Solution hints to problem set 9.

5.1 (2) Easy; describe some subset of $A \times B$.

5.1 (4) B has to be equal to A. or A/B is null.

5.1 (5) Solution in the text. You should get familiar with the proof.

5.1 (7a) $|A \times B| = 5 \times 4 = 20$.

Power set has 2^{20} elements

5.1 (12). Suppose $|A| = m$. Then $|A \times B| = 3m$

$\therefore$ Number of possible relations $= 2^{3m}$

Our m should be such that $2^{3m} = 4096$

i.e. $2^{3m} = 2^{12}$ $\therefore 3m = 12$ i.e. $m = 4$.

$\therefore$ A has 4 elements.

5.2 (1) (e) $R \subseteq A \times B$

$\therefore |R| = 6$ & $|A| = 5$, there exists at least two tuples in R of the type (a, b_1), (a, b_2) for some $a \in A$ & $b_1, b_2 \in B$.

$\therefore$ R is not a function.

5.2 (2) Since $\sqrt{2} \in R$, therefore $f(\sqrt{2})$ & $f(-\sqrt{2})$ are not defined $\left(f(\sqrt{2}) = \frac{1}{(\sqrt{2})^2 - 2} = \frac{1}{0} \right)$.

$\therefore f$ is not a function when the domain is R

However, ~~$f: R \to N$~~ $f: N \to R$ is a function.

5.2 (5) (d) $\overline{B} \cup \overline{C} = \overline{B \cap C}$.

Now $B \cap C = \{(x, y) \mid y = 3x$ & $y = x-7\}$

Since $3x = x - 7 \rightarrow x = -7/2$

$\therefore B \cap C = \{(-7/2, 3\cdot(-7/2))\} = \{(-7/2, -\frac{21}{2})\}$.

$\therefore \overline{B} \cup \overline{C} = \overline{B \cap C} = R \times R - \{(-7/2, -\frac{21}{2})\}$.

5.2 (8) (d) false. Check with $a = 2.3$.

$-\lceil a \rceil = -3$; $\lceil -a \rceil = \lceil -2.3 \rceil = -2$.

5.2 (12) Prove that $\lceil \frac{n}{k} \rceil = \lfloor (n-1)/k \rfloor + 1$, $n, k \in Z^+$

Separate the cases when k divides n & when k doesn't divide n.

k divides n

$n = tk$ for some integer t.

$n-1$ is not divisible by k.

$\therefore \left\lceil \frac{n}{k} \right\rceil = t$ & $\left\lfloor \frac{n-1}{k} \right\rfloor = t-1$

$\therefore \lceil n/k \rceil = \left\lfloor \frac{n-1}{k} \right\rfloor + 1$.

k does not divide n.

Suppose $n = tk + r$, t and integer & r is the residue $0 < r < k$

$\therefore \left\lceil \frac{n}{k} \right\rceil = t+1$

$\left\lfloor \frac{n-1}{k} \right\rfloor = t$

$\therefore \lceil n/k \rceil = \left\lfloor \frac{n-1}{k} \right\rfloor + 1$

5.2 (16)(a) $f(A) = f(\{2, 3\})$

$= f(2) \cup f(3)$

$= \{4, 9\}$.

5.3 (2)(d) $f(x) = x^2$, $x \in \mathbb{Z}$. $f: \mathbb{Z} \to \mathbb{Z}$.

not one-to-one since $f(-1) = 1$ & $f(1) = 1$

5.3 (4)(a) How many onto? $|A| < |B| \to$ # of onto function $= 0$

(b) How many onto? $6^4 - C(6,5)5^4 + C(6,4)4^4 - C(6,3)3^4$

$+ C(6,2) \cdot 2^4 - C(6,1) \cdot 1^4$