1. Suppose that an iterative root-finding method produces a sequence of approximations p_n that has order 3 convergence to the exact solution p_0 with asyptotic error constant 0.4. If the absolute error of the first approximation p_0 is 0.7, estimate the number of iterations that are necessary to achieve accuracy 10^{-9} .

(hints: Given $\lim_{n\to\infty} \frac{|p_{n+1}-p|}{|p_n-p|^3} = 0.4$. Therefore, $|p_n-p| \sim 0.4 |p_{n-1} - p|^3 \sim 0.4^{1+3} |p_{n-2}-p|^{3^2}$. You have to go all the way to the term $|p_0-p|$ which id 0.7.) (Ans: $n \geq 3$)

- 2. How many roots does the equation $cos(x) = -x^2/10$ have? Find one root of the equation using the bijection method. How many iterations are needed to reduce *xtol* to less than 0.1?
- 3. Similar to the previous question. The difference is $f(x) = x 2^{-x}$, the method is Newton-Raphson method, and xtol = 0.01.
- 4. The function $f(x) = \cos x$ is given by the values in the following points: f(0) = 0, f(1) = 0.54030, f(2) = -0.41615, f(3) = -.98999, f(4) = -.65364. Find Lagrange interpolating polynomial of degree 3 and use it to compute the approximation of $\cos(2.1)$.
- 5. Consider the polynomial $x^5 6x^4 + 8x^3 + 8x^2 + 4x 40$. Starting with $p_0 = 3$, perform two iterations of Newton-Raphson methods to find a root of the polynomial.
- 6. Let F(x) = sinx.

(a) Calculate approximations to F'(0.8) using the formula $F(x) = [-F(x+2h) + 8F(x-h) + F(x-2h)]/(12h) + h^4 F^{(5)}(\xi)/30$ with h = 0.1.

(b) Suppose that for any x, we encounter a rounding error e. What is the total error if $e = 5 * 10^{-7}$? (approximation errors and rounding errors)

(c) Use Richardson's extrapolation to deduce the formula in (a) from the following one: $F(x) = [F(x+h) - F(x-h)]/(2h) - h^2 F^{(3)}(\xi)/6$.

7. For a given function f(x), the values f(0.6) = 1.5, f(1) = 2.5, and f(1.5) = 3.5 are given.

(a) Taylor polynomial around basepoint 1 is given by $f(1 + h) = f(1) + hf'(1) + (h^2/2)f''(1) + (h^3/6)f'''(1) + \dots$ Derive the best approximation to calculate f'(1) with the data given above. What is the error term?