Assignment # 5

Section 3.1

2. b

$$f(x) = \sqrt[3]{x-1}$$

$$P_1(x) = 0.6099204008x - 0.1324399760$$

$$P_1(1.4) = 0.7214485851$$

$$P_2(x) = -3.183202832x^2 + 9.682048472x - 6.498845640$$

$$P_2(1.4) = 0.816944669$$

2. d $f(x) = e^{2x} - x$

$$P_1(x) = 34.28581783x - 31.92477833$$

$$P_1(1.4) = 16.07536663$$

$$P_2(x) = \cancel{26}\ 26.853444x^2 - 42.24649756x + 21.78210966$$

$$P_2(1.4) = 15.26976332$$

22.

$$P_{0,1,2}(1.5) = \frac{(x-0)\cdot \overset{P_{12}}{3.5} - (x-2)\,\overset{P_{01}}{2.5}}{2} = \frac{x+5}{2} = 3.25$$

$$P_{0,1,2,3}(1.5) = \frac{(1.5-0)\overset{P_{123}}{4} - (1.5-3)\cdot \overset{P_{012}}{3.25}}{3} = \boxed{3.625}$$

3.4 – 16

$$S(x) = S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

x_i	a_i	b_i	c_i	d_i
0	1	−0.92360	0	0.62086
0.25	0.778801	−0.80718	0.46565	−0.15402
0.75	0.47236	−0.45705	0.23462	−0.31283
1	0.367879	—	0	—

$\int_0^1 S(x)\,dx = 0.631967$

$S'(0.5) = -0.603243$

$S''(0.5) = 0.700274$

$\int_0^1 e^{-x}\,dx = 0.6321205$

$f'(0.5) = -0.6065307$

$f''(0.5) = 0.6065307$

4.1 – 16

$f'(3) = \frac{1}{2}[f(4) - f(2)] = 0.21210$

with error bound

$$\max_{1 \le x \le 5} \frac{|f^{(3)}(x)|\, h^2}{6} \le \frac{4}{6} = 0.\bar{6}$$

$$M = N(h) + k_1 h^2 + k_2 h^4 + k_3 h^6 + \cdots \quad (1)$$

replace h by $h/3$

$$M = N(h/3) + k_1 \frac{h^2}{9} + k_2 \left(\frac{h}{3}\right)^4 + k_3 \left(\frac{h}{3}\right)^6 \cdots \quad (2)$$

$$9(2) - 1 = 9N(h/3) - N(h) + h^4 k_2 \left(-\frac{8}{9}\right) + h^6 k_3 \left(-\frac{80}{81}\right) \cdots$$

$$M = \underbrace{\frac{9}{8} N(h/3) - \frac{1}{8} N(h)}_{N_2(h)} + \frac{1}{8} h^4 k_2 \left(-\frac{8}{9}\right) + h^6 k_3 \left(-\frac{10}{81}\right) \cdots$$

$N(h) = N_1(h)$

$O(h^4)$ approx

$$M = N_2(h) + \frac{1}{9} h^4 k_2 - \frac{10}{81} k_3 h^6 + \cdots \quad (3)$$

replace h with $h/3$

$$M = N_1(h/3) - \frac{1}{9} h^4 \times \left(\frac{1}{3}\right)^4 k_2 - \frac{10}{81} k_3 (h/3)^6 \quad (4)$$

$$81(4) - (3) = 81 N_2(h/3) - N_2(h) + \underbrace{\frac{10 k_3 h^6}{729} + \frac{10}{81} \frac{k_3 h^6}{729}}$$

$$- \frac{10 k_3 h^6}{729} \left[\frac{80}{81}\right]$$

$$O(h^6) \quad M = \underbrace{N_2(h/3) - \frac{1}{81} N_2(h)}_{N3(h)} - \frac{800 \cdot k_3 h^6}{3^{14}}$$

e)

$$① \; f(x+3h) = f(x) + f'(x)(3h) + \tfrac{1}{2} f''(\xi_1)(3h)^2$$

$$② \; f(x-h) = f(x) - f'(x)(h) + \tfrac{1}{2} f''(\varepsilon_2)(h^2)$$

$$①-② = f'(x)3h + f'(x)(h) + \tfrac{1}{2}\left(f^{(2)}(\xi_1)9h^2 - f^{(2)}(\varepsilon_2)h^2\right)$$

