MACM 101 (Discrete Mathematics I)

November 22, 2011

Problems on Pigeonhole Principle (PHP)

Practice Problem Set 11

1. Any subset of size 6 from $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ contains a pair whose sum is 10.

2. Select 101 integers without repetition from 1 to 200. Show that, among the selected ones, there are two integers which are relatively prime, i.e. the greatest common divisor is 1. (Problem 6 of Exercise 5.5)

3. Pick any 7 distict integers from 1 through 126. Show that, among the picked ones, there will be two integers a and b such that $b < a \leq 2b$.

4. Suppose we are given a list of integers $a_1, a_2, ..., a_n$, which need not be distinct. Prove that there is always a set of consecutive numbers $a_{k+1}, a_{k+2}, ..., a_l$ whose sum $\sum_{i=k+1}^{l} a_i$ is a multiple of n.

5. Problems from the text (Exercises 5.5) 5(a), 8 (a)(b)(c)(d), 11.