MACM 101 (D200)
Homework WHW 4
Assignment on Functions and Relations
Due October 29 , 2020; 12:30 pm

October 25, 2020

Exercises from the text (To be handed in)

(1) 4.3.2 (b,e,f,g)

(2) 4.3.6 (b)

(3) 4.4.3 (b)

(4) 4.5.1 (a,b,c,d)

(5) 4.5.5 (c,d,f,g)

(6) 4.5.6 (b,c)

(7) 4.6.3 (c)

(8) 4.6.5 (b)

(9) 6.1.3 (b)

(10) 6.1.4 (a,e,f)

(11) 6.2.1 (b,c,d,g,i,j)

(12) 6.2.3 (c,d)

(13) 6.2.5 (b,c)

(14) 6.3.4 (b,d)

(15) 6.5.2 (a,b,c,d,e)

(16) 6.6.2

(17) 6.6.4

Other Problems (Not to be handed in).

1. Let $\mathbb{Q}$ be the set of rational numbers. Show that $f : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}$ where $f(\frac{a}{b}, \frac{c}{d}) = \frac{a+c}{b+d}$, $b \neq 0$ and $d \neq 0$, does not define a function.

2. We have seen that

$$\lfloor x \rfloor = max \; \{n | (n \in Z) \wedge (n \leq x)\}$$

$$\lceil x \rceil = min \; \{n | (n \in Z) \wedge (n \geq x)\}.$$

   (a) Prove carefully that $\forall x \in Z$,

      i. $\lfloor \frac{x}{2} \rfloor + \lfloor \frac{x+1}{2} \rfloor = \lfloor x \rfloor$.

      ii. $\lfloor \frac{x}{2} \rfloor \times \lfloor \frac{x+1}{2} \rfloor = \lfloor \frac{x^2}{4} \rfloor$

   (b) Are all the identities true $\forall x \in R$?

3. Explain why the function $f : R \to R$ defined by $f(x) = \sin x$ is not invertible. Explain in your arguments the reference to the well-known "inverse sine" function $\sin^{-1} x$ (or arcsin x) and to the fact that

$$\forall x (sin(sin^{-1} x) = x)$$

4. The set $S = \{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}$ can be represented with the structure of a poset in many ways. For example,

   - partial ordering (actually a total ordering) $\leq$ (defined on $S \times S$ by $x \leq y$)
   - partial ordering (actually a total ordering) $\geq$ (defined on $S \times S$ by $x \geq y$)
   - partial ordering $|$ (defined on $S \times S$ by $x|y$)

   Answer each of the following questions for each of the three partial orderings mentioned above.

(a) Find the maximal elements.

(b) Find the minimal elements.

(c) Is there a greatest element?

(d) Is there a least element?

(e) Find all upper bounds of $\{2, 9\}$

(f) Find the least upper bound of $\{2, 9\}$, if it exists.

(g) Find all lower bounds of $\{60, 72\}$.

(h) Find the greatest lower bound of $\{60, 72\}$, if it exists.

(i) Draw the Hasse diagram.

5. Define a relation R on $\mathbb{Z}$ by aRb if and only if $3a + b$ is a multiple of 4.

   (a) Prove that R defines an equivalence relation

   (b) Find the equivalent class [0].

   (c) Find the equivalent class [2].

   (d) What is the partition of $\mathbb{Z}$ by R?

6. Determine whether each of the following defines an equivalence relation R the set A.

   (a) **A** is the set of all circles in the plane; aRb if and only if a and b have the same center.

   (b) **A** is the set of all straight lines in the plane; aRb if and only if a and b are parallel.

   (c) **A** is the set of all straight lines in the plane; aRb if and only if a is perpendicular to b.