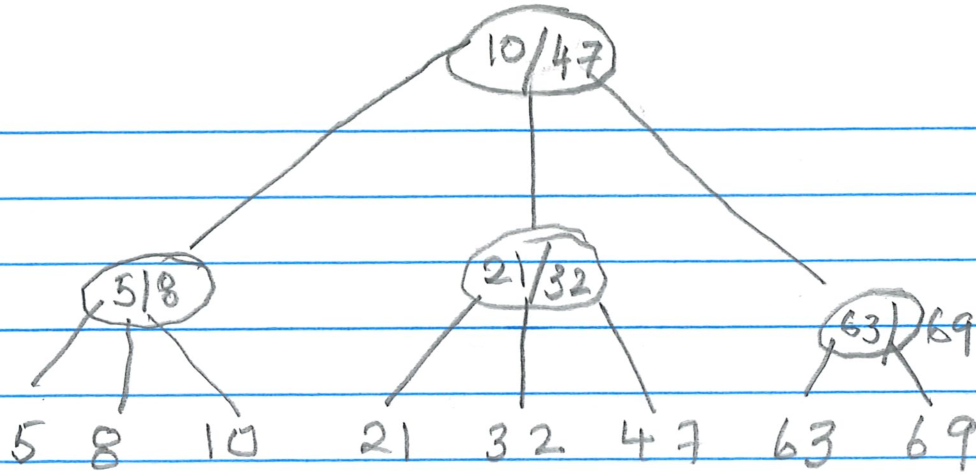
2-3 Trees

- leaf nodes contain data item & all leaves are at the same level.
- each ~~internal~~ nonleaf node
  - has 2 or 3 children
  - two search values
    - largest item in left subtree
    - largest item in middle subtree
- Smallest 2-3 tree
  - only one leaf node.
- balanced & ordered.



height of tree

- n data items
- all nodes have two children
  - height $\le \lfloor \log_2(n+1) \rfloor - 1$
- all nodes have three children
  - height $= \lfloor \log_3(2n+1) \rfloor - 1$

Time - For a general tree, O(height)

$$\log_3(2n+1)-1 \le \text{height} \le \lfloor \log_2(n+1) \rfloor - 1$$

Time to search 1 key: O(height)

Time to list the elements in sorted order : O(n).

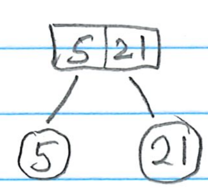
Insertions in 2-3 trees

- insert new leaf in the appropriate place.
- repeat until all nonleaf nodes have 2 or 3 children
  - if there is a node with 4 children, split the parent into two parent nodes with 2 children each
  - if the root node is split, add a new root
- Adjust the search values along the insertion paths.

