MACM 316 Midterm
October 25, 2006

1. TRUE or FALSE? Justify with one line of comments.

   (a) Suppose $p = 0.12345$ approximates $p^* = 0.12344$. Then the relative error is 0.00001.

   (b) The difference between any pair of two consecutive numbers representable exactly in a single precision floating point numbers is the same for all pairs of adjacent floating point numbers.

   (c) Some processors use CHOP because chopping is at least as accurate as ROUNDING.

   (d) All diagonally dominant matrices are positive definite.

   (e) Choleski's method may be applied to any strictly diagonally dominant matrix.

   (f) The Secant method is a variant of Newton's method which uses finite differences of previous iterates for new iterates.

   (g) Newton's method to compute a root of a function $f(x)$ in the interval $[a, b]$ requires that $f(x) \in C^2[a, b]$.

   (h) The main reason for the scaled partial pivoting is to solve $AX = b$ when A has a linear combination of rows equal to zero.

   (i) If a computer solves a 1000×1000 system of linear equations using the Gaussian elimination method in about 4 seconds, it would take 64 seconds to solve a 3000×3000 system.

2. Suppose $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{(x+1)}}$ is to be evaluated for some large x. Explain why computing it directly in floating point arithmetic may give an inaccurate answer, and suggest how to calculate a more accurate answer.

3. (a) Name one advantage that Newton's method has over bisection.

   (b) Name one advantage that bisection has over Newton's method.

   (c) What is meant by quadratic convergence?

4. Let $f(x) = |sin(x)| - 0.5$. In this question we are interested in using the bisection method to find a root on the interval $[\pi, 3\pi]$.

   (a) Find a bracket. Then apply two iterations of the bisection method using 4 decimal digits of computation.

   (b) Write down a pseudocode for the bisection method. Use *xtol, ftol and* N_{max} test for convergence.

   (c) A friend suggests you use Newton's method instead of the bisection method. Explain why this may not be a good idea.

5. (**Bonus question**) The method of bisection for root finding is based on recursive halving of a search interval. The method of quadsection would recursively divide the interval into four pieces.

   (a) How many iterations does it take to compute a root to an accuracy of $\pm 10^{-4}$ via quadsection?

   (b) How does the rate of convergence of quadsection compare to that of bisection? Explain.

6. (a) When can a matrix be factored into the form LDL^T (D is a diagonal matrix, L is a unit lower triangular matrix)?

   (b) Consider the matrix below. Is this matrix diagonally dominant? Show that this matrix satisfies the criteria from part (a). Find the LDL^T for this matrix.

$$A = \begin{bmatrix} 6 & 2 & 2 \\ 2 & 6 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

   (c) Use your factorization found in part (b) to calculate the solution to $Ax = [10, 10, 9]^T$.

7. Use Gaussian elimination with scaled partial pivoting and 3 digit rounding arithmetic to solve the following system:

$$\begin{bmatrix} 4.00 & 40.00 \\ 2.00 & 1.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 60.00 \\ 2.00 \end{bmatrix}$$

8. Suppose $\overline{x}$ is an approximate solution to $Ax = b$. Suppose $r = b - A\overline{x}$ is the residual.

   (a) (**Bonus question**) Show that $\|x - \overline{x}\| \leq \|r\|\|A^{-1}\|$ and $\frac{\|x-\overline{x}\|}{\|x\|} \leq K(A)\frac{\|r\|}{\|b\|}$ where $K(A)$ is the condition number of the matrix A.

   (b) Use the inequalities in (a) to argue that very small residual cannot guarantee an accurate solution, and an accurate solution cannot guarantee a very small residual.