STUDENT NUMBER: ____________ NAME: Version B

SIMON FRASER UNIVERSITY
DEPARTMENT OF MATHEMATICS

October 22, 2003 **MACM 316** 50 MINUTES

FIRST TERM TEST

1. Do all questions. This Test has **FIVE** pages.
2. Answer in the space provided. If you need more space use the back of the page.
3. **Show ALL relevant computations.** Marks will be given for complete solutions more than for correct answers.
4. Non-communicating, non-plotting electronic hand calculators may be used.
5. Marks for each question are displayed in square brackets.

FOR INSTRUCTOR'S USE	
Question	Mark
1 [10]	
2 [10]	
3 [10]	
4 [10]	
Total	

1. This question requires you only to circle T for TRUE or F for FALSE. [10]

(a) The difference between any pair of two numbers "closest together" representable exactly in MATLAB is the same for all pairs of (adjacent) numbers. T F

(b) Some processors CHOP because chopping is either as accurate as or more accurate than ROUNDING. T F

(c) MATLAB ROUNDS all numbers to the nearest number storable by MATLAB. T F

(d) Conversion of every number exactly represented as a binary fraction to a decimal fraction is exact if enough digits are allowed in the converted number. T F

(e) Suppose f is a function with negative slope. Then Newton's method to find a simple zero of f will have oscillatory (cobweb) convergence when convergence occurs. T F

(f) If fixed point iteration converges for a function g, the convergence is always first order. T F

(g) If you need to bracket the zero of a function, the bisection method is always the most efficient choice. T F

(h) The chord method is a variant of Newton's method which uses finite differences of previous iterates for new iterates. T F

(i) When using maximal column pivoting, it is sometimes necessary to scale the rows first to ensure the pivots selected are the best possible or nearly so. T F

(j) If A is a full matrix (very few zero entries) with no special structure, then Gaussian elimination with maximal column pivoting is the most efficient way to solve $Ax=b$ for any n-vector b. T F

(k) The main reason for using maximal column pivoting is to solve $Ax=b$ when A has a linear combination of rows equal to zero. T F

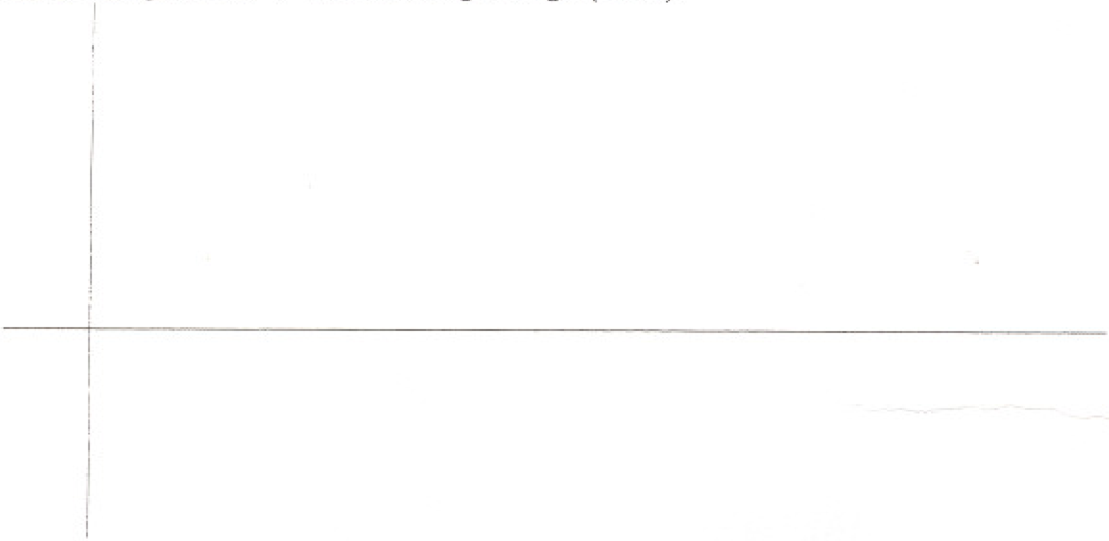
2.(a) Cancellation errors occur when two quantities are close to each other. A formula for the smallest zero of the quadratic function $q_2(x) = x^2 + 201x - 1$ is obtained with the usual formula for solving a quadratic equation. Write down the expression for this smallest zero in this case. Explain how cancellation occurs when evaluating this expression using 5 decimal digits after the decimal point. [3]

(b) Show by rationalizing the numerator you get a different form for computing this zero. Explain why or show how this modified form does not have the same problem with inaccuracy. [2]

(c) Another type of error occurs when tabular values of $F(n) = \int_0^1 x^n e^{-x/3}\,dx$ are computed with the recursion $F(n) = \frac{-3}{\sqrt[3]{e}} + 3nF(n-1)$. For this $F(0) = 3(1 - 1/\sqrt[3]{e})$, can only be computed approximately. State whether the error in F(10) obtained after 10 applications of this recursion is larger or smaller in size than the error in F(0), and exactly how the error changes. [2]

(d) By an accurate quadrature rule, F(10) is approximated as 0.06699615 Find and state a formula which modifies the recursion in part (c) so that it may be used with this value to improve the accuracy in obtaining F(4). Why is this value expected to be more accurate? [3]

3.(a) Suppose f is a decreasing function. A zero z is to be obtained by starting with x_0 close to z, and using Newton's method to find subsequent iterates. Show geometrically (i.e. use a diagram and some statements) how to find x_2 using the co-ordinate axes, the graph of f and a straight edge (ruler). [3]



(b) Use this graph, and the geometry you described in part (a) to construct (obtain or derive) the standard formula for computing the new iterate from the previous iterate. [4]

(c) Starting from $x_0 = 0$, compute the next two iterates by Newton's method to approximate a zero of the function $f(x) = x - \cos(2x)$. [3]

4. Consider the two matrices

$$U = \begin{pmatrix} 2 & 2 & -2 \\ 0 & 3 & 3 \\ 0 & 0 & -2 \end{pmatrix} \quad \text{and} \quad L = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 1 & 1 \end{pmatrix}$$

(a) Find a matrix A such that A=LU. (L and U decompose A.) [1]

(b) For this matrix A and the 3-vector $b=[2,1,-3]^t$, find the solution of Ax=b using forward and backward substitution. [5]

(c) State whether or not maximal column pivoting has been used, and say exactly how you know this has happened. [2]

(d) Find a matrix P that would be used to make PA = LU an LU-decomposition with maximal column pivoting. [2]