

MACM 101 (D200)  
Homework WHW 2  
Due September 28, 2020; 12:30 pm

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**Exercises from the text. Solutions to these problems are already released.**

- (A) 1.6.3, 1.6.4
- (B) 1.7.1, 1.7.3, 1.7.7
- (C) 1.8.3
- (D) 1.9.3, 1.9.5
- (E) 1.10.3, 1.10.7
- (F) 1.11.3

**Other problems**

1. Write the contrapositive, converse, and inverse of the following statement.

If you try hard, then you will win.  
(Inverse = converse of contrapositive.)

**Solution**

Contrapositive: If you will not win, then you do not try hard.

Converse: If you will win, then you try hard.

Inverse: If you do not try hard, then you will not win.

2. Suppose  $P(x, y)$  is the statement  $x + 2y = xy$ , where  $x$  and  $y$  are integers. What are the truth values of
- (a)  $P(1, -1)$  True
  - (b)  $\exists y P(3, y)$  True
  - (c)  $\exists x \forall y P(x, y)$  False  $y = 2$
  - (d)  $\exists y \forall x P(x, y)$  False
3. Suppose the variable  $x$  represents students,  $y$  represents courses and  $T(x, y)$  means “ $x$  is taking  $y$ ”. Match each of the following symbolic statements with all its equivalent English statements in the second list:
- (a)  $\exists x \forall y T(x, y)$
  - (b)  $\forall x \exists y T(x, y)$
  - (c)  $\exists x \forall y \neg T(x, y)$
  - (d)  $\exists y \forall x \neg T(x, y)$
  - (e)  $\neg \exists y \forall x T(x, y)$

The English statements are

- (A) Every course is being taken by at least one student.
- (B) Some student is taking every course.
- (C) No student is taking all courses.
- (D) There is a course that all students are taking.
- (E) Every student is taking at least one course.
- (F) There is a course that no students are taking.
- (G) Some students are taking no courses.
- (H) No course is being taken by all students.
- (I) Some courses are being taken by no students.

(J) No student is taking any course.

**Solution:** aB bE cG dI dF eH

4. Show that the following argument is invalid.

$$p \rightarrow (q \vee r)$$

$$\neg r$$

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$$\therefore \neg p$$

**Solution:**  $p = T$ ,  $q = T$  and  $r = F$  make both hypothesis true but the conclusion false.

5. Show that the following argument is valid.

$$p \vee q$$

$$q \rightarrow r$$

$$p \wedge s \rightarrow t$$

$$\neg r$$

$$\neg q \rightarrow u \wedge s$$

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$$\therefore t$$

**Solution**

- (a)  $\neg q$  Modus Tollens, (ii), (iv)
- (b)  $p$  Disjoint Syllogism, (i), (a)
- (c)  $u \wedge s$  Modus Ponens, (a), (v)
- (d)  $s$  Simplification, (c)
- (e)  $p \wedge s$  Conjunction, (b), (d)
- (f)  $t$  Modus Ponens, (e), (iii)

6. Determine whether the following argument is valid.

Every cat is striped.

No cat is friendly.

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$\therefore$  nothing striped is friendly.

**Solution** If the universe contains striped animals other than cats, then it is invalid. May be zebras are friendly.