1. (a) Write nth order Taylor polynomial approximation for the function $f(x)$ expanded about $x = a$ and indicate a bound on the error or the remainder term by this approximation.

   (b) Estimate a bound on the error for a 2nd order approximation of $f(x) = sin(x)$ when $a = 0$ and $x = 1$.

   (c) Solve the quadratic equation $x^2 + c^2x - 1 = 0$ where c is a constant, using the standard formula. Comment on what computational error might exist with either or both of the roots for certain values of the costant c. In the case that a problem does exists give an alternate form of the root that removes this difficulty.

2. (a) if the bisection method is applied to the solution of the equation, $x^3 + x - 4 = 0$ with the initial search interval $[1, 2]$ then what is the bound on the number of iterations needed to achieve an approximate solution with an accuracy of 10^{-3}.

   (b) Starting from the formula for Newton's method, derive the Secant method.

   (c) Starting at $p_0 = 1$, perform two iterations of Newton's method on the equation in (a). Retain at least 5 significant figures in your calculations.

3. (a) Show graphically what a fixed point iteration $p_{n+1} = g(p_n)$ does.

   (b) Give conditions which guarantee that a fixed point exists and is unique, and also that the fixed point iteration converges linearly.

   (c) Apply the results of (b) to the iteration $p_{n+1} = g(p_n)$, where $g(x) = \frac{e^{x/2}}{4}$ and show that it converges linearly to a unique fixed point in the interval $[0, 0.5]$.

4. (a) Suppose a sequence $\{p_n\}_0^\infty$ converges to p^* with $p_n \neq p^*$ for all n, then state the mathematical condition that implies $\{p_n\}_0^\infty$ converges to p^* withorder α, with asymptotic error constant λ.

   (b) For the iteration $p_{n+1} = g(p_n)$ where $g(x) = (x^3 - 2)(x - 2^{1/3}) + x$, state whether the iteration will converge to $2^{1/3}$ given p_0 "close" to $2^{1/3}$.

5. Denote the successive intervals that arise in the bisection method by $[a_0, b_0]$, $[a_1, b_1]$, $[a_2, b_2]$, and so on.

   (a) Show that $a_0 \leq a_1 \leq a_2 \leq \ldots$ and that $b_0 \geq b_1 \geq b_2 \geq \ldots$

   (b) Show that, for all n, $a_nb_n + a_{n-1}b_{n-1} = a_{n-1}b_n + a_nb_{n-1}$.

6. Using a calculator, observe the sluggishness with which Newton's method converges in the case of $f(x) = (x - 1)^m$ with $m = 8$. Use $p_0 = 1.1$.