MACM 316 Assignment 6

Date: November 24, 2006

Date due: 5pm, Monday, December 4, 2006

This is an optional assignment. This assignment can be used to replace any one of your previously submitted assignments.

Please submit your answers, stapled together with your name and student id cover-page, in the assignment box marked MACM 316.

- 1. Let $f(x) = 2x^2 \sin(x) e^x \cos(x)$. For h = 0.1 and h = 0.01, approximate f'(2.19) using two 3-point formulas for f'(x). 2.092.182.192.22.292.39x2.2111.59654 12.8569112.9972 14.39606 f(x)13.1374813.2777115.7739
- 2. Let $f(x) = 2x^2 \sin(x) e^x \cos(x)$. Approximate f''(x) with h = 0.1 and h = 0.01. Find the exact value of f''(2.19) and compare your approximations with the exact value, i.e. find relative errors.
- 3. Using the Taylor polynomial around basepoint x_0 , approximate $f'(x_0)$ using the values $f(x_0)$, $f(x_0 + h)$, $f(x_0 2h)$, and $f(x_0 4h)$ so that the approximation error is $O(h^3)$. Of course, we are assuming that the function is a well-behaved function.
- 4. Our objective is to approximate the integral

$$I = \int_{1}^{2} (\cos x)^2 dx.$$

- (a) Use trapezoidal and Simpson's rules to approximate I.
- (b) What is the error term for each rule? Determine the upper bound of the absolute value for each error term.
- (c) Approximate the integral using the composite trapezoidal and Simpson's rules for n = 4.
- (d) Determine the value of n (and therefore of h in the case of uniform spacing) needed for the approximation to be within 10^{-8} for both the rules.