$$= 4h\, f'(x) + \tfrac{1}{2}h^2\left(9f^{(2)}(\varepsilon_1) - f^{(2)}(\varepsilon_2)\right)$$

$$\Rightarrow f'(x) = \frac{1}{4h} f(x+3h) - f(x-h) - \tfrac{1}{8}h\left(9f^{(2)}(\varepsilon_1) - f^{(2)}(\varepsilon_2)\right)$$

f) let $f'(x) = \frac{1}{2h}\left[f(x+2h) - f(x)\right]$

$$f(x+2h) = f(x) + f'(x)(2h) + \tfrac{1}{2} f''(\varepsilon)(2h)^2$$

$$\Rightarrow f'(x) = \frac{1}{2h}\left[f(x+2h) - f(x)\right] - \frac{4}{4h} f''(\varepsilon)h^2$$

$$f'(x) = \frac{1}{2h}\left[f(x+2h - f(x)\right] \underbrace{- h f''(\varepsilon)}_{\text{error term}}$$

$$f'(0) = \frac{1}{2h}\left[f(2h) - f(0)\right]$$

$$\boxed{\text{error term} \quad -h f''(\varepsilon) \quad \cdot < \varepsilon < 2h}$$

g) By adding the ~~two~~ equation,

we obtain :

$$f(x+h) + f(x-h) \cancel{+}\, 2f(x) = h^2 f''(x)$$

however by adding these two equation we are missing a point. ~~added~~ The error term in these two equations are not the same.

$$① \quad f(x+h) - f(x) = h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(\xi_1)$$

$$\boxed{x < \xi_1 < x+h}$$

$$② \quad f(x-h) - f(x) = -h f'(x) + \frac{h^2}{3} f''(x) - \frac{h^3}{6} f'''(\xi_2)$$

$$\boxed{x-h < \xi_2 < x}$$

So when you add these two, you get:

$$f(x+h) + f(x-h) - 2f(x) = h^2 f''(x) + \frac{h^3}{6} \left[f'''(\xi_1) - f'''(\xi_2) \right]$$

Assignment #6

1. Suppose $x_0 = 2.19$. & $h = 0.1$.

Then
$$\left.\begin{aligned} x_0 + h &= 2.29 \\ x_0 + 2h &= 2.39 \\ x_0 - h &= 2.09 \end{aligned}\right\} \quad (A)$$

Three-point formula:

$$f'(x_0) \approx \frac{1}{2h}\left[f(x_0+h) - f(x_0-h)\right] \quad (1)$$

or
$$f'(x_0) \approx \frac{1}{2h}\left[-3f(x_0) + 4f(x_0+h) - f(x_0+2h)\right] \quad (2)$$

We can easily plug in the values of (A) in $f(x)$ & evaluate (1) & (2) as

(1) : $f'(2.19) = 13.9976$

(2) : $f'(2.19) = 14.0937$

For $h = 0.01$; ~~then~~

$x_0 + h = .22$; $x_0 + 2h = 2.21$ & $x_0 - h = 2.18$

$\therefore$ (1) becomes $f'(2.19) = 14.0285$

(2) becomes $f'(2.19) = 14.0305$

2. $f(x) = 2x^2 \sin x - e^x \cos x$

$f'(x) = 4x \sin x + 2x^2 \cos x - e^x \cos x + e^x \sin x.$

$f''(x) = 4 \sin(x) + 8x \cos x - 2x^2 \sin x + 2e^x \sin x.$

$\therefore f''(2.19) = -0.16972.$

We use Taylor polynomial to show that

$$f''(x_0) \approx \frac{1}{h}\left[f(x_0 - h) - 2f(x_0) + f(x_0 + h) \right] \quad (1)$$

For $h = 0.1$, (1) gives

$$f''(2.19) \approx \frac{1}{0.01}\left[f(2.09) - 2f(2.19) + f(2.29) \right]$$

$$\doteq -0.1800$$

The relative error is $\left| \frac{-0.16972 + 0.1800}{-0.16972} \right|$ ~~$= 6.05704$~~

$= 6.05704\,\%$

for $h = 0.01$,

$f''(2.19) = -0.1$

The relative error $\left| \frac{-0.16972 - (-0.1)}{-0.16972} \right| = 41.07942\%$

Questions 3 & 4 will be provided later.

(Computer programs)