MACM 101 (D200)
Homework WHW 6
Chapter 8 (Counting)
Due November 26, 2020; 12:30 pm

November 24, 2020

Exercises from the text

Exercises: 8.4.4; 8.5.2; 8.9.4; 8.10.2; 8.10.8

Other Problems

1. Using combinatorial arguments to show that

$$\binom{2m}{m} = \sum_{k=0}^{m} \binom{m}{k}^2$$

.

Hint: Consider we have m distinguished white balls and m distinguished black balls. We want to select a subset of m balls from a set of $2m$ balls. Also use the fact that $\binom{m}{j} = \binom{m}{m-j}$.

2. In how many ways can we place r red balls and w white balls in n boxes so that each box contains at least one ball of each color?

3. (a) How many sequences (lists) of m 0s and n 1s are there?

   (b) How many sequences are there in which each 1 is separated by at least two 0s? (Assume that for this part $m \geq 2(n-1)$.)

4. We are given a red box, a blue box and a green box. We are also given 10 red balls, 10 blue balls, and 10 green balls. Balls of the same colour are indistinguishable. Consider the following constraints:

1 No box contains a ball that has the same colour as the box.

2 No box is empty.

Determine the number of ways in which we can put 30 balls into boxes so that:

(a) No constraint has to be satisfied. Every combination is allowed.

(b) Constraint 1 is satisfied.

(c) Constraint 2 is satisfied.

(d) Constraints 1 and 2 are satisfied.

5. Five rooms of a house are to be painted in such a way that rooms with an interconnecting door have different colors. If there are n colors available, how many different color schemes are possible when the rooms in the house are arranged in the following way?

   (a) Connected rooms form a linear order with one door interconnecting two adjacent rooms.

   (b) Connected rooms form a linear order with one door interconnecting two adjacent rooms. The first and last rooms must be colored differently.

   (c) Connected rooms form a circular order with one door interconnecting two adjacent rooms.