

## Macm 316 Midterm Test: Oct 28, 2002

Instructions: Answer all questions. Closed book. 115 Minutes. Explain all steps.

Do not expect marks to be granted for a correct answer if the intermediate steps are omitted.

1. (2 marks) Compute the absolute and relative error in approximating  $p$  by  $p^*$  when  $p = 110$  and  $p^* = 112.2$ .
2. (3 marks) Find the rate of convergence of the following function as  $h \rightarrow 0$ :

$$\lim_{h \rightarrow 0} (2 \cos(h) + h^2) = 2$$

3. (4 marks) True or False (no explanation required):
  - (a) Choleski's method may be applied to any strictly diagonally dominant matrix.
  - (b) Pivoting is not required when applying Gaussian elimination to a diagonally dominant matrix.
  - (c) Crout factorization can be directly applied to any tridiagonal nonsingular matrix.
  - (d) All diagonally dominant matrices are positive definite.
4. (5 marks) Some questions on iterative methods.
  - (a) Show that the following matrix is positive definite.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 4 & 2 \\ 0 & 2 & 2 \end{bmatrix}$$

- (b) If

$$b = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$$

name an iterative method that can be used to solve  $Ax = b$ .

- (c) Find the first two iterations of the method you proposed in (b) using the zero vector as an initial guess.
5. (4 marks) Some questions on direct methods for solving linear systems follow.
    - (a) Factor the following matrix into the LU decomposition using the LU factorization Algorithm with  $l_{ii} = 1$  for all  $i$ .

$$A1 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 10 \\ 3 & 14 & 28 \end{bmatrix}$$

- (b) Consider the following matrix  $A_2$ . Find the permutation matrix  $P$  so that  $PA$  can be factored into the product  $LU$ , where  $L$  is lower triangular with 1s on its diagonal and  $U$  is upper triangular.

$$A_2 = \begin{bmatrix} 2 & 6 & 10 \\ 1 & 3 & 3 \\ 3 & 14 & 28 \end{bmatrix}$$

6. (3 marks) Your friend works for the city's environmental enforcement unit. They have determined that somewhere between the source of a creek and 2 km downstream a chemical is slowly leaking into the stream from a source along the stream. His equipment detects the presence of the chemical but cannot determine the amount present. Assuming the chemical source is at a fixed location and that the chemical only flows downstream, describe how he should carry out the testing to locate the chemical to within 50 meters. What is the maximum number of tests needed using your approach? Efficiency counts as the environmental unit is, rather sadly, on a strict budget!
7. (2 marks) If  $g \in C[a, b]$  and  $g(x) \in [a, b]$  for all  $x \in [a, b]$  then we know from class that  $g$  has a fixed point in  $[a, b]$ .

Suppose, in addition, that  $g'(x)$  exists on  $(a, b)$  and a positive constant  $k < 1$  exists with

$$|g'(x)| \leq k, \text{ for all } x \in (a, b).$$

Prove that the fixed point in  $[a, b]$  is unique.