Homework 3
MACM 101-D1
October 4, 2019
Date due: October 11, 2019 in the Class.

1 Practice Problems (Not to be handed in)

1. Problems (pages 116-117) 4, 6, 8, 10, 13, 20.

2. Prove the following statements using either direct or contrapositive proof, which ever is easer.

   (a) If $a, b \in Z$ (set of all integers) and a and b have the same parity, then $3a + 7$ and $7b - 4$ do not.

   (b) If n is odd, then $8|(n^2 - 1)$.

   (c) If $x^5 - 4x^4 + 3x^3 - x^2 + 3x - 4 \geq 0$, then $x \geq 0$.

3. Prove the following statements using proof by contradiction method.

   (a) There exist no integers a and b for which 21a + 30b = 1.

   (b) Every non-zero rational number can be expressed as a product of two irrational numbers.

4. Write the following compound statements in symbols. Use the following letters to represent the statements:

c	:	It is cold.
d	:	It is dry.
r	:	It is rainy.
w	:	It is warm.

   (a) It is neither cold nor dry.

   (b) It is rainy if it is not dry.

   (c) To be warm it is necessary that it be dry.

   (d) It is cold or dry, but not both.

2 Homework Problems (To be handed in)

1. Suppose the variables x, y represent students and courses, respectively. $T(x, y)$ is an open statement "x is taking y". For each of the following symbolic statements state its equivalent English statements.
   (a) $\exists y \forall x T(x, y)$
   (b) $\neg \exists x \exists y T(x, y)$
   (c) $\forall y \exists x T(x, y)$
   (d) $\neg \forall x \exists y T(x, y)$
   (e) $\forall x \exists y \neg T(x, y)$

2. Give an example to show that

$$(\forall y)(\exists x)\ p(x, y) \leftrightarrow (\exists y)(\forall x)\ p(y, x)$$

3. Prove or disprove the following statements about integers whose domain is non-zero integers.
   (a) If $a|b$ and $c|d$, then $(a+b)|(c+d)$.
   (b) If $a|b$ and $b|c$, then $a|c$.
   (c) If $a|b$ and $b|c$, then $(a+b)|c$.
   (d) If $a|b$ and $b|c$, then $ab|c^2$.

4. Suppose n is an arbitrary integer.
   (a) Show that $n(n+1)$ is divisible by 2.
   (b) Show that $n(n+1)(n+2)$ is divisible by 3!.

5. (a) Prove that $\sqrt{7}$ is an irrational number.
   (b) Show where your arguments in (a) get violated if you want to show in a similar manner that $\sqrt{9}$ is an irrational number.
   (c) Find a counterexample to the statement that every positive integers can be written as the sum of the squares of three integers. What is the smallest integer for which it is a counterexample.