MACM 101

Discrete Mathematics I

Date: October 11, 2011

Solution Hints for Problem Set 5

Excercise 2.3

2.

The logical implication is given by x→y. So for each sub-part, find truth table for implication where in
a) x is $(\mathtt{p} \rightarrow \mathtt{q}) \wedge (\mathtt{q} \rightarrow \mathtt{r})$ and y is $\mathtt{p} \rightarrow \mathtt{r}$. Follow the same for the other sub-parts. The implication will be verified when x→y is a tautology for the given expressions.

4.

a) Janice's daughter Angela will check Janice's spark plugs. (modus ponens)
b) Brady did not solve the first problem correctly. (modus tollens)
c) This is a repeat until loop. (modus ponens)
d) Tim watched Telivision in the evening. (modus tollens)

8.

1. Premise
2. Conjunctive Simplification i.e. if $\mathtt{a} \wedge \mathtt{b}$ is true, then both $\mathtt{a}$ and $\mathtt{b}$ are true
3. Premise
4. Rule of Detachment on steps 2 and 3.
5. same argument as in 2
6. Rule of Conjunction on step 4 and 5
7. Premise
8. Step 7 and the rule that says $p \rightarrow q \Longleftrightarrow \neg q \rightarrow \neg p$
9. DeMorgan's Law on step 8
10. Rule of Detachment on 6 and 9

11. Premise
12. Step 11 and the rule that says $p \to q \Longleftrightarrow \neg q \to \neg p$
13. Applying DeMorgan's Law and Double negation($\neg\neg p\ is\ p$) on step 12.
14. Rule of Detachment on 10 and 13
15. Step 14 and Rule of Conjunctive Simplification

10.

a)

1	$p \wedge \neg q$	premise
2	p	Step 1 and Conjunctive Simplification
3	r	Premise
4	$p \wedge r$	Step 2,3 and Conjunctive Simplification
5	$\therefore (p \wedge r) \vee q$	Step 4 and Disjunctive Amplification

b)

1	$p, p \to q$	premise
2	q	Step 1 and Rule of Detachment
3	$\neg q \vee r$	Premise
4	$q \to r$	Step 3 and $q \to r$ is equivalent to $q \to r$
5	$\therefore r$	Steps 2 and 4 and Rule of Detachment

c)

1	$p \to q, \neg q$	premise
2	$\neg p$	Step 1 and Modus Tollens
3	$\neg r$	Premise
4	$\neg p \wedge \neg r$	Step 2,3 and Rule of Conjunction
5	$\therefore \neg(p \vee r)$	Step 4 and DeMorgan's Law

d)

1	$r, r \to \neg q$	premises
2	$\neg q$	Step 1 and Rule of Detachment
3	$p \to q$	premise
4	$\therefore \neg p$	Step 2, 3 and Modus Tollens

12.

a)

p: Rochelle gets the supervisor's position
q: Rochelle works hard
r: Rochelle gets a raise
s: Rochelle buys a new car

$(p \wedge q) \rightarrow r$
$r \rightarrow s$
$\neg s$

$\therefore \neg p \vee q$

1	$\neg s$	premise
2	$r \rightarrow s$ premise	
3	$\neg r$	1,2 and modus tollens
4	$(p \wedge q) \rightarrow r$	premise
5	$\neg(p \wedge q)$	3,4 and modus tollens
6	$\neg p \vee \neg q$	$\neg(p \wedge q) \iff \neg p \vee \neg q$

b)

p: Dominic goes to the race track
q: Helen gets mad
r: Ralph plays cards all night
s: Carmela gets mad
t: Veronica is notified

$p \rightarrow q$
$r \rightarrow s$
$(q \vee s) \rightarrow t$
$\neg t$

$\therefore \neg p \wedge \neg r$

1	$\neg t$	premise
2	$(q \vee s) \rightarrow t$ premise	
3	$\neg(q \vee s)$	1,2 and modus tollens
4	$\neg q \wedge \neg s)$	DeMorgan's Law
5	$\neg q$	3,4 and Conjunctive Simplification
6	$p \rightarrow q$	premise
7	$\neg p$	5,6, modus tollens
8	$\neg s$	4 and Rule of Conjunctive Simplification
9	$r \rightarrow s$	premise
10	$\neg r$	8,9 and modus tollens
11	$\neg p \wedge \neg r$	7,10 and Rule of Conjunction

c)

p: There is a chance of rain.
q: Lois' red head scarf is missing
r: Lois does not mow her lawn
s: The temperature is over 80° F

$(p \vee q) \rightarrow r$
$s \rightarrow \neg p$
$s \wedge \neg q$

$\therefore \neg r$

Invalid : (p,q,r,s) - (0,0,1,1)

Excercise 2.4

7.

a)

Easy

b)

True : (i),(iv),(v),(vi)
False: (ii),(iii) for x=10

c)

Easy

d)

(i) x=0
(iii) x=20

8.

It will be easy to examine these statements, if you first find the values of x that satisfy p(x);i.e. x=(3,5). Next it is important whether each statement applies which of the two quantifiers on the statements. Then try to see if the given statements are true or false.

14.

In a hurry to find the solutions, one might start looking for logically equivalent statements by examining their validity. Of course, this is a wrong. So, try to find what statements convey the same semantics. Although, many times it is possible to express the same semantic in different ways. Still, looking at the ways to express the statements helps in finding logically equivalent statements. Nevertheless, when you have the semantics as well as the symbolic form of statements, it is comparatively easier to find logically equivalent statements.
Logically equivalent statements
(a,b,e,f)
(c,g)
d is not equivalent to any other.

16.

Easy

18.

(a),(b)and(c) are easy.
d)$\forall x[p(x) \vee q(x) \wedge \neg r(x)]$ Try converting the implication into its other form with disjunction.

24.

a)True (if x keeps on increasing y can be decreased to keep the sum constant or vice versa)
b)False (for x=18 there is no y for which x+y=17 ; positive integers only)
c)False (consider x=20, say)
d)True