MACM 101 DISCRETE MATHEMATICS I QUIZ 1 (VERSION 1) DATE JANUARY 19, 2018

Answer questions worth 20 points

- (1) (5 points) Which of the following questions has the solution 15!/5! ? Justify.
 (a) How many ways are there to distribute 10 identical objects into 6 distinct boxes? C(10+6-1,6-1)
 - (b) How many ways can 5 out of 15 people be arranged in a lineup? **P(15,5)**
 - (c) How many bit strings of length 10 begin with the sequence "01101"? 2^5
 - (d) How many ways can a committee of 5 people be formed with 15 people? C(15,10)

(e) How many ways are there to distribute 15 different books to 5 children? 5^{15} The answer is NONE.

- (2) (5 points) In a given competition, there are twenty one countries (including Canada and USA) competing for nine different medals. How many different ways are there to award these medals if
 - (a) there is no restriction? P(21,9)
 - (b) Canada and USA must get one of the top three spots? P(3,2)P(19,7)
- (3) (5 points) We are given 100 distinct objects. We want to arrange objects in a circle where arrangements are considered the same if one can be obtained from the other by rotation. Determine how many ways
 - (a) all these objects can be arranged in a circle. 99!
 - (b) 40 of these objects can be arranged in a circle. $C(100,40)^{*39!}$; or P(100,40)/40
- (4) (5 points) An apartment building has three floors and four families can live on each floor. There are twelve families who will occupy the building. In how many ways one can assign 12 families if
 - (a) there are no restrictions? 12!; or (if order is not important) C(12,4)C(8,4)C(4,4)
 - (b) two particular families (say, families X and Y) must be assigned to the first floor? P(4,2). 10!; or (no order) C(10,2)C(8,4)C(4,4)
 - (c) family X and family Y must be assigned to different floors? 12! P(3,1)*P(4,2).10!; or (no order) C(12,4)C(8,4)C(4,4) - C(3,2)C(10,2)C(8,4)C(4,4);
- (5) (5 points) Find the number of integral solutions to the equation $x_1 + x_2 + x_3 + x_4 + x$ $x_5 + x_6 = 20$ subject to the conditions

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- (a) $x_i \ge 1, i = 1, 2, 3, 4, 5, 6.$ Ans: C(14+6-1,6-1)
- (b) $x_1 \ge 5, x_i \ge 1, i = 2, 3, 4, 5, 6.$ Ans: C(10+6-1,6-1)
- (c) $0 \le x_1 \le 4, x_i \ge 1, i = 2, 3, 4, 5, 6.$ **Ans = number of integral solutions to** $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20, x_1 \ge 0, x_i \ge 1, i = 2, 3, 4, 5, 6$ minus the number of integral solutions to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20, x_1 \ge 5, x_i \ge 1, i = 2, 3, 4, 5, 6$ = C(15 + 6 - 1, 6 - 1) - C(10 + 6 - 1, 6 - 1).