MACM 101 : Tutorial (November 6-7, 2019)

1. Problem 8 (page 241), 14 (page 237)
   Solution to Problem 8 Look at the solution by following the link provided in the homepage for the even-numbered questions.
   Solution to Problem 14. You might see a problem like this in the quiz
   We compute GCD(33,29) as follows:
   $33 = 1 \times 29 + 4$
   $29 = 7 \times 4 + 1$
   $4 = 4 \times 1 + 0.$

   Therefore,

   $gcd(33,29) = 1 = 29 - 7 \times 4$; $i.e.$ $1 = 29 - 7 \times (33 - 1 \times 29) = 29 \times 8 + 33 \times (-7)$.

   Since $gcd(33,29)$ divides 2490, we can write $2940 = 29 \times (8 \times 2490) + 33 \times (-7 \times 2940)$. We want to find a and b, both nonnegative, such that $2940 = 29 \times a + 33 \times b$. We first write the above identity as

   $$2940 = 29 \times (8 \times 2940 - 33 \times t) + 33(-7 \times 2940 + 29 \times t)$$

   for any integer t. We note that $8 \times 2940 - 33 \times t \geq 0$ implies that $t \leq 712.7272...$, and $-7 \times 2940 + 29 \times t \geq 0$ when $t \geq 709.655....$ Thus when $t = 710, 711,$ *or* 712, both $a = 8 \times 2940 - 33 \times t \geq 0$ and $b = -7 \times 2940 + 29 \times t \geq 0$. Therefore, there are three ways to express $2490 = 33 \times a + 29 \times b$ where both a and b are non-negative.

2. Use the Euclidean algorithm - no other method will be acceptable - to determine integers m and n such that $gcd(341,527) = 341m + 527n$.

   Solution We apply the Eulidean algorithm:
   $527 = 1x341 + 186$
   $341 = 1x186 + 155$
   $186 = 1x155 + 31$
   $155 = 5x31 + 0$

   Hence
   $$gcd(341,527) = 31 = 186 - 155$$

$$31 = 186 - (341 - 1 \times 186)$$

$$31 = 2 \times 186 - 1 \times 341$$

$$31 = 2 \times (527 - 1 \times 341) - 1 \times 341$$

$$31 = 527(2) + 341(-3)$$

3. (a) **Inclass Test** Showing all your work, use the Euclidean algorithm - other method is acceptable here - to determine the greatest common divisor of the integers 243 and 198.

   (b) Using your computations above, determine two integers, x and y, such that gcd(243, 198) = 243x + 198y.