MACM 316-3
TEST 2 SOLUTIONS

10:30 – 11:20am, July 8, 2002
Instructor: Adrian Lewis

Family name: _____ **Initials:** _____

Student ID number: _____

READ INSTRUCTIONS CAREFULLY:

- **Do not lift the cover page until instructed!**
- Fill out your name and ID in the space provided.
- You may use an approved graphing calculator. No other aids.
- Answer all questions, **explaining your answers** carefully in the space provided. If you run out of space, use the back of the preceding page.
- This exam consists of 4 questions and 7 pages (including this one).

Question	1	2	3	4	Total
Grade	/4	/6	/9	/6	/25

3. (a) **[2 marks]** If your computer solves a 1000×1000 random system of linear equations using Gaussian elimination in about 3 seconds, roughly how long would you wait to solve a 5000×5000 system?

The time taken to solve an $n \times n$ system by Gaussian elimination is roughly proportional to n^3. Hence you expect to wait 5^3 times as long, namely 375 seconds.

(b) **[3 marks]** Factor the matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 6 & 2 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

into a product LU, where L is lower triangular with 1's on its diagonal and U is upper triangular.

Apply Gaussian elimination:

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix} \quad m_{21} = 3.$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \quad m_{32} = 2.$$

Hence

$$\begin{bmatrix} 2 & 1 & 0 \\ 6 & 2 & -1 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Question 3 (continued).

(c) **[2 marks]** Prove there do not exist lower triangular L and upper triangular U satisfying

$$\begin{bmatrix} 0 & -2 & 0 \\ 2 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix} = LU.$$

$$\begin{bmatrix} 0 & -2 & 0 \\ 2 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ b & d & 0 \\ c & e & f \end{bmatrix} \begin{bmatrix} g & h & i \\ 0 & k & l \\ 0 & 0 & m \end{bmatrix}.$$

implies $0 = ag$, $-2 = ah$, and $2 = bg$. The first equation shows either $a = 0$, contradicting the second equation, or $g = 0$, contradicting the third equation.

(d) **[2 marks]** Find a permutation matrix P so

$$P \begin{bmatrix} 0 & -2 & 0 \\ 2 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix}$$

has an LU factorization. (You may use part (b).)

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 \\ 2 & 1 & 0 \\ 6 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 6 & 2 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

and this matrix has an LU factorization, by part (b). Hence we can choose

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$