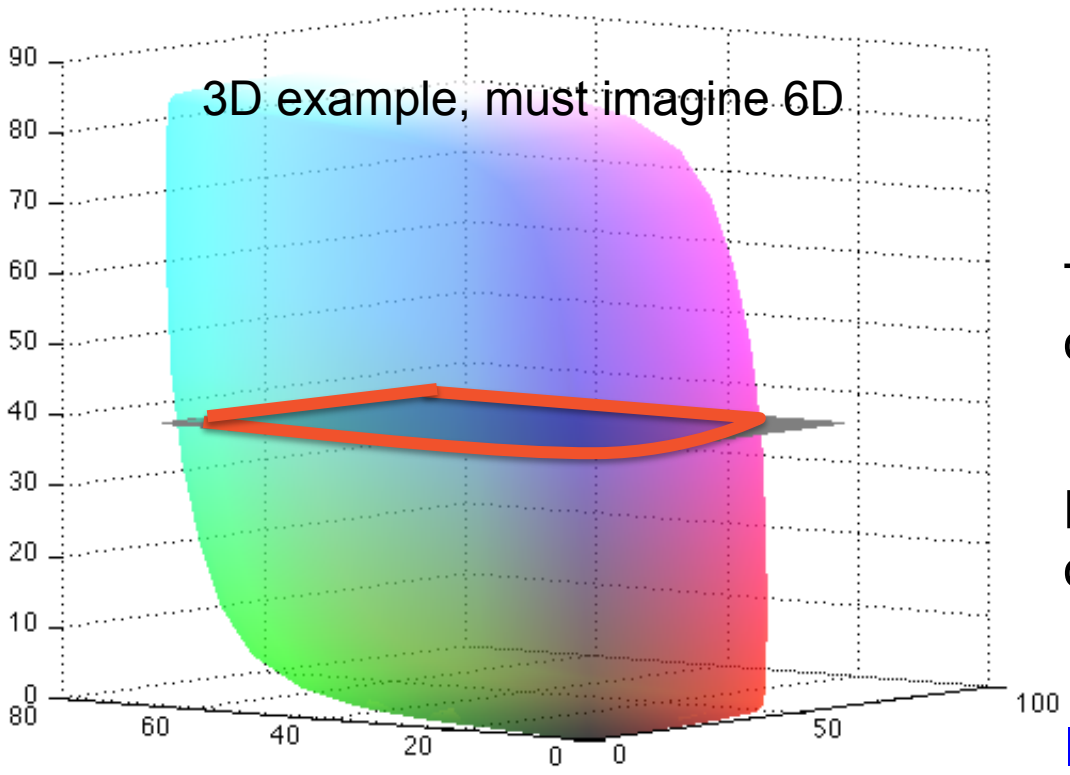


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Points on the boundary are 5-transition optimal reflectances.

In other words, the 5 transition points are 5 unknowns.

