Chapter 1 solutions

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1.2. True. If m was the first to propose to w, w would've paired up with m. If w was already paired with a different man $\bar{m}$ at the time m proposes, w would've left $\bar{m}$ and paired up with m, as m ranks higher in her preference list. Any pairing of m and w, other than to each other, would result in instability.

1.3. As defined in the problem, a pair of schedules is ***stable*** if neither networks can unilaterally change its schedule and win more time slots. Each show has a fixed rating and is independent of the slot its playing. It is easy to come up with an example involving two shows. Suppose Network A schedules a show, say a_1 (rating 100), for the first slot and network D plays show d_1 (rating 50), at the same time. Clearly, Network A wins the first slot. Suppose the show a_2 playing in Network A during the second time slot has a rating of 200, and competing show d_2 from Network D has a rating of 150; network A would win the second slot. But network D can change its schedule to pair its show d_2 with a_1 to win one of the slots. This shows instability.

1.6. The problem can be solved using the Gale-Shapley algorithm. Ships and ports can be set up as sets of men and women from the GS algorithm. Ships maintain a preference list of ports in the order they visit and ports rank the ships in the reverse order they visit. We claim that a stable matching of ships and ports satisfy the condition (†). To prove this, we assume that the claim is false and work towards a contradiction. Assume that the assignment of ships to ports violates the required condition (†). So, a second ship $\bar{s}$ visits port p even when ship s is stationed at p . That should imply that $\bar{s}$ and p prefer each other over their current partners. This suggests an instability in our matching. But from theorem 1.6 (refer text book), we know that G-S algorithm returns a stable matching. Hence a contradiction. So condition (†) is satisfied by the matching.

1.7. In a valid switching, there is no crossing of resulting streams at a junction. We keep this in mind while creating our preference list.

So, input wires show a higher preference for output lines earliest in their route and output wires rank input lines in the reverse order. We now claim that "A stable matching between input and output wires define a valid switching". To prove this, we assume that the claim is false and work towards a contradiction. This part is similar to problem 1.6

Problem 3. Best case would be when everyone gets their most preferred partner in the first attempt. All men choose a different woman as their highest preference.

Worst case takes $\Theta(n^2)$ iterations. One possible scenario involves n iterations for the first man to find a stable pairing, $n-1$ for the second man, $n-2$ for the next and so on. Such an example is shown below. Here men are competing for the same women and would take $\frac{1}{2}n(n+1)$ iterations($\Theta(n^2)$) to arrive at a stable matching.

$m_1:$	w_1	w_2	w_3	$w_1:$	m_3	m_2	m_1
$m_2:$	w_1	w_2	w_3	$w_2:$	m_3	m_2	m_1
$m_3:$	w_1	w_2	w_3	$w_3:$	m_3	m_2	m_1