MACM 101 : Homework 5 (November 1, 2019)
Homework is due (in the tutorial class) Nov 13-14, 2019
Exercises on Properties of Integers (Chapter 4).

Practice Problems

1. Problems from the text: (page 219) 1,3, 10, 11
2. Problems from the text: (page 230) 7, 8, 28
3. Problems from the text: (page 236) 3, 10, 14, 15, 19

Homework Problems

1. Consider $n+2$ distinct point from the circumference of a circle.If consecutive points along the circle are joined by line segments creating a polygon with $n+2$ sides then the sum of interior angle of the resulting polygon equals $180n$ degree.

2. Suppose that a sequence a_n $(n = 0,1,2,\ldots)$ is defined recursively by $a_0 = 1$, $a_1 = 7$, $a_n = 4a_{n-1} - 4a_{n-2}$ $(n \geq 2)$. Prove by induction that $a_n = (5n+2)2^{n-1}$ for all $n \geq 0$.

3. Show that, for any positive integer n, n lines "in general position" (i.e. no two of them are parallel, no three of them pass through the same point) in the plane $\mathbb{R}^2$ divide the plane into exactly $\frac{n^2+n+2}{2}$ regions. (Hint: Use the fact that an nth line will cut all $n-1$ lines, and thereby create n new regions.)

4. Give a recursive definition of the sequence $\{a_n\}, n = 1,2,3,\ldots,$ if
   (a) $a_n = 4n$
   (b) $a_n = 4^n$
   (c) $a_n = 4$

5. Give a recursive definition for the set of all
   (a) positive even integers
   (b) positive odd integers
   (c) nonnegative even integers

6. Let $n \in \mathbb{Z}^+$ with $n = r_0 + r_1 \times 6^1 + r_2 \times 6^2 + \ldots + r_k \times 6^k$. Prove that
   (a) $2|n$ if and only if $2|r_0$.
   (b) $4|n$ if and only if $4|(r_0 + r_1 \times 6)$.

(c) $8|n$ if and only if $8|(r_0 + r_1 \times 6 + r_2 \times 6^2)$

7. (a) Determine the prime factorization of 374544.

   (b) Determine the number divisors of 374544 of types $a^i, i = 1,2,3,4$ where a is an integer.

8. (a) Use Euclidean algorithm to determine the greatest common divisor of the integers 243 and 198.

   (b) Use your computations above, determine two integers, x and y, such that $gcd(243, 198) = 243x + 198y$.

   (c) Determine a value of c such that $c = 243a + 198b$ where $a, b, c \in \mathbb{Z}^+$.

9. Determine the value of $c \in \mathbb{Z}^+$, $30 < c < 39$, for which the Diophantine equation $243a + 198b = c$ has no solution. Determine the solutions of the remaining values of c.