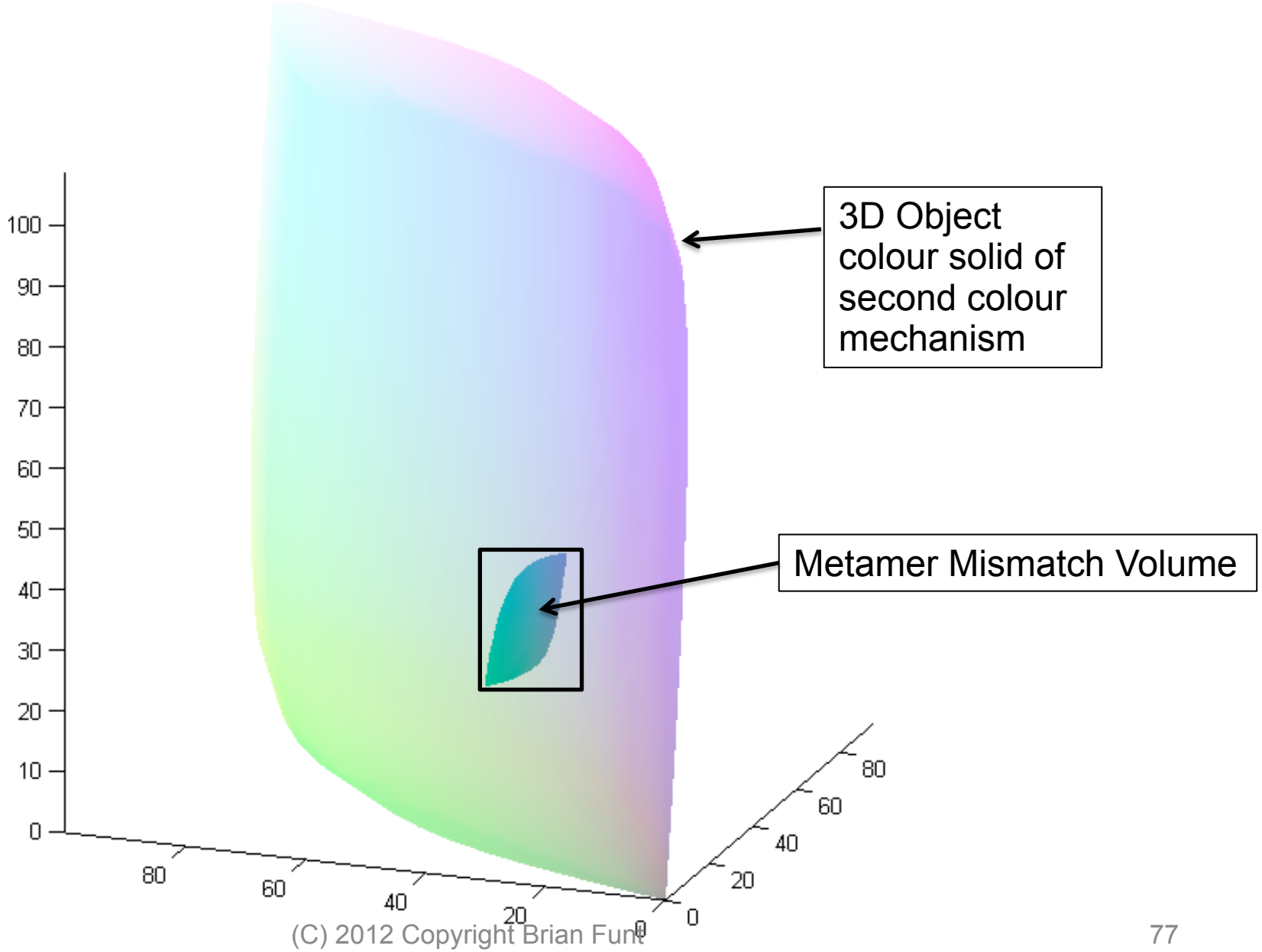
# Six Equations

First three constrain the solution to be a metamer to (20,40,50)