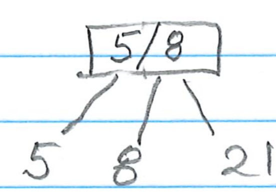
Example

Insert 5

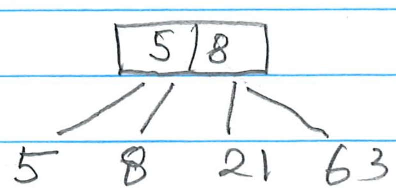
Insert 21

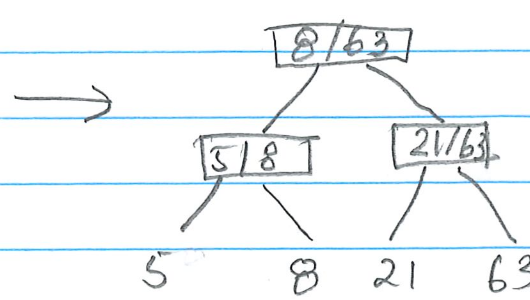


Insert 8

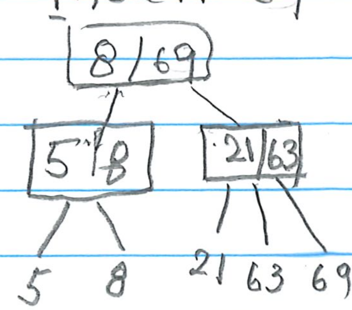


Insert 63

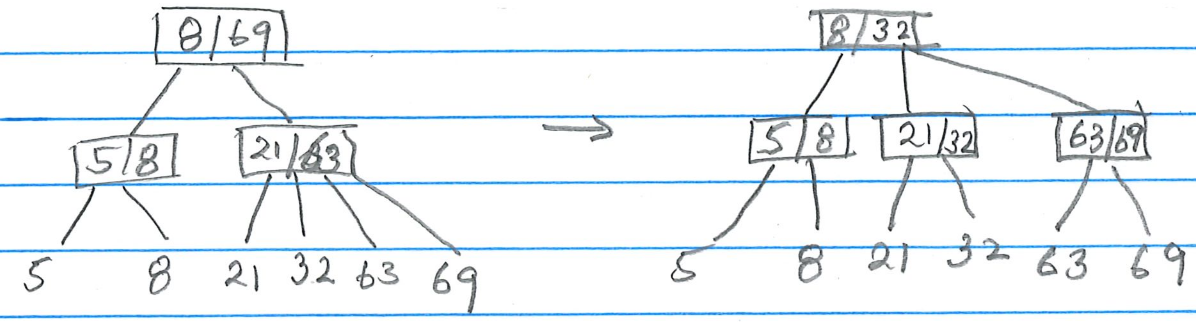




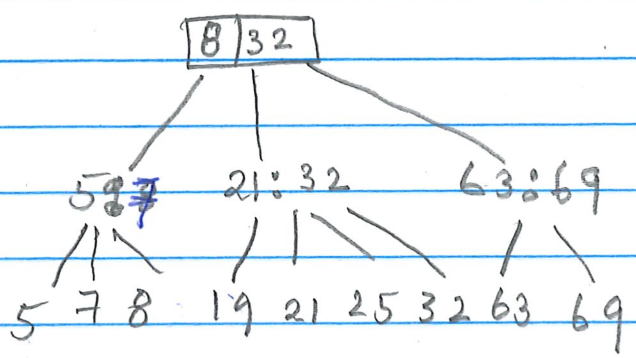
