Problem set on pigeonhole principle.

1: Discussed in the class

2. Pigeons : 101 integers selected (without repetition from 1 to 200.

Holes : 100 pairs

$\{1,2\}, \{3,4\}, \{5,6\}, \cdots, \{199,200\}$

$\gcd(i, i+1) = 1.$

3. Pigeons : 7 distinct integers from 1 through 126.

Holes : $\{1,2\}, \{3,4,5,6\}, \{7,8,9,10,11,12,14\},$

$\{15,16,17,\cdots,30\}, \{31,32,\cdots,62\}, \{63,64,\cdots,126\}$

6 groups

4. Discussed in the Class.

5.

Exercises 5.5

5(a): Solved in the book.

I has used the fact that every integer x in $\{1, 2, \cdots, 300\}$ can be expressed as

$x = 2^k . m$ where m is an odd number, $1 \le m \le 299$ & $k \ge 0$. For example

$17 = 2^0 . 17$; $34 = 2^1 \times 17$, $64 = 2^6 \times 1$, $160 = 2^5 \times 5$

8(a) holes : Odd, even.

(b) holes : {odd, odd}, {odd, even},

{even, even), (even, odd)

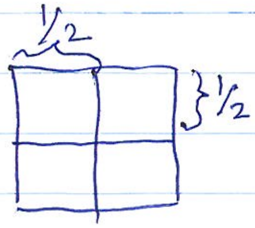
$|S| = 5$ why?

(c) holes : 2^3 number of holes.

(d) holes : 2^n number

$S \subseteq \underbrace{Z^+ \times Z^+ \times \cdots \times Z^+}_{n \text{ times}}$

11. Holes:



Problem Set 12

Exercise 7.1

6. Check which ones are reflexive, antisymmetric & transitive.

8. a) R_1 & R_2 are relations on A, which are reflexive. Clearly, $\forall a \in A$, $(a,a) \in R_1$ & R_2

$\therefore (a,a) \in R_1 \wedge (a,a) \in R_2 \rightarrow (a,a) \in R_1 \cup R_2$.

b) $R_1 \cup R_2$ is symmetric if R_1 & R_2 are symmetric.

c) $R_1 \cup R_2$ is not antisymmetric even if R_1 & R_2 are.

$$\underset{\text{antisymmetric}}{\underset{R_1}{\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}} \cup \underset{\text{antisymmetric}}{\underset{R_2}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}} = \underset{R_1 \cup R_2}{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \text{ not antisymmetric}$$

d) $R_1 \cup R_2$ is not transitive even if R_1 & R_2 are transitive.

$$\underset{\text{transitive}}{\underset{R_1}{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}} \cup \underset{\text{transitive}}{\underset{R_2}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}} = \underset{\text{not transitive.}}{\underset{R_1 \cup R_2}{\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}}}$$

16. a) $R \subseteq \mathbb{Z} \times \mathbb{Z} = \{(a,b) \text{ if } a < b\}$

b) Let $R \subseteq \mathbb{Z} \times \mathbb{Z}$.

- R satisfies symmetric & transitive

  i.e. $(a,b) \in R \rightarrow (b,a) \in R$

  – $(a,b) \in R$ & $(b,a) \in R \rightarrow (a,a) \in R$ by transitive property

  $\therefore$ R is not irreflexive.

- R satisfies irreflexive & symmetric.

  i.e. $(a,a) \notin R$ for any $a \in A$.

  & $(a,b) \in R \rightarrow (b,a) \in R$.

  If transitive property is satisfied,
  $(a,b) \in R \wedge (b,a) \in R \rightarrow (a,a) \in R$, a contradiction.

17 $A = \{1,2,3,4,5,6,7\}$

	1	2	3	4	5	6	7
1		a_1	a_2	a_3	a_4	a_5	a_6
2			a_7	a_8	a_9	a_{10}	a_{11}
3				a_{12}	a_{13}	a_{14}	a_{15}
4					a_{16}	a_{17}	a_{18}
5						a_{19}	a_{20}
6							a_{21}
7							

- 7 diagonal elements
- 21 off diagonal.

– $\binom{7}{4}$ different ways to place 4 relations on the diagonals

$R = \{(1,1),(2,2),(3,3),(4,4)\}$ is symmetric

17 (continued)

(c) Total #

$$\binom{7}{7}\binom{21}{0} + \binom{7}{5}\binom{21}{1} + \binom{7}{3}\binom{21}{2} + \binom{7}{1}\binom{21}{3}$$

7 on the diagonal

5 on the diagonal

being symmetric, by selecting the first, you select the 2nd as well

Exercises 7.2

4. $R_1 \subseteq A \times A$, a poset $= \{(a_1, a_2) \mid a_1 \le a_2\}$

$R_2 \subseteq B \times B$, a poset $= \{(b_1, b_2) \mid b_1 \le b_2\}$

$R \subseteq (A \times B) \times (A \times B)$

$= \{((a_1, b_1), (a_2, b_2)) \in R$ if $a_1 \le a_2$ & $b_1 \le b_2\}$

- R_1 & R_2 are total orders

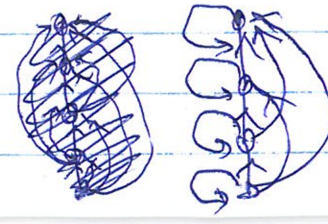
- R is not a total order since $((1,2),(2,1)) \notin R$ if $A = B = Z^+$

6 (a)

$$\begin{array}{c|ccccc} & a & b & c & d & e \\ \hline a & 1 & 1 & 1 & 1 & 1 \\ b & 0 & 1 & 0 & 1 & 1 \\ c & 0 & 0 & 1 & 1 & 1 \\ d & 0 & 0 & 0 & 1 & 1 \\ e & 0 & 0 & 0 & 0 & 1 \end{array}$$

12.

For every $x, y \in G$, either $(x, y) \in R$ or $(y, x) \in R$, but not both; $(x, x) \notin R$ $\forall x$, if (x, y) has an arc & (y, z) has an arc, (x, z) has an arc. You should be able to draw (G, R) as follows:



7.4

2. a) We need to determine the class 8 belongs to.
   – 3 choices

   b) A_1 needs one of 7, 8

   Once 7 (or 8) goes to A_1, the other 8 (or 7) has two choices

   $\therefore$ there are 4 different partitions.

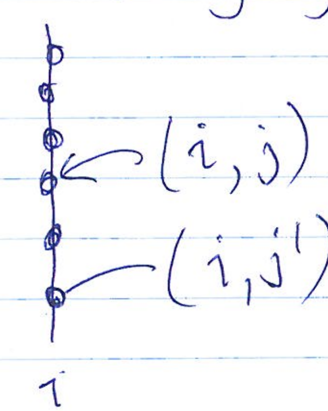
4. $[1] = \{1, 2\}$.

6. (b) $A \subseteq R \times R$

   Each equivalent class is a set of points on a vertical line. This provides partition of the plane.

   The case gets clearer if $A \subseteq Z^+ \times Z^+$

   In this case an equivalent class contains all the points lying on a vertical line



8 (b) The equivalence classes are: $\{1, 4, 7\}, \{2, 5\}, \{3, 6\}$