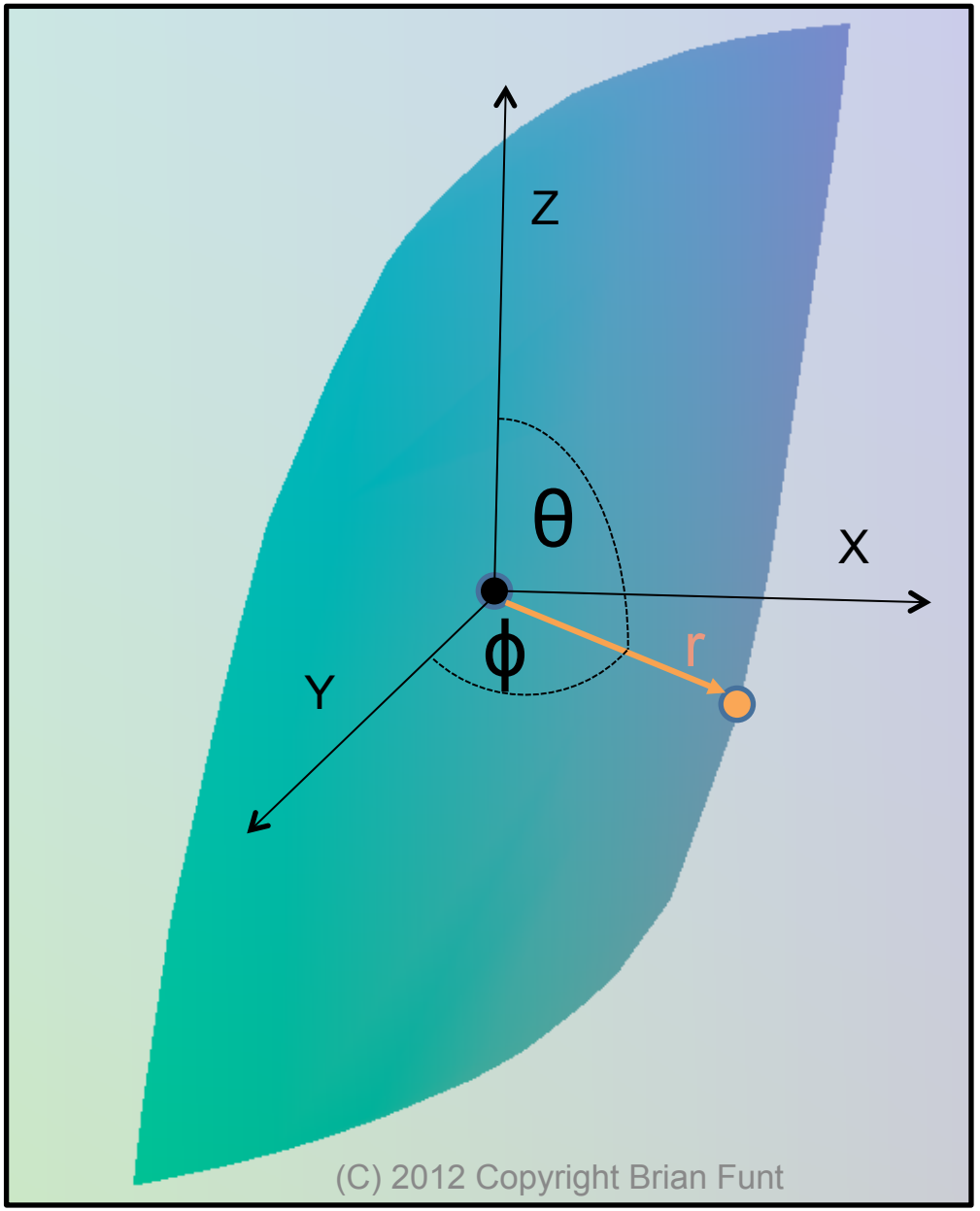
$$\int R(opt(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5)d\lambda = 20$$
$$\int G(opt(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5)d\lambda = 40$$
$$\int B(opt(\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5)d\lambda = 50$$

