MACM 101-D1 (HW 2) September 23, 2019 Date due: October 2-3, 2019 in the Tutorial.

## 1 Practice Problems (Not to be handed in)

- 1. Problems (Exercises 1.4) 2, 8, 16, 18
- 2. Problems (Exercises 1.6) 2, 3, 4, 8, 10, 14, 20, 22
- 3. Problems (Exercises 2.1) 4, 5, 8, 12
- 4. Find the # of integral solutions to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 21$  subject to the conditions
  - (a)  $x_i \ge 0, i = 1, 2, 3, 4, 5, 6.$
  - (b)  $x_3 \ge 5, x_i \ge 1, i = 1, 2, 4, 5, 6.$
  - (c)  $0 \le x_1 \le 3, x_i \ge 1, i = 2, 3, 4, 5, 6.$

## 2 Homework Problems (To be handed in)

- 1. In how many ways can we place r red balls and w white balls in n boxes so that each box contains one ball of each color?
- 2. (a) How many sequences (lists) of m 0s and n 1s are there?
  - (b) How many sequences are there in which each 1 is separated by at least two 0s? (Assume that for this part  $m \ge 2(n-1)$ .)
- 3. We are given a red box, a blue box and a green box. We are also given 10 red balls, 10 blue balls, and 10 green balls. Balls of the same colour are indistinguishable. Consider the following constraints:

A: No box contains a ball that has the same colour as the box.

**B:** No box is empty.

Determine the number of ways in which we can put 30 balls into boxes so that:

- (a) No constraint has to be satisfied. Every combination is allowed.
- (b) Constraint A is satisfied.
- (c) Constraint B is satisfied.

- (d) Constraints A and B are satisfied.
- 4. Construct a truth table for each statement.
  - (a)  $p \to \neg q$
  - (b)  $[p \land (p \to q)] \to q$
  - (c)  $[p \to (q \land \neg q)] \leftrightarrow \neg p$