Insert 69

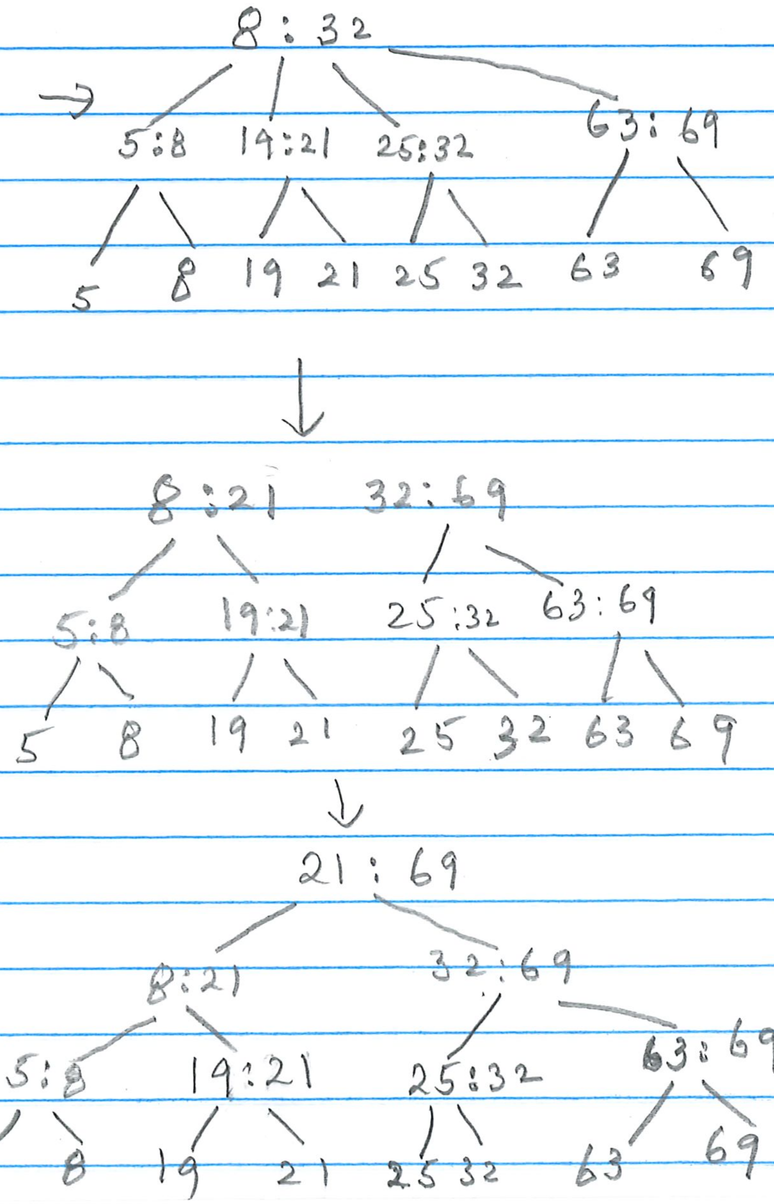


Insert 32



Insert 7, 19, 25



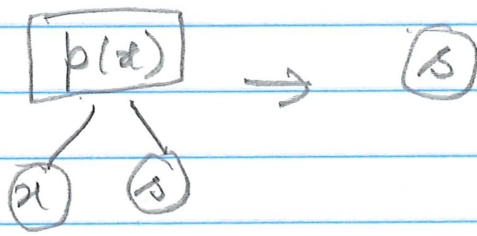


Time to insert
is O(height)

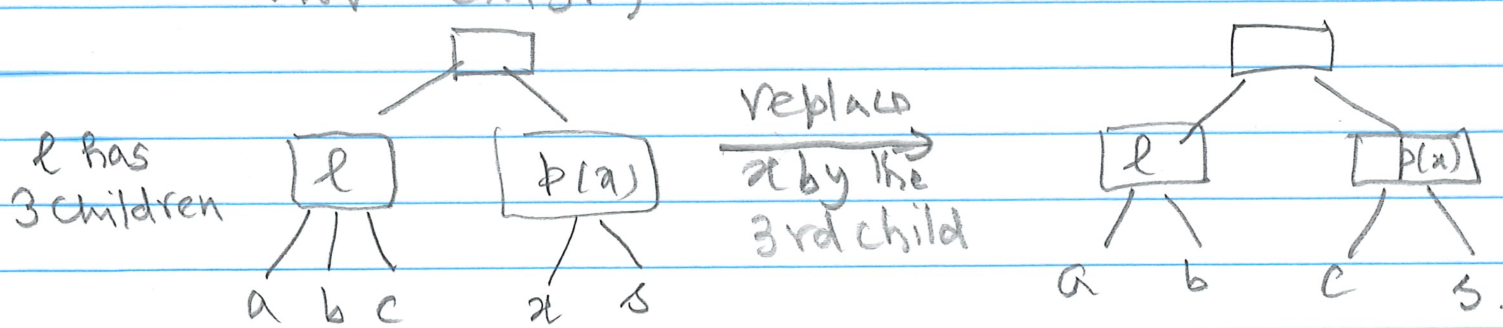
Deletions in 2-3 trees

Delete x from the tree; $p(x)$: parent of x.

