

MACM 101
Discrete Mathematics I
Date: September 21, 2011

1. Solution hints of Problem Set 3

(a) Problems (Exercises 1.4) 2, 8, 16 18

- i. Problem 2: You need to consider when the youngest gets 1 and the youngest gets two candies separately. Here there are 5 types, and the number of objects to be distributed is 12. It is a case of combination with repetition.
- ii. Problem 4: It is equivalent to getting integral solution $x_1 + x_2 + x_3 = 8$ where $x_i \geq 1, i = 1, 2, 3$. Why?
- iii. Problem 16: $C(n-1, n-19)$ is the answer for the first equation. Remember that the solutions are positive, ie, $x_i \geq 0$ for all i . $C(n-1, n-64)$ is the solution for the second equation. You need to find n such that $C(n-1, n-19) = C(n-1, n-64)$. Show that $n = 82$.
- iv. Problem 18: We need to find solutions that satisfy both the equations. Substituting the second equation to the first one, we get $x_4 + x_5 + x_6 + x_7 = 37 - 6$. The answer is $C(31+3-1, 3-1)$.

(b) Problems (Exercises 1.5) 2, 3, 4, 8, 10, 14, 20, 22

- i. Problem 2:(b) The order is important; each dial, except the first, has 4 choices. The first one has 5 choices.
- ii. Problem 3: Each intersection point is determined by two chords and each chord is determined by two points. Thus any 4 points determine an intersection point.
- iii. Problem 4: (b) Enumerate all possible breaks and solve for each one. Some of the cases: 4 from one and one from each of the other book; one from one book, two from a second one and the rest three from the thirs book There are $3!$ ways to order the books.

- iv. Problem 8: We select two vowels, say OE, as one and remove I from the letters. There are $7!/2!$ permutations. We can now add I to each permutation in 5 different ways since I cannot be placed to either side of 'OE'.
- v. Problem 10: Observe that the last letter has to be R, the 18th alphabet. The first and the second initials must come from A through Q.
- vi. Problem 14: (b) the answer is $P(4,2) \cdot 10!$. (c) total possible way (question(a)) - the number of ways H and T can be assigned to the same floor (question(b)).
- vii. Problem 20: There are $C(n,r)$ ways to select r objects. Now each such r objects can be arranged around a circle in $2 \cdot (r-1)!$ ways.
- viii. Problem 22: Easy