These equations define the MMV boundary implicitly

# Introduce Spherical Coordinates in MMV



# Expanded View of MMV



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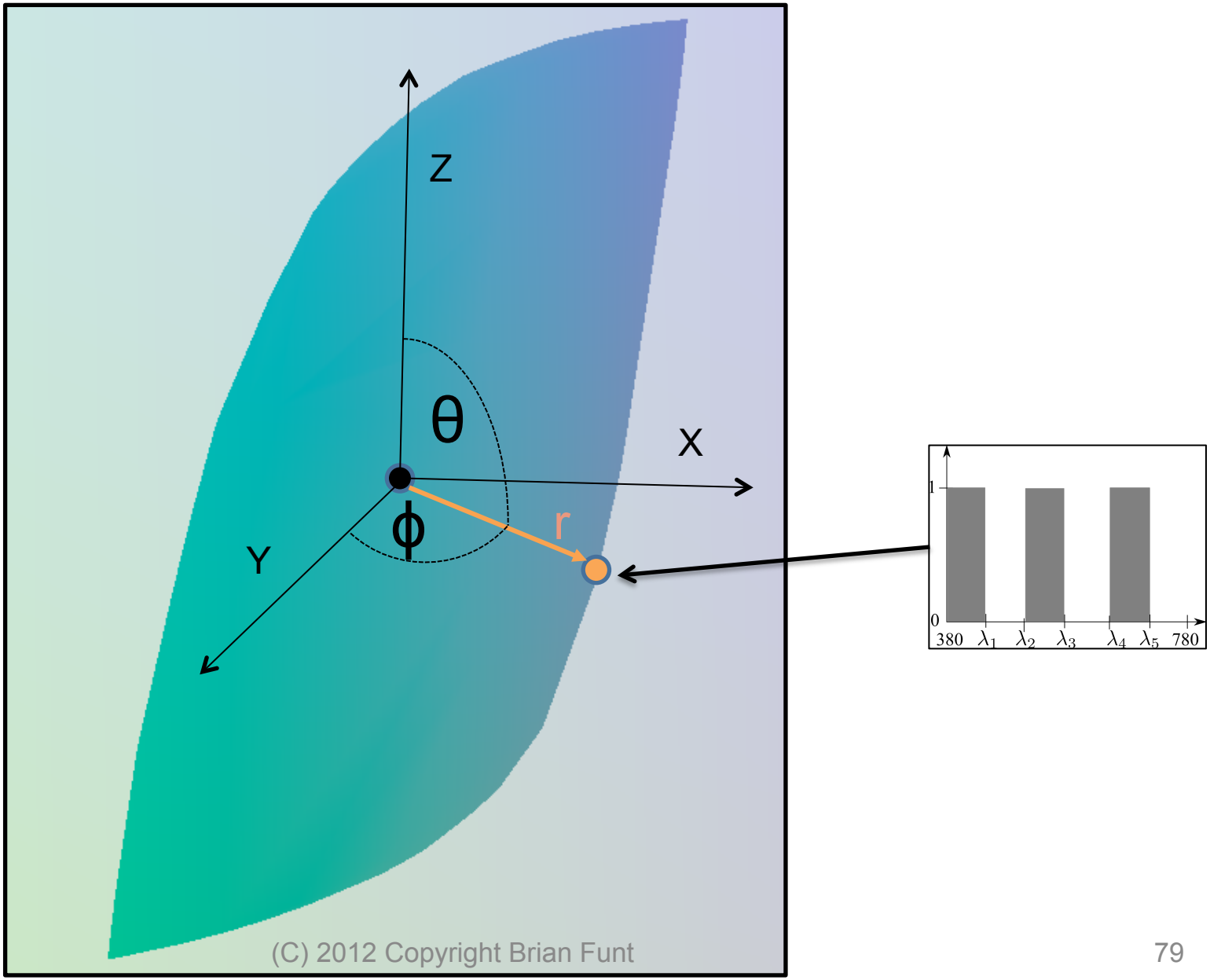

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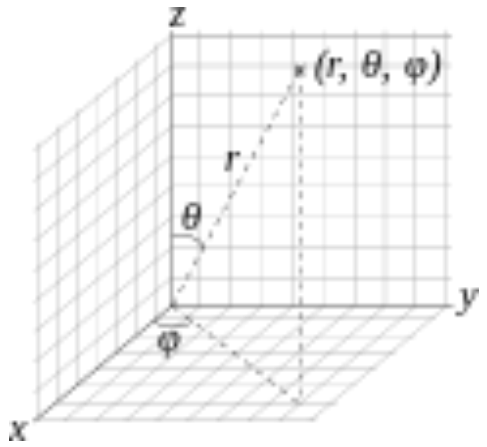


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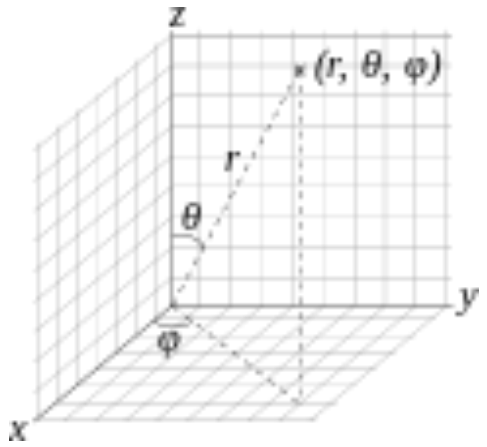


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Specify direction (θ,ϕ) and solve for r, $\lambda_1$,.., $\lambda_5$

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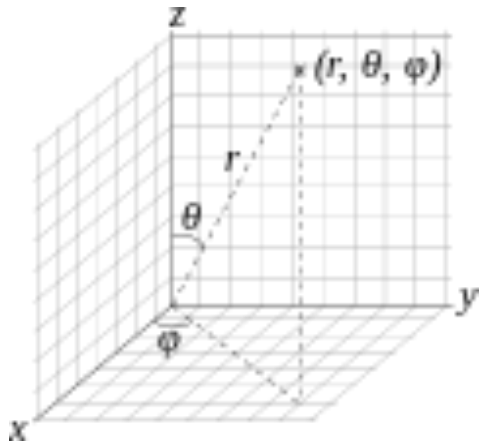