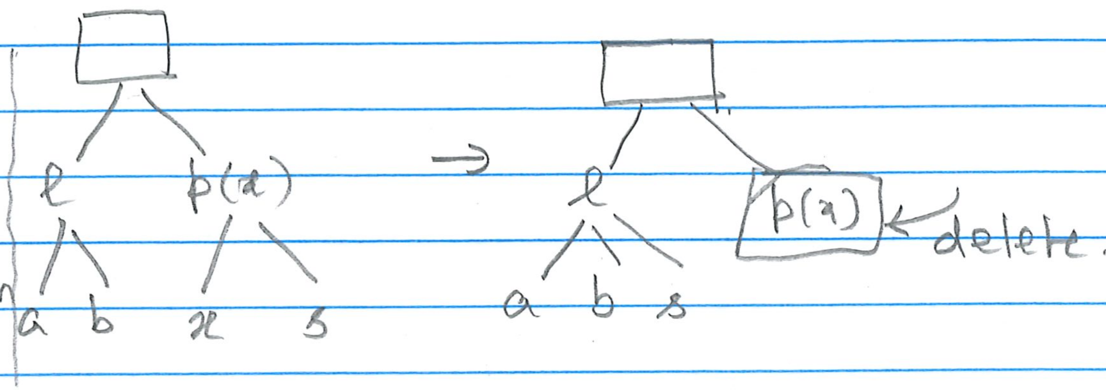
- if x is the root: delete x
- if $p(x)$ has 3 children, delete x.
- if $p(x)$ has 2 children (one is x & the other child is s)
  - if $p(x)$ is the root



- $p(x)$ is not the root node. Let ℓ be the left sibling of $p(x)$ & r be the right sibling of $p(x)$; (note ℓ or r may not exist)

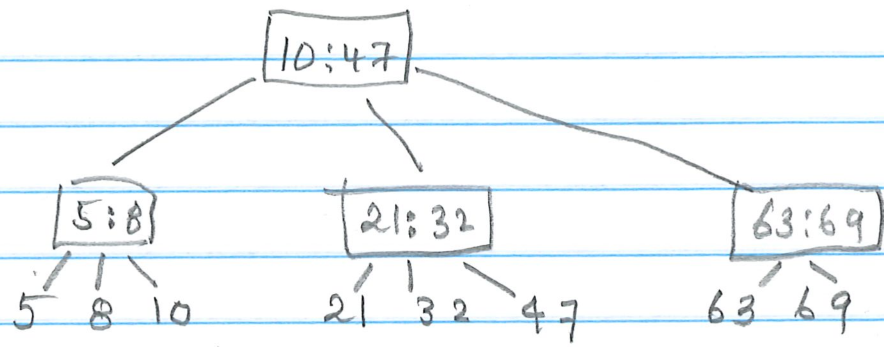


l has two children

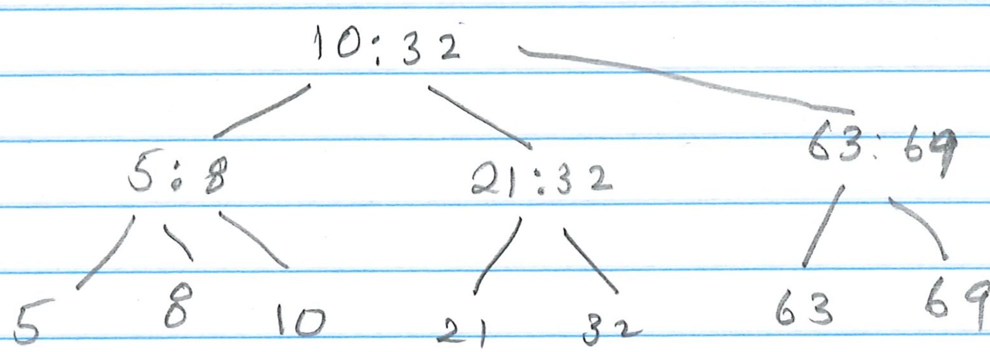


steps:
- remove x
- Combine p(x) with l
- make s a child of l
- rename p(x) to x
- recursively remove x

