CMPT 307 : Quiz 4 (For practice only)
November 28, 2019

1. A natural sequence alignment problem is the following. Given a list $a = (a_1, a_2, \ldots, a_n)$ of n integer elements and a list $b = (b_1, b_2, \ldots b_m)$ of m integer elements, find the longest aligned subsequence, i.e. $((i_1, j_1), (i_2, j_2), \ldots, (i_k, j_k))$ such that $a_{i_l} = b_{j_l}, i_l < i_{l+1}$, and $j_l < j_{l+1}$ for all l. For example, suppose $a = (16, 17, 18, 11, 19, 12)$ and $b = (17, 11, 12, 16, 29)$. The longest aligned subsequence is $((2,1),(4,2),(6,3))$. This implies that the largest subsequence is $(17, 11, 12)$. Design a DP solution by following the following steps.

   - Determine the subproblems of the above problem.
   - Describe the memoized version of the recurrence formula to solve a subproblem. Analyze the running time.
   - How do you retrieve the optimal solution?

2. **Partition Problem** Suppose that m readers are given the assignment of reading through a shelf of books. For a fair assignment of responsibility, the books are partitioned among the m readers. The books on the shelf are not allowed to be rearranged. The objective is to partition the shelf into m sections and assign each section to one worker thereby minimizing the maximum sum of pages of of the readers. Suppose $m = 3$ and shelf has 9 books. The books are ordered left to right. The ith book has $i \times 100$ pages. The optimal partition is

   $$100 \;\; 200 \;\; 300 \;\; 400 \;\; 500 \mid 600 \;\; 700 \mid 800 \; 900$$

   where the largest workload of a reader is 1700 pages. This is the best possible.

   In general, we have the following partition problem:
   Input: An arrangement S of nonnnegative integers $< s_1, s_2, \ldots, s_n >$ and an integer m.
   Output: Partition S to m ranges, to minimize the maximum sum over all the ranges without rearranging the the numbers.

When $m = 1$, the problem is easy.

Design algorithms to solve the following variants of the partition problem.

1. Solve the partition problem when $m = 2$ by formulating it as a DP problem.
2. Generalize the previous result when $m = 3$ or more.