MACM 101 : Homework 4 (October 21, 2019)
Homework is due by Oct.30, 2019
Exercises on Sets and Probability.

The lecture slides file contains a few problems from the text. Please hand in the solutions to the following problems.

1. Write the following set by listing their elements between braces.

   (a) $\{x \in \mathbb{R} : x^2 = 7\}$
   (b) $\{x \in \mathbb{Z} : | 2x | < 5\}$
   (c) $\{x \in \mathbb{Z} : -2 < x \leq 7\}$
   (d) $\{x \in \mathbb{Z} : -5 < x \leq 2\}$

2. Write the following set in set-builder notation.

   (a) $\{-3, -2, -1, 0, 1, 2, 3\}$
   (b) $\{0, 1, 8, 27, 64, 125, \dots\}$
   (c) $\{0, -1, -4, -9, \dots\}$

3. Let $\mathbb{R}$ be the universal set. Let $A = \{1\}$, $B = (0,1) = \{x : 0 < x < 1\}$ and $C = [0,1] = \{x : 0 \leq x \leq 1]$. Write down the following sets.

   - $A \cup B$; $A \cap B$; $B \cap C$; $A \cup C$; $A \cap C$

   Are any of the pairs of sets A, B and C disjoint?

4. Let A, B, C, D be nonempty sets. Prove that$A \times B \subseteq C \times D$ if and only if $A \subseteq C$ and $B \subseteq D$.

5. Let A, B and C be three arbitrary subsets of the universal set U. Use an element containment proof (i.e. prove that the left side is a subset of the right side and the right side is a subset of the left side) to prove the following:

   - $\overline{A \cap B \cup C} = \overline{A} \cup \overline{B} \cap \overline{C}$.
   - $\overline{A \cup B \cap C} = \overline{A} \cap \overline{B} \cup \overline{C}$.

6. Use the membership table method to determine which membership $\subseteq, =, \supseteq$ is true for the following pair of sets.

   - $(B - C)$, $(B - A) - (C - A)$

7. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ by using the set builder notations.

8. Two fair six-sided dice are rolled and the sum s of the numbers coming up is recorded. What is the probability of $s \geq 10$? Show your work for the case when the dice are distinguished and when they are not.

9. A random experiment consists of rolling an unfair, six-sided die. The digit 6 is three times as likely to appear as the numbers 2 and 4. The numbers 2 and 4 are twice as likely to appear as one of the numbers, 1, 3, and 5.

   Assign appropriate probabilities to the six outcomes in the sample space.