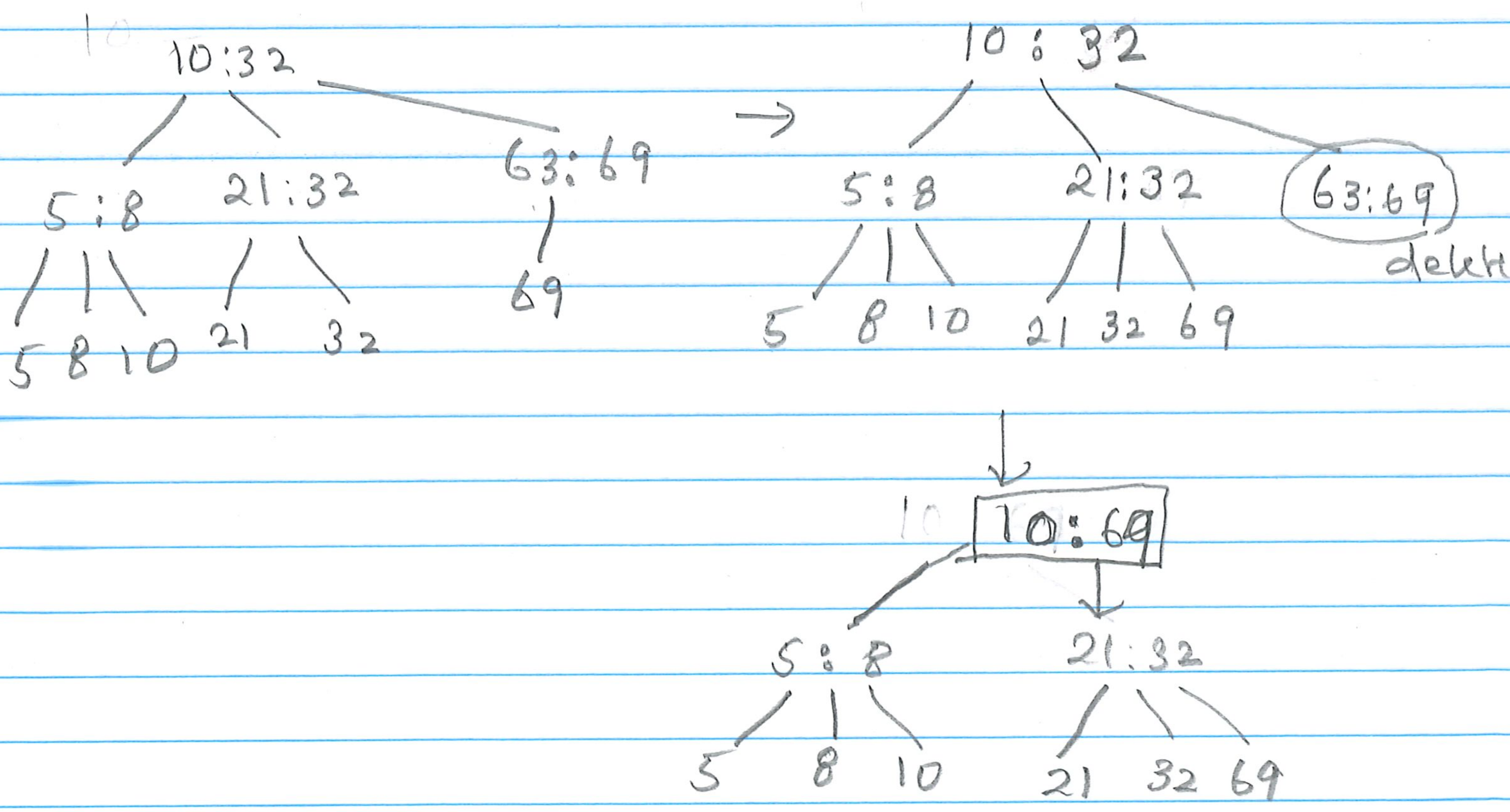
Example



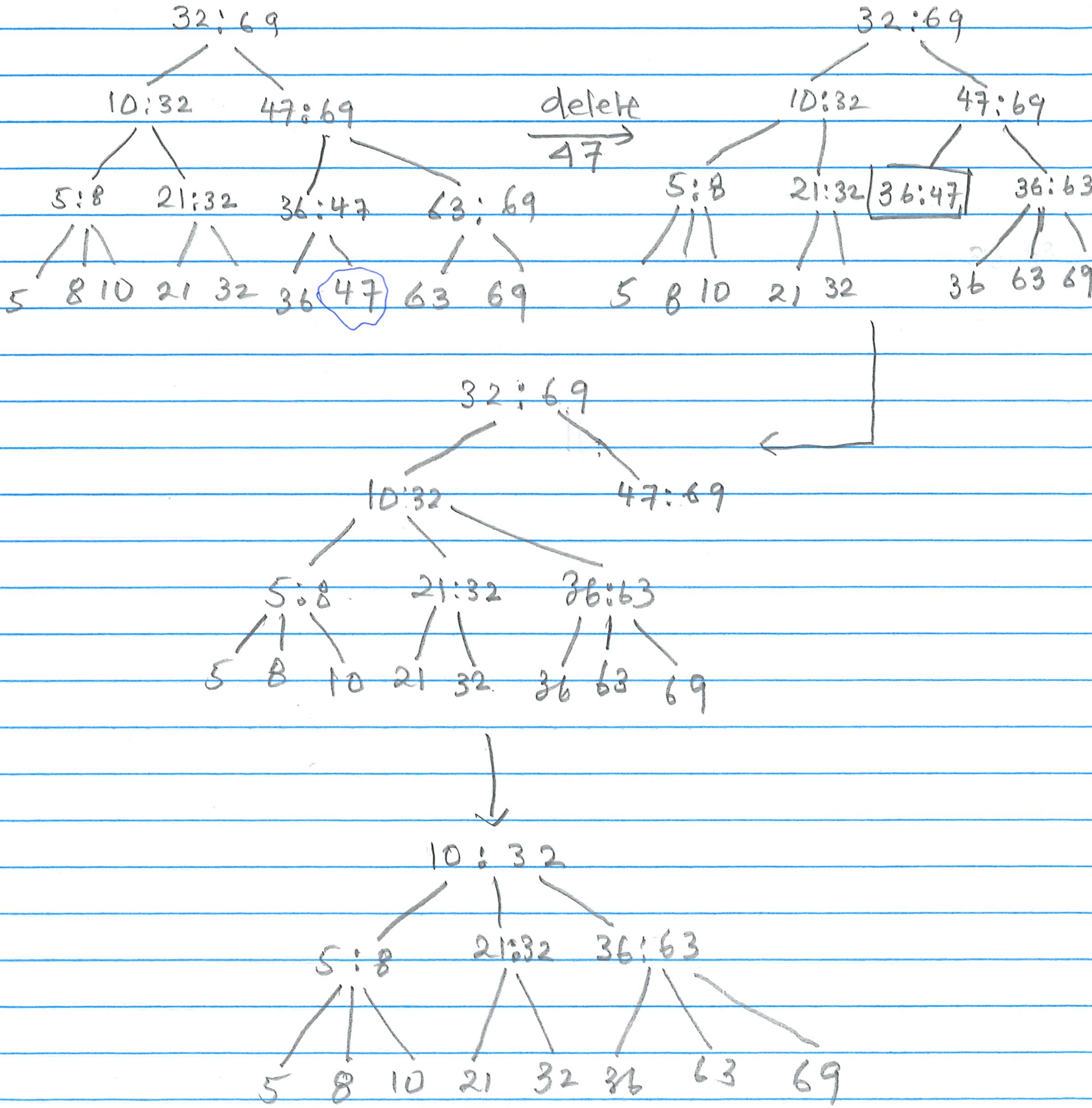
delete 47



delete 63



Example



32:69
10:32
47:69
5:8
21:32
36:47
63:69
5 8 10 21 32 36 47 63 69
delete 47
32:69
10:32
47:69
5:8
21:32
36:47
36:63
5 8 10 21 32
36 63 69
32:69
10:32
47:69
5:8
21:32
36:63
5 8 10 21 32 36 63 69
10:32
5:8
21:32
36:63
5 8 10 21 32 36 63 69