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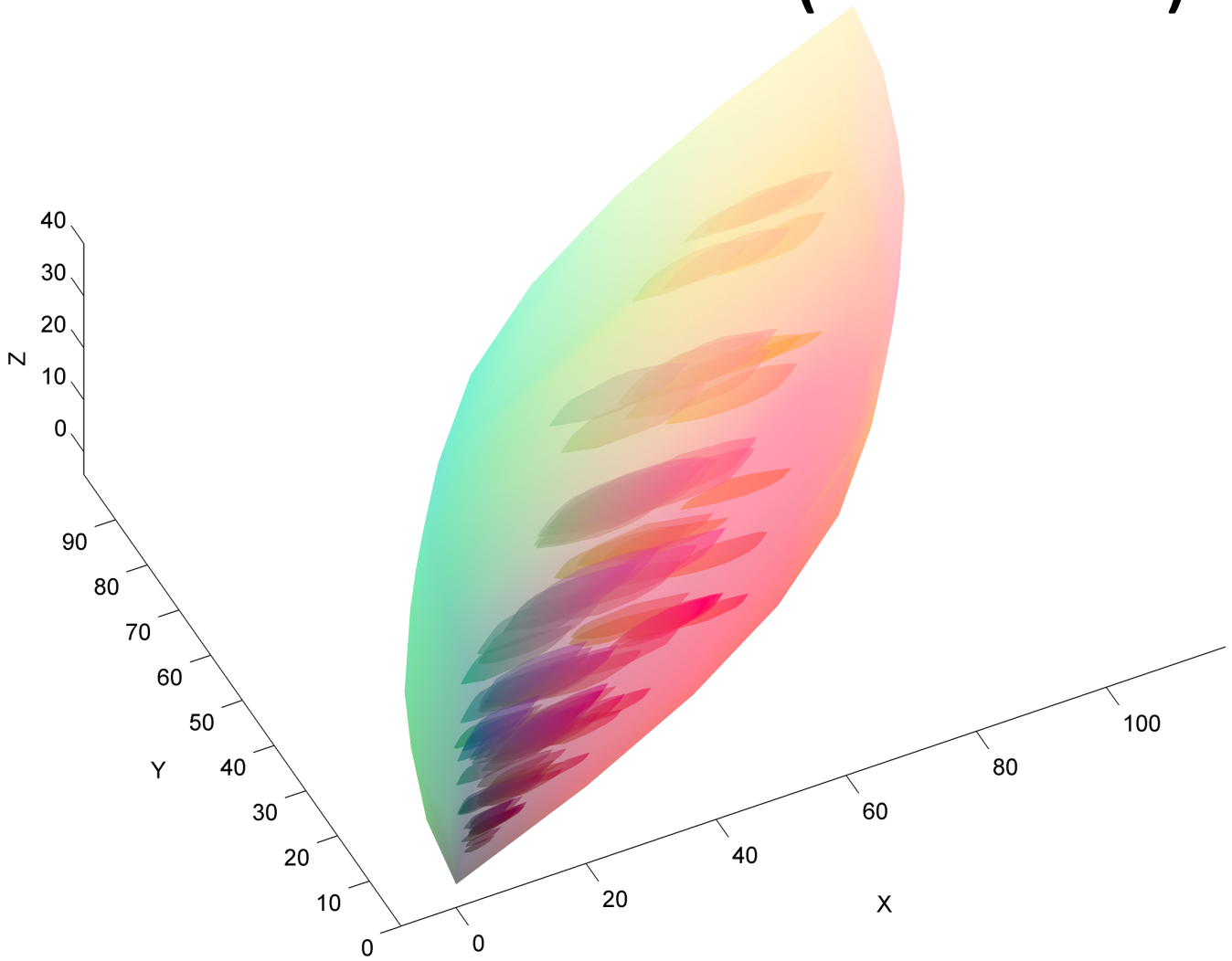
MMV described as r(θ,ϕ)

