Exercises from the text

(A) 1.6.3, 1.6.4
(B) 1.7.1, 1.7.3, 1.7.7
(C) 1.8.3
(D) 1.9.3, 1.9.5
(E) 1.10.3, 1.10.7
(F) 1.11.3

Other problems

1. Write the contrapositive, converse, and inverse of the following statement.

   If you try hard, then you will win.
   (Inverse = converse of contrapositive.)

2. Suppose $P(x, y)$ is the statement $x + 2y = xy$, where $x$ and $y$ are integers. What are the truth values of
   
   (a) $P(1, -1)$
   (b) $\exists y P(3, y)$
3. Suppose the variable \( x \) represents students, \( y \) represents courses and \( T(x, y) \) means “\( x \) is taking \( y \)”. Match each of the following symbolic statements with all its equivalent English statements in the second list:

(a) \( \exists x \forall y T(x, y) \)
(b) \( \forall x \exists y T(x, y) \)
(c) \( \exists x \forall y \neg T(x, y) \)
(d) \( \exists y \forall x \neg T(x, y) \)
(e) \( \neg \exists y \forall x T(x, y) \)

The English statements are

(A) Every course is being taken by at least one student.
(B) Some student is taking every course.
(C) No student is taking all courses.
(D) There is a course that all students are taking.
(E) Every student is taking at least one course.
(F) There is a course that no students are taking.
(G) Some students are taking no courses.
(H) No course is being taken by all students.
(I) Some courses are being taken by no students.
(J) No student is taking any course.

4. Show that the following argument is invalid.

\[
p \to (q \lor r) \\
\neg r \\
\therefore \neg p
\]
5. Show that the following argument is valid.
   \[ p \lor q \]
   \[ q \rightarrow r \]
   \[ p \land s \rightarrow t \]
   \[ \neg r \]
   \[ \neg q \rightarrow u \land s \]
   \[ \therefore t \]

6. Determine whether the following argument is valid.
   Every cat is striped.
   No cat is friendly.
   \[ \therefore \text{nothing striped is friendly.} \]