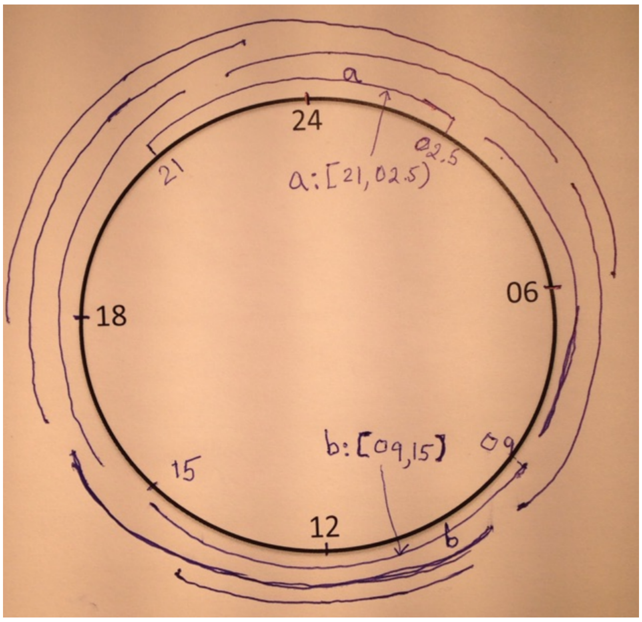
CMPT 307 Homework 5
November 2, 2019
Due date: November 13, 2019

Practice Problems from the text

1. **Chapter 4:** 4.7, 4.9, 4.10, 4.13, 4.14, 4.17, 4.20
2. **Chapter 5:** 5.5, 5.6, 5.7, 5.8, 5.11, 5.12, 5.13, 5.23

Homework problems

1. Consider a set S of activities in a 24 hour day schedule (see the figure) where the ith activity a_i has starting time s_i and finish time f_i. An activity a_j contains an activity a_i if the interval $[s_j, f_j)$ contains the interval $[s_i, f_i)$. An activity a_i is called a proper activity if it does not contain another activity. Two activities a_i and a_j are compatible if $[s_i, f_i)$ and $[s_j, f_j)$ do not overlap. The problem we are solving is to select a maximum size subset (A^*) of mutually compatible activities.



(a) Show that all the activities of A^* are proper activities.

(b) Design an algorithm to determine all the proper activities of S. Let S' denote the set of all proper activities of S.

(c) Design a greedy algorithm to select a maximum size subset of mutually compatible activities that contains a proper activity x. Let $A^*(x)$ denote this set.

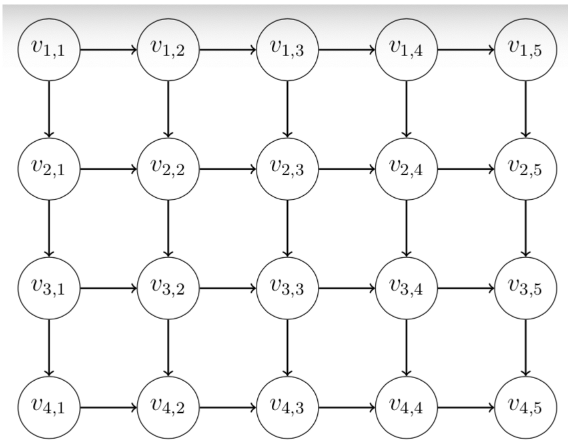
(d) It is possible for two proper intervals a and b realizing $A(a)$ and $A(b)$ such that $|A^*(a)| \neq |A^*(b)|$. (see the figure)

(e) Show the size difference of $A^*(a)$ and $A^*(b)$ is at most one.

(f) Design an algorithm to compute A^*, given S.

2. We are given a directed graph $G = (V, E)$ where the vertices are weighted (could be positive or negative) and the arcs are all weighted (non-negative). The cost of a path (cycle) is the sum of the weights of the edges and the vertices of the path (cycle). We are interested in determining if there is a negative cost cycle in G. Describe an algorithm with a complete analysis to perform such a task.

3. Consider a grid graph $G = (V, E)$. A 4×5 grid graph is shown below. There is a directed edge from vi, j to $v_{i,j+1}$ for $1 \le i \le n$ and $1 \le j \le m - 1$. Similarly there is a directed edge from vi, j to $v_{i+1,j}$ for $1 \le i \le n - 1$ and $1 \le j \le m$.



We are interested in computing the shortest path distance from $v_{1,1}$ to $v_{n,m}$ where the cost of edge e is c_e, possibly negative. Describe the most efficient algorithm to compute the shortest path. Provide a clear analysis of the algorithm.

4. The following questions are on minimum spanning tree.

(a) Suppose we have an undirected graph with weights that can be either positive or negative. Do Prim's and Kruskal's algorithim produce a MST for such a graph? Explain.

(b) Prove that for any weighted undirected graph such that the weights are distinct (no two edges have the same weight), the minimal spanning tree is unique.

5. We are given a directed graph $G = (V, E)$ with non-negative edge weights. Design an $O(|V| \log |V| + |E| \log |V|)$ algorithms to compute the shortest from s to t where one of the edge cost in the shortest path is allowed to be ignored.

6. Problem 5.9 of the text.