# MMVs of 100 Munsells (D65 to A)

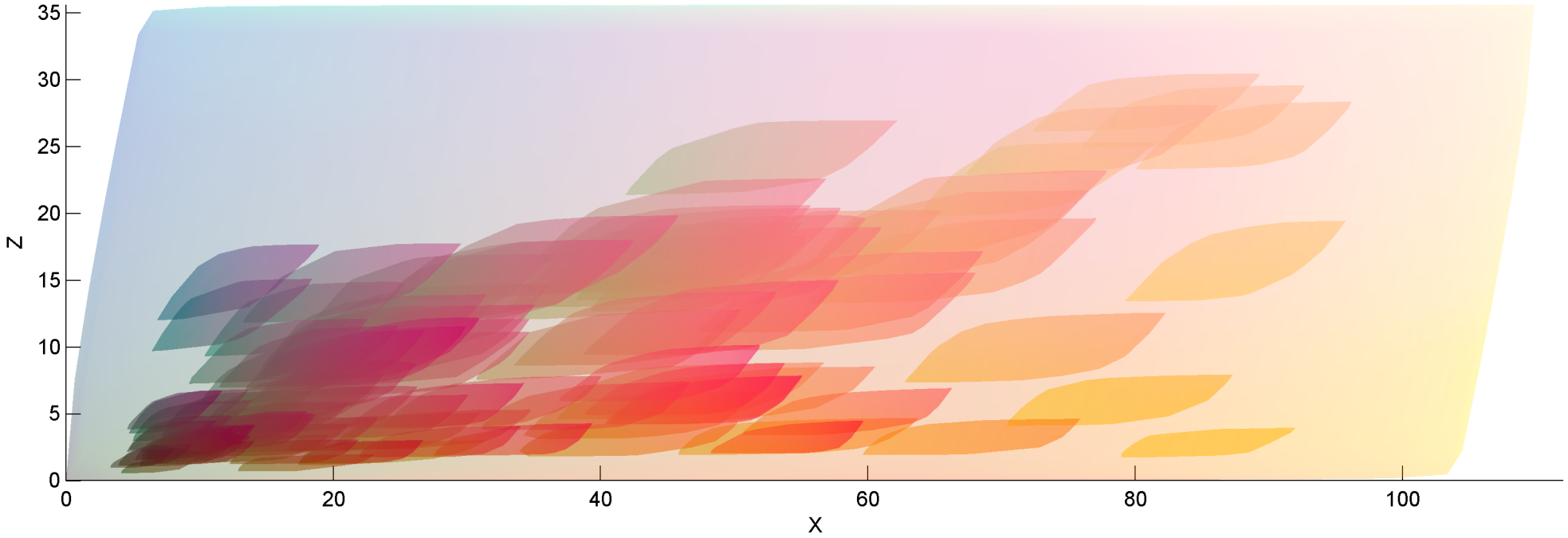



Biggest volumes are near the achromatic axis

Smaller towards the object colour solid boundary

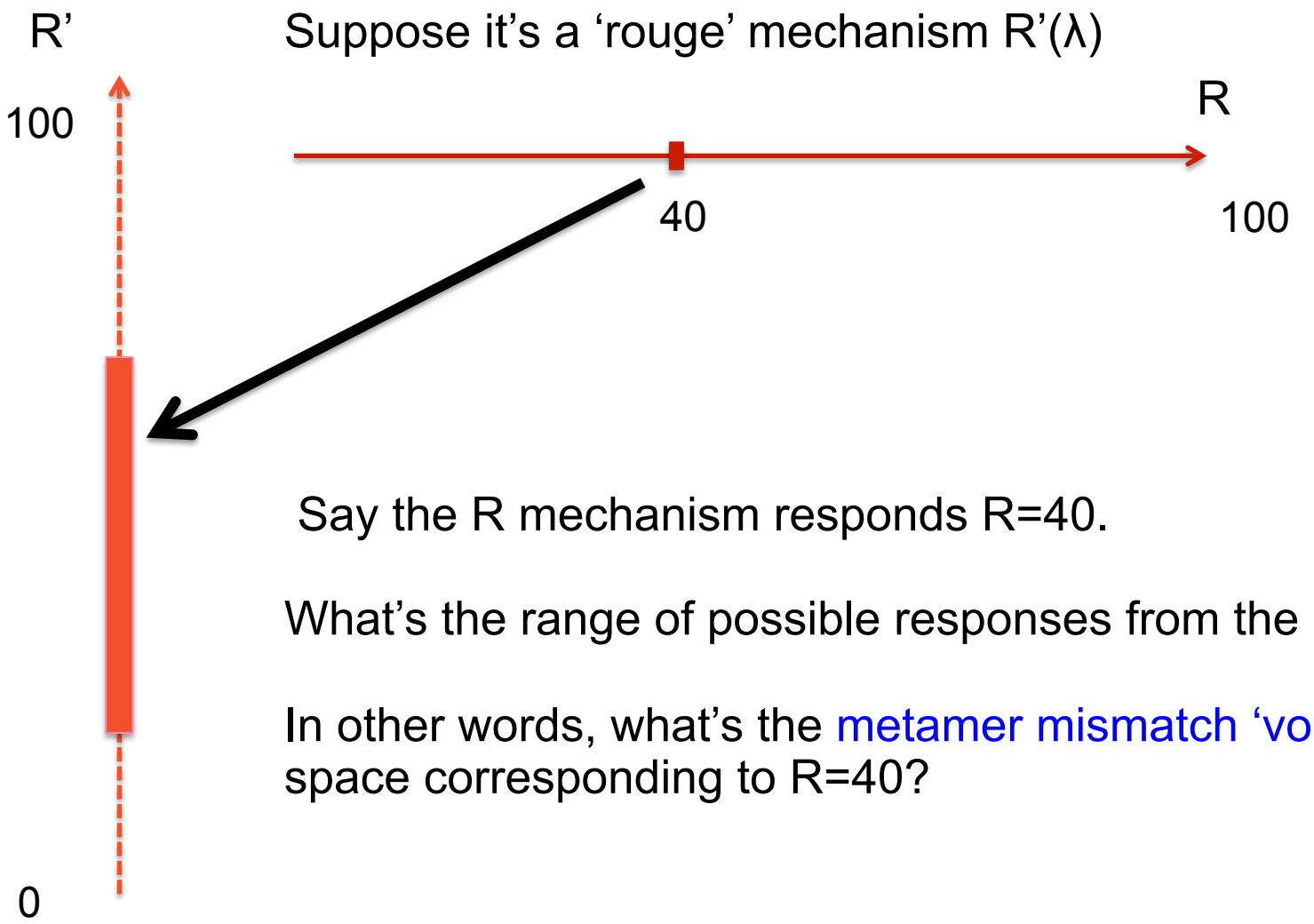
Volume goes to zero at the boundary

# 2D Projection of previous slide



# Review of Single-Channel Case

# Two Single-Channel Mechanisms

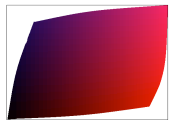


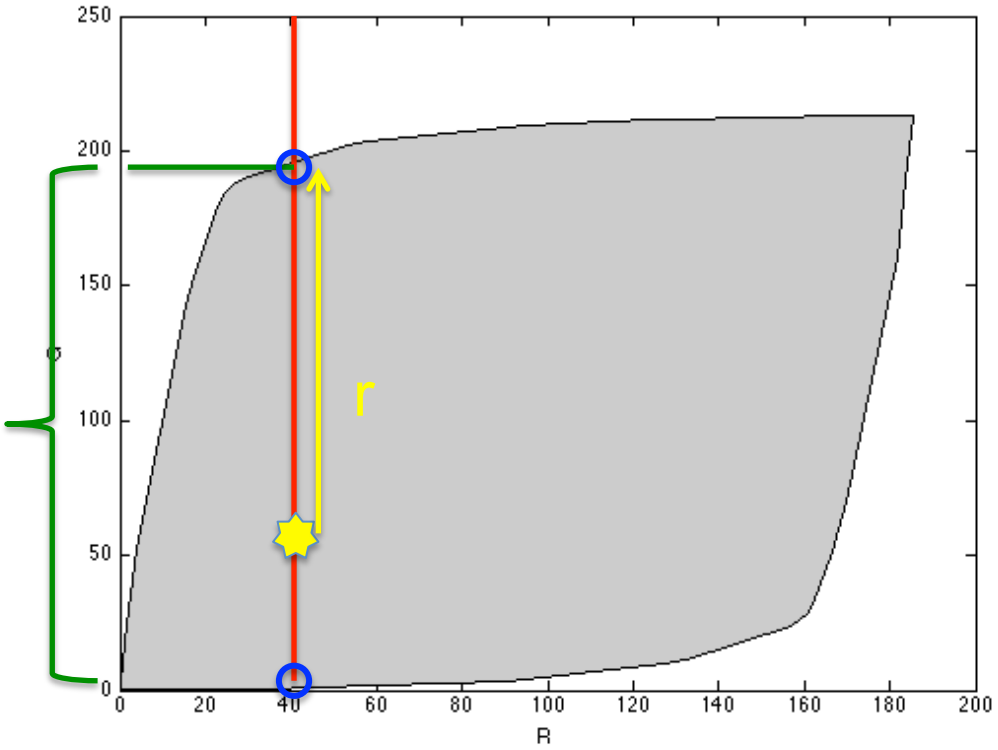

