MACM 101 (D200)
Homework WHW 5
(Chapter 7)
Inductions and recursions
Due November 10, 2020; 12:30 pm

November 2, 2020

Exercises from the text (To be handed in)

(1) 7.1.2 (c,d,e)

(2) 7.1.4 (a,b,c,d)

(3) 7.2.1 (b,c,g)

(4) 7.2.2 (a,b)

(5) 7.3.3 (b)

(6) 7.3.4 (b,d)

(7) 7.3.6 (b)

(8) 7.4.1 (g)

(9) 7.4.2 (b,d)

(10) 7.4.3 (e,f)

(11) 7.5.2 (b)

(12) 7.5.3 (c,d)

Other Problems (Not to be handed in).

1. Consider $n+2$ distinct point from the circumference of a circle.If consecutive points along the circle are joined by line segments creating a polygon with $n+2$ sides then the sum of interior angle of the resulting polygon equals $180n$ degree.

2. Suppose that a sequence a_n $(n = 0, 1, 2, \ldots)$ is defined recursively by $a_0 = 1$, $a_1 = 7$, $a_n = 4a_{n-1} - 4a_{n-2}$ $(n \geq 2)$. Prove by induction that $a_n = (5n+2)2^{n-1}$ for all $n \geq 0$.

3. Show that, for any positive integer n, n lines "in general position" (i.e. no two of them are parallel, no three of them pass through the same point) in the plane $\mathbb{R}^2$ divide the plane into exactly $\frac{n^2+n+2}{2}$ regions. (Hint: Use the fact that an nth line will cut all $n-1$ lines, and thereby create n new regions.)

4. Give a recursive definition of the sequence $\{a_n\}, n = 1, 2, 3, \ldots,$ if
   (a) $a_n = 4n$
   (b) $a_n = 4^n$
   (c) $a_n = 4$

5. Give a recursive definition for the set of all
   (a) positive even integers
   (b) positive odd integers
   (c) nonnegative even integers