Say the R mechanism responds R=40.

What's the range of possible responses from the R' mechanism?

In other words, what's the metamer mismatch 'volume' in R' space corresponding to R=40?

# Single-Channel MMV






This solid represents all possible (R,R') pairs that can occur.

MMV for R=40

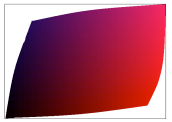
MMV: the R' lying on the R=40 line

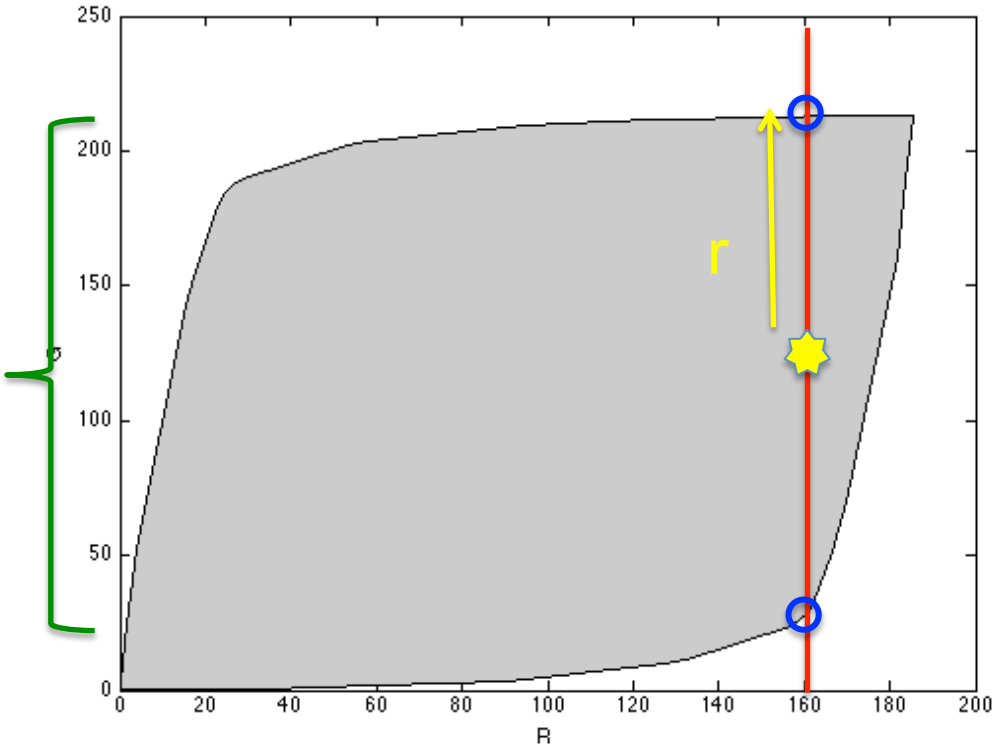
MMV induced by R=40 is $0 \leq R' \leq 198$

MMV is completely specified by its boundary, R'=0 and R'=198

MMV boundary:
intersection of Object Colour Solid Boundary with the R=40 cross-section

# MMV for R=160






MMV induced by R=260 is 25 ≤ R' ≤ 210

MMV is completely specified by its boundary, R'=25 and R'=210

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  - Six equations in six unknowns for each location on MMV boundary